FRACTURE OF HETEROGENEOUS TWO-DIMENSIONAL LATTICES UNDER THE DIFFERENT REGIMES OF LOADING

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ABSTRACT

The fracture of heterogeneous materials under the different modes of loading (i.e., steady-state, linear and cyclic modes) is studied within the simple deterministic approach. The material is simulated by a 2-D inhomogeneous square lattice. Characteristics of the fracturing process with variation of inhomogeneity parameter dG and of deformation conditions are investigated. The material durability and fracturing degree as the functions of the parameters of simulation are discussed. The fractal dimensionality of the system is shown to be practically independent of the changes in any of conditions and equal to Df=1.10±0.04. The fracture percolation clusters being formed are of highly anisotropic form and the parameter of their anisotropy (i.e., the value of cluster width to length ratio) increases with dG and lies within δ=0-0.18(±0.10).

KEYWORDS

Crack, durability, fractal dimensionality, fracture, percolation

INTRODUCTION

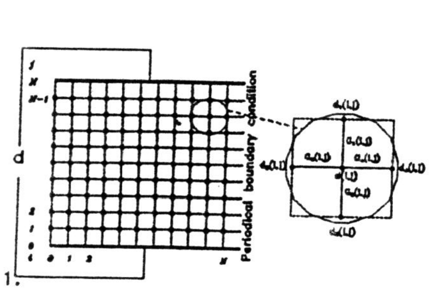
The growth of the disordered dendrite structures with fractal properties can be observed in the processes of different nature such as the flowing of fluids through porous medium, the aggregation , the formation of polymer gels, the dielectric breakdown, the material failure under the outer stress, etc. Different characteristics of structures with fractal geometry are widely discussed in literature (see Mandelbrot, 1982; Meakin, 1988a, 1989a, for example). The nonequilibrium processes of heterogeneous materials' fracture at the different modes of variation of the outer stress are of particular interest (Regel et al., 1972; Poirier, 1985).

In recent years, some different computer models were developed for the investigation of the material failure (Dobrodumov & El'jashevich, 1973; Herrmann et al., 1989). All these models were based on approximation that the cracking is a nonlocal process controlled by a Navier equation. The simple stochastic models of material failure and peculiarities of cracking processes with different rules of growth were discussed by Meakin (1988b, 1989b). Also, the computer model of the surface film cracking processes was studied (Meakin, 1988c). The deterministic model of crack growth was developed by Takayasu (1985). In this model the small initial fluctuation increases according to the deterministic laws, which results in the growth of the random fractal configurations. Recently, in the frame of such approximation the diagrams of material failure were obtained and the kinetics of crack growth under the steady deformation was thoroughly studied (Lebovka et al., 1990; Lebovka & Mank, 1992).

Here we present the results of computer simulation of the processes of heterogeneous material failure at different modes of deformation variation.

COMPUTER MODEL

We used calculation algorithm proposed by Takayasu (1985). Disordered media is represented by two-dimensional square lattice of NxN nodes. The lattice is periodic in the i-th direction (Fig. 1). All neighboring nodes are connected by brittle sticks. The nodes belonging to the lower rows are kept fixed and all of the upper rows are displaced by a value d.



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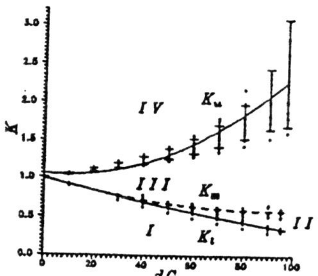

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Fig. 1. Idealized model of a heterogeneous material. The lattice is periodic in the horizontal direction.

Fig. 2. Phase diagram of the brittle fracture for inhomogeneous 2D square lattice. The different points correspond to different initial configurations and solid lines corresponds to the standard configuration. The bars denote standard deviations. I - no fracture; II - multiple fracture without percolation cluster formation; III - fracture with percolation cluster formation; IV - complete fracture.

Rigidity modula of sticks are distributed randomly in the range of $G = 1 \pm dG/100$, where dG(%) is the degree of lattice inhomogeneity. The breaking condition for single stick is defined by relation $\Delta d_K(i,j) > d_c$, where $\Delta d_K(i,j)$ is the absolute value of the stick deformation, d_c-is the critical deformation. It is assumed that after the stick failure its rigidity modulus decreases gradually $G \to \epsilon G$, where $\epsilon \ll 1$ ($\epsilon = 0.01$). We define the deformation factor as

$$K = d/(N * d_c) \qquad (1)$$

Different initial configurations can be choosen by introducing the different initializing numbers N_s of a randomizer. The main results of this work were obtained for large enough lattices of 50x50 and the same N_s. It is shown in (Lebovka & Mank, 1992), that for lattices of such a size the dimensional effects are practically absent. The kinetics of the process of a deformational failure was carried out by an iterative solving of the equations, defining the equilibrium of forces in each node. To improve the convergence of the iterative procedure, the relaxation method with the empirical choice of proper coefficient was used(Anderson et al., 1984). The time of dilatone formation in Zurkov's(1983) model can serve as a physical analogue of the time step we used in our model.

The outer deformation was changing with time according to different laws:

$$K = K_o \qquad \text{(const)} \qquad (2\text{-}1)$$

$$K = K_o + \Delta K * \sin(2\pi(t-1)/t_p) \qquad \text{(cyclic)} \qquad (2\text{-}2)$$

where K_o-is the mean deformation factor value, ΔK is an amplitude of K variations and t_p is a period. The choice of K_o and ΔK values, defining the mean deformation factor and of the amplitude of K variations, was done with the help of fracturing diagram for a given system(see fig. 2).

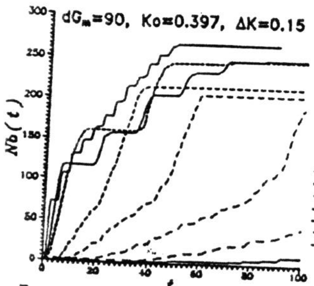
To initialize the failure process, the choice of Ko>Kl is necessary, and to observe the formation of the percolation clusters the high enough amplitude of deformation factor variations $\Delta K > K_m - K_o$ must be used.

RESULTS AND DISCUSSION

<u>The Kinetics of Failure and Durability at Different Modes of Loading.</u> The number of broken elements versus time dependencies Nb(t) under the cyclic mode of deformation variation are presented in fig. 3. Here, the periodical changes reflecting the sinusoidal character of deformation variations are obvious in all the cases, besides, the number of broken elements increases mainly during the first periods of failure.

The important characteristic of the failure process is the time defining the system durability. In the case of steady state or linear variations of deformation, the durability analysis, based on the extended exponential distribution law, can be used (Lebovka et al., 1990, 1991). In order to characterize the system durability in the case of cyclic mode of deformation variations, we introduce the time t_d, which is necessary for a system to achieve a state when 90% of the total number of elements of the system become broken after the process ends ($Nb(t=t_d) = .9 * N_{oo}$).The t_d versus t_p curves at different values of amplitude ΔK are presented in fig. 4.

At small values of amplitude, the td versus tp dependencies have a complex character: first the sudden rise of t_d (at $t_p<100$) is observed, then it decreases (at $100<t_p<250$) and grows moderately(at $t_p>250$). At high enough values of amplitude, the moderate increase of t_d is observed. It was found (Lebovka et. al., 1991) that in the case of unsteady changes of outer deformation the passive and active (the duration time of Δt =10- 20 steps) phases of fracture may be distinguished and the durability of the inhomogeneous system is controlled by the rate of a deformation increase. Taking this into account in the case when $t_d<t_p/4$



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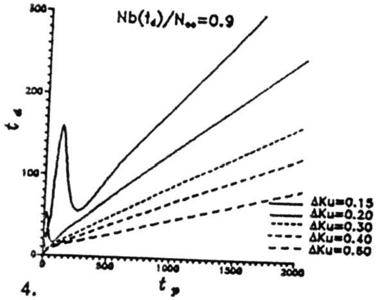

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Fig. 3. Number of destroyed elements Nb versus time t, with dG=90 and K_o=.397

Fig. 4. The system destruction time t_d (at $Nb=.9*N_{oo}$) as a function of t_p, with dG=90.

(i.e., when the process developes during the first quarter of period) we can evaluate the system durability with the help of linear relation

$$t_d = t_p * \lambda/4 + \Delta t \,, \tag{3}$$

where $\lambda = 2*\arcsin(((K_m-K_o)/\Delta K)\ (0<\lambda<1)$.

At low values of amplitude($\Delta K=K_m-K_o$) the durability increases significantly in the range of high frequencies(at small t_p). For such conditions the fracture takes place mostly in the periods when $K>K_m$. In this case

$$t_d = t_p * \Delta t. \tag{4}$$

Thus, the transition between the regions of high (Δt) and low ($\lambda/4$) angular coefficients must be noticeable on $t_d(t_p)$ curves, which is in agreement with data presented in fig. 4.

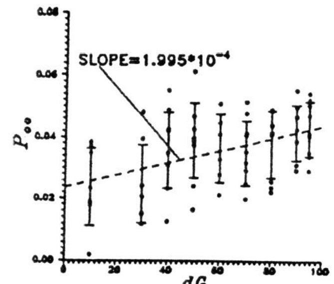
The Degree of System Failure at Different Modes of Loading. The degree of system failure is defined as a ratio of the number of broken elements to the net number of elements (N_t) $P(t)=Nb(t)/N_t$. Fig. 5 shows the failure degree at the finite stage of evolution of P_{oo} as a function of the inhomogeneity parameter dG in the steady mode of deformation. Note that with decreasing of inhomogeneity degree, the value of Poo in the percolation point also increases.

Fig. 6. shows the failure degree P_{oo} as a function of a period t_p for the cyclic mode of external deformation. Failure degree decreases quickly with t_p increase. The $P(t_p)$ dependencies can be approximated by the relation

$$P_{oo} = a t_p^{\,b} \tag{5}$$

where a and b are adjustable parameters. The dashed curve in Fig. 6 represents the corresponding least-mean-square fit to this relation and the insert depict the variations of a and b coefficient versus amplitude ΔK.

Note that the value of a corresponds to the failure degree in the "high frequency" region (t_p=1), and thus, we can conclude that at given conditions the failure degree increases



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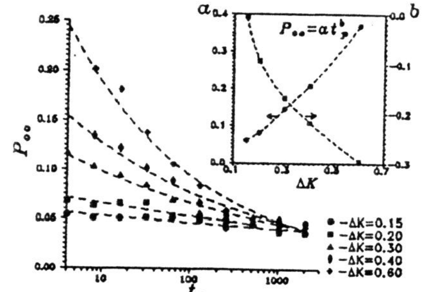

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Fig. 5. The plot of Poo versus dG with $K:K_m$ at the steady mode of external deformation. The different points correspond to different initial configurations. The line represents the corresponding least-mean-square fit. The bars denote standard deviations.

Fig. 6. The plot of P_{oo} versus t_p for cyclic mode of external deformation. The insert depicts the plot of a and b parameters in Eq. 5 versus ΔK.

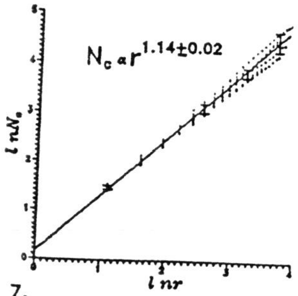
with increasing of ΔK. We can understand such behavior if we take into account, that at large amplitudes and at low frequencies the great number of individually broken elements appear in the initial moments of time and these elements define mostly the final cracking pattern at large time intervals. In the "low frequency" region (t_p>>1) the failure degree practically doesn't depend on the deformation amplitude. In this case, the values of tp and ΔK define only the time of the active phase appearance and have no any noticeable effect on the geometry and cracking patterns of growing clusters.

Fractal Properties of Percolation Clusters. For multiple labeling of broken elements clusters the modified algorithm for the bond percolation problem was used (Hoshen & Kopelman, 1976). The Hausdorff-Besicovitch fractal dimensionality D_f was estimated with the help of "box with sand" method (Nittman et. al., 1988). In this method, we calculate the number of cluster elements N_c in a square of rxr dimension and this number is averaged over all the elements of cluster. For the fractal cluster the function $N_c(r)$ is given by

$$N_c \propto r^{Df} \tag{6}$$

and from double logarithmic $\ln(N_c)$ vs. $\ln(r)$ relation(Fig.7) we can estimate the value of D_f. The calculations of D_f were made only for a maximal cluster of bound elements.

The data presented in this figure prove the automodel character of failure percolation cluster configurations with mean fractal dimensionality of D_f=1.14±0.02 for this series of data. Fig.8 shows the D_f vs. inhomogeneity parameter dG dependence. The values of D_f are rather insensitive to the conditions of deformation variation



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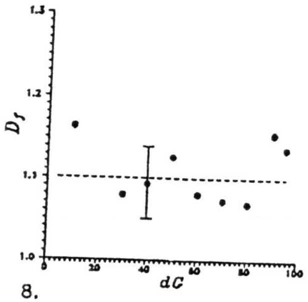

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Fig. 7. Plot of $\ln(N_c)$ versus $\ln(r)$ for percolation clusters, with dG=90, $K=K_m$. The different points correspond to different initial configurations. The straight line is obtained from a linear fit to data. The bars denote standard deviations.

Fig. 8. The fractal dimensionality of the percolation clusters D_f as a function of dG, with $K=K_m$. The bars denote standard deviations.

(K_0, ΔK, t_p). We have obtained the following averaged on different initial configuration and different loading conditions value of $D_f=1.10\pm0.04$.

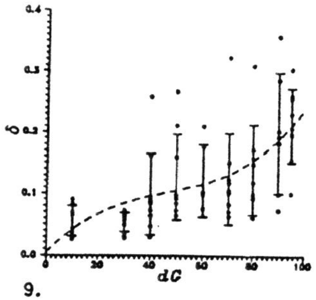
Yet, we must note that according to Takaysu estimations (1985,1988) the percolation cluster fractal dimensionalities for the same task is $D_f=1.65\pm0.05$. This value is very close to $D_f=1.7$ for the diffusion-limited-aggregation (DLA) model, but differs significantly from the value obtained in this work. Such an inconsistency may be a result of difference in approaches used for evaluation of fractal dimensionality. In the works of Takayasu (1985,1988) the D_f value was estimated from the dependence of the number of percolation cluster broken elements versus the size of lattice (within the range of $5<N<50$). But in this case the effects connected with the limited sizes of system under investigation weren't taken into account and therefore this method of D_f evaluation can give the wrong results.

For the square lattice Meakin (1988) found the small enough value for the fractal dimensionality of cracks $D_f=1.27\pm0.02$. In this work the D_f value was estimated by measuring the number of linear elements needed to outline the cluster perimeter (so called, the fractal dimensionality of growth perimeter). In general, D_f may depend on the detailes of lattice geometry, on the mechanism of breakage growth and also on the character of load applying(i.e., whether it is shear strain, uniaxial extension, etc.).

Meakin et.al.(1989) have noted that the value of D_f for 2D cracking pattern can vary from 1.22 to 1.95, depending on the given conditions. The anisotropic fractals observed in that case are rather self-affine than self-similar (Mandelbrot, 1982) and they have different scaling properties in the i-th and j-th directions. We think the cracking cluster growth is rather defined by the rules of the anisotropic bonding of new

elements, and the degree of their geometry anisotropy is an important characteristic for such cluster.

Clusters Geometry Anisotropy. The cluster anisotropy parameter was defined as ratio of cluster root mean square thickness in the transverse and longitudinal directions to the tension applied $\delta=\xi_y/\xi_x$. The value of cluster anisotropy parameter δ appears to be more sensitive characteristic to differences in the conditions of rapture than the value of D_f and it depends markedly on K_0, ΔK, t_p and dG. Fig 9 shows δ as a function of dG for percolation clusters at different initial configurations. For highly inhomogeneous materials increasing of value δ with dG correlates with the increase of cluster branching degree.



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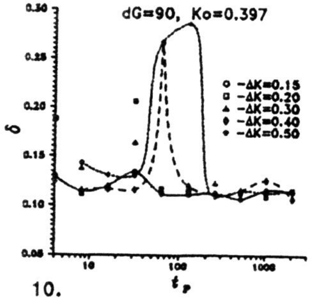

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Fig. 9. Parameter of anisotropy δ for maximal cluster as a function of dG, with $K=K_m$. The different points correspond to different initial configurations. The bars denote standard deviations.

Fig. 10. Parameter of anisotropy δ for a percolative cluster as a function of tp, with $K=.397$ ($K_0=K_l$) and dG=90.

The time dependencies $\delta(t)$ for maximal cluster in the cyclic mode of deformation variation are also remarkable. In the initial moments of time (within the induction period) only individual linear elements are being formed, and in this case $\delta=0$. The formation of leading cluster is characterized by the appearance of relatively compact weakly anisotropic failure structures with high value of δ. During the evolution of fracture δ decreases with time. The d versus tp dependence for percolation cluster is somewhat unexpected (Fig. 10). At low enough value of $\Delta K=0.15$ the curves of $\delta(t_p)$ pass through the maximum corresponding to the formation of weakly anisotropic and highly branched percolation clusters.

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