FRACTURE AND EQUILIBRIUM DAMAGE ACCUMULATION ON POSTCRITICAL DEFORMATION STAGE

A.A. TASHKINOV, V.E. WILDEMAN and A.N. ANOSHKIN
Perm Politechnical Institute,
29-a Komsomol Av., Perm, 614000, Russia

ABSTRACT

The object of the present paper is to consider the mathematical modeling of postcritical deformation processes. They correspond to the descending branch of stress-strain diagram and are connected with the processes of structural fracture and crack generation. The stability criteria of these processes were obtained and boundary conditions with finite stiffness of loading system were formulated. Postcritical deformation conditions for composites were investigated. The equilibrium of damage accumulation being satisfied for damage zones, there is a possibility to involve strength reserves and increase the vitality of structures. This is illustrated by calculation results.

KEYWORDS

Structural fracture, damage accumulation, postcritical deformation, descending branch of stress-strain diagram, critical state criterion, stiffness of loading system, composite materials.

STRUCTURAL FRACTURE AND DESCENDING BRANCH OF THE DIAGRAM

The top point on the stress-strain diagram corresponds to the critical state. At the same time a failure of material results from the lack of damage accumulation stability on postcritical deformation stage. In this case the failure means the final brief nonequilibrium stage of the process connected with avalanche defect growth. The resistance to the failure on the postcritical deformation stage depends on the loading system stiffness. Hence the deformation diagram doesn't break at the top point but the descending branch takes place. Every point on the descending branch may correspond to the moment of failure which depends on the loading conditions. The realization of the postcritical deformation stage leads to the utilization of the carrying capacity reserves. This can be applied at optimal designing of special materials and structures. It is the structural fracture that is the cause of existence of the descending branch on the deformation diagram of heterogeneous media. Fig. 1, as an example, gives the results of the theoretical investigation of mechanical behavior of elastic-plastic laminar composites. The components strength

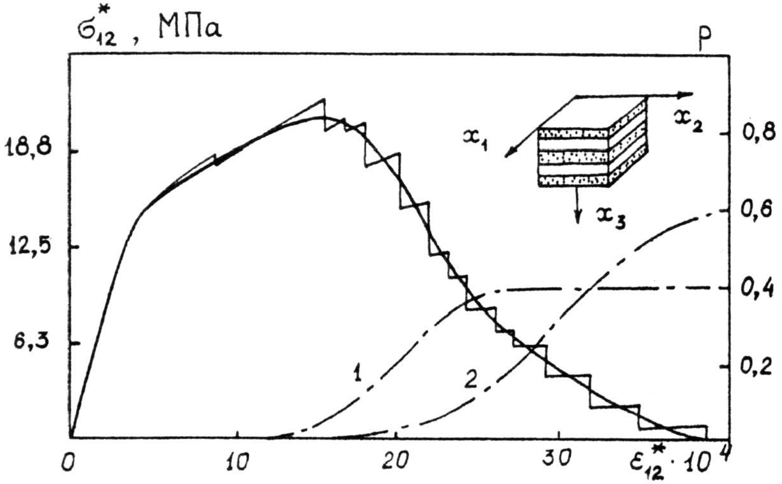

Fig.1 Calculated deformation diagrams of the laminar composite and volume ratio of fractured aluminum (1) and magnesium (2) layers curves

properties are assumed to follow Weibull three-parameter distribution function. The smooth diagram in Fig. 1 corresponds to this case. The stepped diagram corresponds to the deformation of one realization of random set with limited number of layers. The fracture of layers by shear or tear modes resulting from stress redistribution at complex stress state of composite led to the descending branch and to the "tooth" on the deformation diagram as well. Some macrostress-macrostrain relations have already been interrupted on the rising branch of the diagram.

FORMULATION OF BOUNDARY CONDITIONS IN FRACTURE MECHANICS PROBLEM

To evaluate the stability of structural fracture and crack generation processes the examination of the energy balance is necessary. A new formulation of boundary conditions

proposed for the boundary-value problems of fracture mechanics takes into account the finite stiffness of the loading system. This formulation allows to describe energy redistribution between deformable body and loading mechanism.The external forces applied to the part Γ_S of the body boundary are assumed as:

$$S_i(t)|_{\Gamma_S} = \sigma_{ij}(t)n_j|_{\Gamma_S} = S_i^\circ(t) + (\partial S_i/\partial u_j)u_j(t), \qquad (1)$$

where $S^\circ(t)$ is a vector of external forces assumed by loading program, u is a vector of the boundary point displacement, $R_{ij}(u) = -\partial S_i/\partial u_j$ is stiffness of loading mechanism, t is the time. Displacement at Γ_u - boundary are given as:

$$u_i(t)|_{\Gamma_u} = u_i^\circ(t) + (\partial u_i/\partial S_j)S_j(t), \qquad (2)$$

were $Q_{ij}(S) = -\partial u_i/\partial S_j$ is the compliance of the loading mechanism and u° is prescribed according to the loading program. It is quite obvious, that $R_{ik}Q_{kj} = \delta_{ij}$, eq.(1) and eq.(2) are mutually inverse. At $R_{ij}=0$ or $Q_{ij}=0$ the boundary conditions correspond to "soft" or "rigid" loading respectively, and coincide in form with the boundary conditions, commonly used in mechanics of deformable solids.

STABILITY OF POSTCRITICAL DEFORMATION

To estimate the stability of the equilibrium damage growth process on the postcritical stage, the relation between the spent energy (sum of increments of the elastic deformation energy δW and the work of fracture δA_p) and the supplied one (the work done by external forces δA_b) should be considered at virtual infinite small deformation increment. The work of fracture and the increase of the potential energy of elastic deformation compose the specific work of deformation for elementary material volume. Such work on any deformation interval is defined as the area under the equilibrium diagram curve, received by experiment on "stiff" testing machine. The work of external forces is connected with displacement of the points of a deforming body caused by the decrease of its stiffness in the process of fracture :

$$\delta A_b = \int_{\Gamma_S}(S_i - 1/2\,R_{ij}\delta u_j)\delta u_i\, d\Gamma_S + \int_{\Gamma_u}(S_i - 1/2\,\delta S_i)Q_{ij}\delta S_j\, d\Gamma_u \qquad (3)$$

In this case the inequality

$$\delta A_b < \delta W + \delta A_p \qquad (4)$$

is a postcritical deformation stability condition. Evidently, the spontaneous damage growth without the increase of the external load is impossible, since there isn't enough supplying and releasing energy for the work of fracture. Non-fulfillment of the inequality corresponds to avalanche defect growth i.e. dynamic failure. At any material point of the body subjected to external loading the local postcritical deformation stability condition is

$$(V_{ijmn} - D_{ijmn})\,\delta_{mn}\,\delta_{ij} > 0 \qquad (5)$$

where D – tangential modulus tensor on postcritical stage, V – loading system stiffness tensor. Components of tensor V can be obtained by expression

$$V_{ijmn} = (1/2)(\partial\sigma_{ij}/\partial u_k)(dx_m\delta_{kn} + dx_n\delta_{km}) \qquad (6)$$

Interrelation between the internal forces and the displacements reflects stiffness characteristics of all set of the material points and the loading system elements.

FRACTURE MECHANISMS AND CRITICAL STATE CRITERIA

The traditional strength criteria do not include the loading system stiffness and correspond to "zero" stiffness. Similar criteria can be used for the evaluation of the critical stress-deformed state. The critical state is characterized by the combination of two conditions: the postcritical deformation condition and the stability fracture condition for the deformation process. Concerning the mechanical properties of the material at the damage zones any assumptions connected with evaluation of the invariants of stress/strain tensor can be accepted. For example, with an isotropic material it can be accepted that damage zones are able to resist to hydrostatic compressive loading if the first invariant of stress tensor $I_\sigma \leq 0$; and the damage zones are not able to resist to any loading if $I_\sigma \leq 0$. In case of that assumption the components of the elasticity modula tensor are

$$C_{ijkl} = \begin{cases} K\delta_{ij}\delta_{kl} + G(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk} - (2/3)\delta_{ij}\delta_{kl}), & \Phi_\sigma < 0 \\ K\delta_{ij}\delta_{kl}, & \Phi_\sigma \geq 0,\quad I_\sigma < 0 \\ 0, & \Phi_\sigma \geq 0,\quad I_\sigma \geq 0, \end{cases} \qquad (7)$$

where Φ_σ – any strength criterion (for damage zone $\Phi_\sigma \geq 0$).

Application of this model for calculation of stress-deformed state of unidirectional fiber-reinforced composites subjected to complex transversal loading demonstrated possibility of the stable growth of damage zones in the structure of a composite. Fig.2 gives results of calculation for transversal compression of the unidirectional glass-reinforced plastic of regular model. On the Fig.2 the stress intensity σ_i ($\sigma_i = (3/2 S_{ij} S_{ij})^{1/2}$) is: in the non-shading zone $\sigma_i < 35$ MPa, in the single-shading zone 35MPa < σ_i < 56MPa, in crossing-shading zone 56MPa < σ_i , damage zone with $I_\sigma < 0$ marked by points.

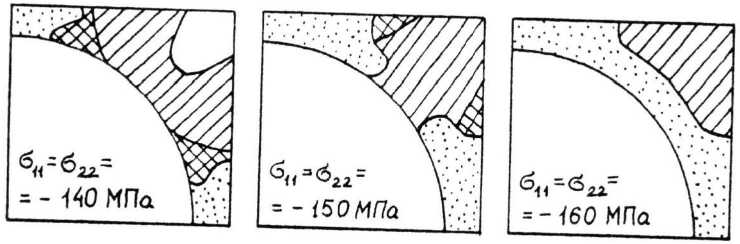

Fig.2 Damage zones in structure of the unidirectional glass-reinforced plastic under transversal compressive loading

POSTCRITICAL DEFORMATION OF THE STRUCTURAL ELEMENTS OF COMPOSITES AND STRUCTURES

The stability conditions of postcritical deformation are analyzed for the elements of bar systems and components of granulated, laminated and fiber-reinforced composites. The obtained conditions set limitations in the relations between stiffness parameters and parameters of the descending branch of the diagram. For instance, the equilibrium fracture conditions for spherical inclusion in infinite matrix are:

$$G > (3/4)K_p^s, \quad G(9K+8G) > G_p^s\, G(K+2G) \qquad (8)$$

where G and K – shear modulus and bulk modulus of the matrix, G_p^s and K_p^s shear and bulk modula of the spherical inclusion at the postcritical deformation stage. For two component laminated composite subjected to active deformation($\varepsilon_{13}^* \neq 0$) the postcritical deformation stability condition of the "first" component layers deformation is:

$$G^{(2)} > G_p^{(1)}(1-p^{(1)})/p^{(1)}. \qquad (9)$$

For the laminar packet loaded to increase the macrostress σ_{12}^{*} the similar condition will be:

$$G^{(2)} > G_p^{(1)} p^{(1)}/(1-p^{(1)}) \qquad (10)$$

The stability and instability zones are illustrated on Fig.3. On the account of more full utilization of strength reserves the zone 2 is preferable. The sudden dynamic layers failure corresponds to zone 4.

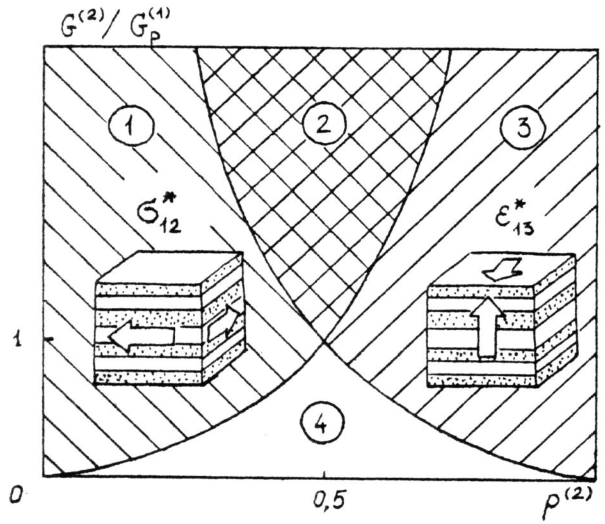

Fig.3 Zones of stability of layers postcritical deformation

At uniaxial loading the conditions of equilibrium fracture of the fiber-reinforced composites are:

$$R > E_f^p p - E_m(1-p) + 4G_f^p G_m p(1-p)(\nu_m - \nu_f^p)^2 (G_f^p + G_f^p p(1-2\nu_m) -$$
$$- G_m(1-p)(1-2\nu_f^p))^{-1}, \qquad (11)$$

where ν - Poisson's ratio, f and m - indexes refer to fiber and matrix respectively , p - fiber volume ratio, R - loading system stiffness characteristic. In case of extension test R

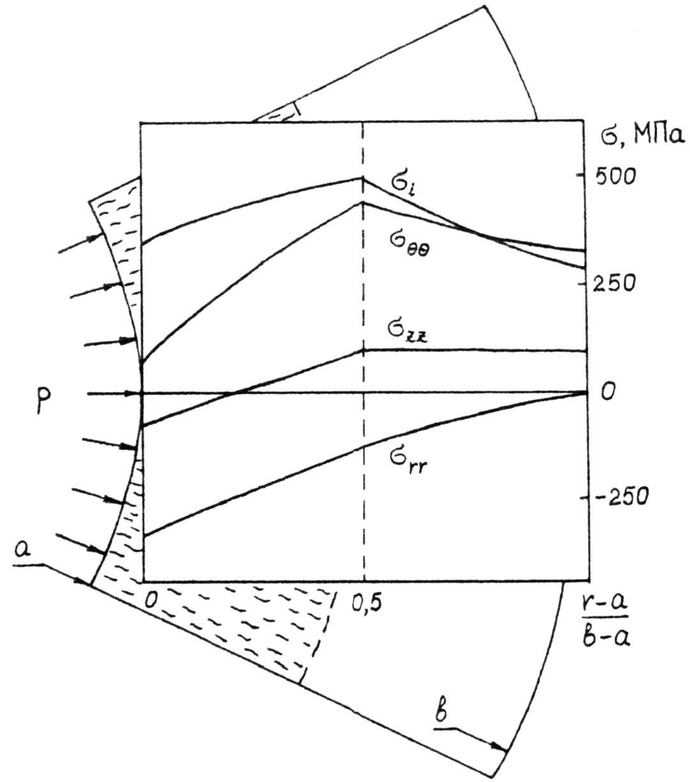

Fig.4 Critical stress state of the thick-wall tube at postcritical deformation

is connected with the stiffness R_m of the test machine by the formula:

$$R = R_m l/F,$$

where l and F are the length and the cross-section area of specimen working zone. The satisfaction of the condition of postcritical deformation is a means of utilization of additional carrying capacity reserves and also a method to increase the vitality of a structure, i.e. the capability to resist to external loading on the stage of crack generation and growth. It is illustrated on Fig.4 by analytical calculation results of carrying capacity of the thick-wall tube subjected to internal pressure taking into account the disstrengthening of material.Reserve of the carrying capacity calculated for this case is 50%.