ENERGY MODEL OF MICROCRACKING AND FRACTURE OF CONTINUOUSLY REINFORCED COMPOSITE MATERIALS BY LONG-TERM LOAD

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ABSTRACT

Model describing process of single and multiple fracture layer composite materials composed with microheterogeneous components under long-term load is developed. According to this model, we represent each layer of composite in sense of characteristics rigidity as a continuous set of ternary structural elements involving of fiber cuts, bonding agent and adhesive layer (dimensions of each element are equal) and as continuous, totality of its energy levels, in sense of characteristics strength. The conception of damage accumulation as sequence fracture of structural elements with simultaneous change in elastic and strength characteristics of undamaged components is laid in the base of model. This process could be accomplished in concentrated or scattered damage accumulation direction completing so-called single (by one magistral crack development) or multiple (dispersive) fracture. The strength and elastic properties of each separate microvolume component are probable quantities distributed correspondingly by normal and Weibull laws.

KEYWORDS

model, composite, simulating, vector, layer, random, rigidity, strength, damage, crack.

MATHEMATICAL SIMULATING OF COMPOSITE MATERIALS

Composite materials or materials of class (K) are known as materials consisting from two or more interinsoluble components with different physical and mechanical properties. There are three factors as necessary condition to produce composite materials: reinforcing elements, bonding agent and adhesive layer. Denote them with symbols $(\mathring{K}, K, K)$. Triplet of these parameters one can consider as vector $\tilde{\tilde{K}}$ corresponding with material under consideration. By varying parameters

$(\mathring{K},\tilde{K},K)$ in direction of its mechanical and structural properties change one receive a new composite material and corresponding vector $\tilde{K}$. A population of all such vectors forms space $\{\tilde{K}\}$. Uniting got by various ways vector spaces one receive a notion of hyperspace (Delyavskii, 1990a),

$$\{\vec{\tilde{K}}\} \in \{\tilde{K}^{(1)}\} \cup \{\tilde{K}^{(2)}\} \cup \ldots \cup \{\tilde{K}^{(n)}\}, \qquad (1)$$

which is a mathematical model of composite material. We consider the class of layered composites on which there is a sequence in one direction of successive layers having thicknesses $h^{(k)}$. Let h be the thickness of the complete material, and then $\omega^{(k)} = h^{(k)}/h$ is the relative volume content of layer k in the composite $(k = 1, 2, \ldots, n)$. We consider the vector for the volume content of the material $\tilde{\omega} = \{\omega^{(1)} \ldots \omega^{(n)}\}$. The set of those vectors constitutes a vector space $(\tilde{\Omega})$, which we call the space of geometrical characteristics for the composite. Similarly, we have the space of mechanical characteristics $\{\tilde{M}\}$, Let's introduce a vector space

$$\{\tilde{K}\} = \{\tilde{M}\} \cap \{\tilde{\Omega}\} , \qquad (2)$$

as an intersection $(\cap)$ of the considered spaces and corresponding to it a scalar space

$$\{K\} = \{\tilde{M}\} \cdot \{\tilde{\Omega}\} , \qquad (3)$$

made by spaces $\{\tilde{M}\}$ and $\{\tilde{\Omega}\}$ scalar product. Let's denominate relation (3) as effective transformation of the second kind. Making use of the effective transformation of the first kind (Delyavskii, 1990a) let's present considered plate as continuous totality of structural elements - parallelepipeds with the volume υ. We separate one layer with composite material and introduce for its following assumptions.

1. For each composite material it is always possibly to find such effectivly homogeneous material that displacement vector components for both materials in the same points are equal.
2. Rigidity and strength tensor components at each point of material are random quantities with the distribution describing by the threeparametrical Weibull law.
3. Averaged with respect to the structural element in any material point the rigidity parameters $b_{(\upsilon)}$ are deterministic and strength parameters $r_{(\upsilon)}$ are probably quantities with the distribution describing by the normal law.
4. Strength tensor effective components distribution with respect to material region (from point to point) is been described by the uniform law.

__Effective Structural Element Rigidity and Base Volume.__ Let the distribution for the random rigidity tensor component be described by a Weibull law (Weibull,1939; Korolyuk et. al.,1985) in triparametrical representation:

$$P(\mathfrak{b}) = 1 - \exp\{-(\upsilon/\text{э})[(\mathfrak{b}-b)/(\mathbf{b}-b)]^{m}\}. \qquad (4)$$

We now consider the physical significance of the parameters in Eq. (4). It is evident (Weibull,1939) that m characterizes the microstructure inhomogeneity: the larger m, the more homogeneous the material. In any real material, $1 < m < \infty$ (the case $m = 1$ represents an absolutely heterogeneous material, while $m = \infty$ represents an absolutely homogeneous one). If there are some n^* elements with partial or complete damage among n composite material structural elements, then relation n^*/n completely corresponds to the parameter m interpretation. So given parameter one can consider as material damage measure

$$m=\theta=n^*/n. \qquad (5)$$

Volume (э) is a measure of material structural nonhomogeneity We'll define it as average volume of pore (defect) and denote as a base composite volume. We assume that in some volume υ may have with identical probabilities not only microcracks, whose rigidity is zero, but also microinclusions, whose rigidity may be taken as infinitely large, which gives us the tensor for the effective rigidity for the semiinfinite range for the elastic characteristics (Korolyuk et. al.,1985)

$$b_{(\upsilon)} = b + (\mathbf{b} - b)(\upsilon/\text{э})^{-1/m}\, \Gamma(1+1/m) \qquad (6)$$

It is obvious that each of these distinct volumes υ cannot be less than the elementary volume э. We consider a volume υ such that the effective rigidity vector is equal to **b**:

$$\upsilon = \text{э}[\Gamma(1+1/m)]^{m}, \qquad (7)$$

in which the expression in [·] is a Γ-function of fractional argument. As $\Gamma(1+1/m) < 1$ for $1 < m < \infty$, **b** cannot be attained for any real material, and we call it the theoretical rigidity tensor; b - rigidity tensor for real materials.

__Statistical Distribution of Internal Energy and Energy Levels of Composite Material.__ We consider the critical (failuring) potential energy as random quantity $w_{(\upsilon)}$ distributed by normal law. Mathematical expectation $Mw_{(\upsilon)}$ and dispersion $Dw_{(\upsilon)}$ of this quantity we determine on test notched spesimens with material at issue according to technique (Delyavskii, 1990a) The potential energy of structural element near notch tip determine as

$$\langle W\rangle = \sum_{r,s,p=1}^{2} b_{ij(v)}\Bigg\{ \sum_{k=1}^{2} T^{(2)}_{k\,r(i)} T^{(2)}_{p\,s(j)} +$$

$$+ \left[T^{(1)}_{1\,r(i)} T^{(2)}_{p\,s(j)} I(\vartheta_r) + T^{(2)}_{1\,r(i)} T^{(1)}_{p\,s(j)} I(\vartheta_s) \right] \sqrt{l} + \tag{8}$$

$$+ \left[T^{(1)}_{p\,r(i)} T^{(2)}_{p\,s(j)} + T^{(1)}_{p\,r(i)} T^{(2)}_{3-p\,s(j)} \right] I(\vartheta_r,\vartheta_s)\, l\ ,$$

where

$$I(\vartheta_r) = \frac{1}{4\rho_1\rho_2}\int_{-\rho_1}^{\rho_1}\int_{-\rho_2}^{\rho_2}\frac{dx_1 dx_2}{\sqrt{\vartheta_r}}\ ;$$

$$I(\vartheta_r,\vartheta_s) = \frac{1}{4\rho_1\rho_2}\int_{-\rho_1}^{\rho_1}\int_{-\rho_2}^{\rho_2}\frac{dx_1 dx_2}{\sqrt{\vartheta_r\vartheta_s}}\ ; \tag{9}$$

$$\vartheta_r = r_0 + x_1 + s_r x_2\ ;$$

$$\langle W\rangle = \tilde{W}\cdot\tilde{\omega}\ , \tag{10}$$

symbol <> denote average (scalar) value $\tilde{W}$; r_0 - curvature radius at the notch tip; $2l$ - its length; b_{ij} - coefficients of matrix rigidity; $T^{(j)}_{p\,r(i)}$ - factors expansion in terms of fraction power ϑ_r strain tensor components $\langle\varepsilon_i\rangle$. We assume that damage to a structural element distinguished near notch tip occurs (in average) always when the next condition is satisfied

$$\langle W\rangle^{*}_{(v)} = M w_{(v)}, \tag{11}$$

where (*) denote critical vale of quantity at issue. Equating this criterial condition to the experimentally determined relation

$$1/p_* = c_0 + c_1\sqrt{l} + c_2 l, \tag{12}$$

where p_* is limit of proportionality notched specimen; c_j $(j=\overline{0,2})$ - experimental constants, we get three transcendent equtions system with respect to the unknown structural element dimensions and the quantity $\left[M w_{(v)}\right]$. Dispersion $\left[D w_{(v)}\right]$ we find as (Delyavskii, 1990a) . Having

$$D w_{(v)} = \frac{1}{n}\sum_{n}\left[w^{(n)}_{(v)} - M w_{(v)} \right]^2, \tag{13}$$

found dispersion we determine minimum $w^{(min)}_{(v)}$ and maximum

$w^{(max)}_{(v)}$ random quntity $w_{(v)}$ distributed by the normal law. Let we introduce the notion of structural element internal energy

$$U_{(v)} = \tfrac{1}{2} b_{ij(v)} \langle r_i\rangle_{(v)} \langle r_j\rangle_{(v)}\ , \tag{14}$$

as potentional energy expressed in terms second kind of strength (Delyavskii, 1990b).

$$\langle r\rangle_{(v)} = \tilde{r}_{(v)}\cdot\tilde{\omega} \tag{15}$$

We assume that

$$w_{(v)} \equiv U_{(v)}. \tag{16}$$

Let divide up an interval of change $w_{(v)} \in \left[W^{(min)}_{(v)}, W^{(max)}_{(v)}\right]$ into subintervals so to at each one the quantity $\left[D w_{(v)}\right]$ would be minimal. Upper boundaries of subintervals we call the composite material energy levels. Then we find probability G_s of hit arbitrarily chosen structural element on energy level [s]. It allows to determine probable number of elements $n_{[s]}$ related to each of the energy level.

$$n_{[s]} = nG_s. \tag{17}$$

Concluding stage material simulating is a procedure of placing of structural elements, each of them belongs to a some energy level occasionally on the material region.

SIMULATION OF THE DAMAGE SET AND CRACK INITIATION AND PROPAGATION IN COMPOSITE MATERIALS

Let introduce a notion of the damage set mechanism. For each mechanism there acts only one component $\langle W_i\rangle_{(v)}$ of potentional energy, which characterizes the damage to the two components of a structural element: the fiber and bonding agent, under loading applied along reinforcement - mechanism I; bonding agent and adhesive layer under normal and shear forces applied in the plane of layer and the perpendicular direction - mechanism II and mechanism III. Let us call these components as low-strength and high-strength component of structural element. In line with that we represent total potential energy as sum energy for each fracture mechanism

$$\langle W\rangle_{(v)} = \sum_{i=I}^{III} \langle W_i\rangle_{(v)}, \tag{18}$$

where

$$\langle W_I\rangle_{(v)} = \tfrac{1}{2} a_{11(v)} \langle\varepsilon_{11}\rangle^2_{(v)}\ ,$$

$$\langle W_{II}\rangle_{(v)} = \tfrac{1}{2} a_{22(v)} \langle\varepsilon_{22}\rangle^2_{(v)} + \tag{19}$$

$$+ a_{12(v)} \langle\varepsilon_{11}\rangle_{(v)} \langle\varepsilon_{22}\rangle_{(v)} + \tfrac{1}{2} a_{66(v)} \langle\varepsilon_{66}\rangle^2_{(v)}\ .$$

$$\langle W_{III}\rangle_{(\upsilon)} = \tfrac{1}{2}a_{33(\upsilon)}\langle\varepsilon_{33}\rangle^2_{(\upsilon)} +$$
$$+ a_{13(\upsilon)}\langle\varepsilon_{11}\rangle_{(\upsilon)}\langle\varepsilon_{33}\rangle_{(\upsilon)} + a_{23(\upsilon)}\langle\varepsilon_{22}\rangle_{(\upsilon)}\langle\varepsilon_{33}\rangle_{(\upsilon)} +$$
$$+ \tfrac{1}{2}a_{44(\upsilon)}\langle\varepsilon_{44}\rangle^2_{(\upsilon)} + \tfrac{1}{2}a_{55(\upsilon)}\langle\varepsilon_{55}\rangle^2_{(\upsilon)} .$$

The subsript (υ) denotes averaging with respect to the structural element; $a_{ij(\upsilon)}$ - coefficients of compliance matrix. Let consider the triad of parameters $\langle W_i\rangle_{(\upsilon)}$ $(i=I,II,III)$ as vector $\langle\vec{W}\rangle_{(\upsilon)}$ and put it in relation to the vector for internal energy $\vec{U}_{(\upsilon)}$. The lexicographic order between these vectors is (Bronshtein and Semendyaev,1981; Delyavskii,1991)

$$\langle\vec{W}\rangle_{(\upsilon)} \prec \langle\vec{U}\rangle_{(\upsilon)} : \langle W_i\rangle_{(\upsilon)} = U_{i(\upsilon)} \quad (i\leqslant n);$$
$$\langle W_i\rangle_{(\upsilon)} < U_{i(\upsilon)} \quad (i>n), \tag{20}$$

which is called the damage accumulation and their collocation over all components

$$\langle W_i\rangle_{(\upsilon)} \underset{i}{\equiv} U_{i(\upsilon)} , \tag{21}$$

is taken as the damage to a structural element. It is assumed if component of potential energy $\langle W_i\rangle_{(\upsilon)}$ reach corresponding energy level all the structural elements belonging to this level will be completely or partially damaged. The damage set for each layer in the composite is taken as the vector

$$\langle\vec{\theta}\rangle^{*(k)} = \vec{n}^{*(k)}/n^{(k)}, \tag{22}$$

whose components are the ratios of the numbers $\vec{n}^{*(k)}$ of the partially or completely damaged structural elements to the total number $n^{(k)}$. We make the following assumptions:
1) the damage accumulation is a continuous vector random process developing in time and space;
2) the damage to the structural elements occurs at those points in the material where conditions have attained a state of limiting equilibrium; and
3) the damage is accompanied by change in the elastic and strength characteristics of the undamaged structural components. Those characteristics are zero for the damaged elements.

We represent the damage accumulation for each layer of the composite as a vector damage sequence of strength levels subject to the criteria (20) and (21) with simultaneous replacement of the mechanical properties of the material at the various energetical levels. We take the vector kinetic equation for the damage at the $[s^{(k)}]$ level as (Delyavskii,

1991)

$$\frac{d\langle\theta_i\rangle^{(k)}_{[s]}}{dt} = A_i^{(k)} \frac{\Phi_i}{\left[1 - \langle\theta_i\rangle^{(k)}_{[s]}\right]^n} , \tag{23}$$

in which Φ_i is a criterial function which corresponds to the conditions $\Phi_i = 0$ for the unloaded state and $\Phi_i = 1$ at the instant of failure.

Multiples Fracture. We split up the damage accumulation process in view its the nonuniformity into υ stages, each of which corresponds to damage at the energy level $[s^{(k)}]$ (on one of the components) with simultaneous changes in the elastic and strength characteristics of the composite at the other energy levels. We specify the law followed by the Young's moduli $b^{(k)}_{ij[s]}$ and the effective material strength parameters $r^{(k)}_{[s]}$ because of the damage in the realization of stage υ at each level $[s^{(k)}]$ in the form

$$\underset{\nu}{b}{}^{(k)}_{ii[s]} = \underset{\nu-1}{b}{}^{(k)}_{ii[s]} \left\{ 1 - \frac{\underset{\nu}{\langle\theta_i\rangle}{}^{(k)}_{[s]} + \sum\limits_{q=1}^{\nu-1} \underset{q}{\langle\theta_i\rangle}{}^{*(k)}_{[s]}}{\langle\theta_i\rangle^{*(k)}_{[s]}} \right\} , \tag{24}$$

$$\underset{\nu}{b}{}^{(k)}_{[s]} = \underset{\nu}{b}{}^{[k]} + \left[b^{(k)} - \underset{\nu}{b}{}^{(k)}\right]\left(V^{(k)}_{[s]}/\mathrm{Э}\right)^{\theta^{*(k)}_{[s]}} \Gamma\left[\theta^{*(k)}_{[s]}\right], \tag{25}$$

$$\underset{\nu}{\langle r_i\rangle}{}^{(k)}_{[s]} = \underset{\nu-1}{\langle r_i\rangle}{}^{(k)}_{[s]} \, \underset{\nu}{b}{}^{(k)}_{ii[s]} \,/\, \underset{0}{b}{}^{(k)}_{ii[s]} . \tag{26}$$

The (*) means that the value of the parameter θ is calculated at the end of stage q; $V^{(k)}_{[s]}$ - energy level $[s^{(k)}]$ volume. The law Poisson's ratio change is taken from the condition for symmetry in the rigidity tensor $b_{ij[s]}$. Let's consider the first stage $\nu = 1$. We provide a small increment to parameter t and calculate the damage to all the layers of the composite at the energy levels $[s^{(k)}]$. From (25) we calculate the reductions in rigidity at the energy levels and for a layer as a whole as functions of the damage; we derive the changes in the effective moduli for the layered composite and establish the redistribution of the macroscopic stresses and macrostrain for each energy level in the scalar and vector spaces. We test (21) on the low-strength component of a energy level $[1^{(k)}]$. If the test is not obeyed, we increase the increment and repeat the procedure. If the criterion is met, we recalculate the strength parameters for each energy levels, we derive the redistribution of the stresses in the unfailured parts of the structural elements and again test (21) for the high-strength component of the first energy level. There are two possibilities: the test is not met, so we increase t and repeat the procedure, or the test is met, in which case we repeat the described operation . We thus get the critical time for the mechanism after which the failure in the structural elements in the layer occurs spontaneously because of the stress redistribution in the unfailured elements. The time at which the damage attains the critical value in the layers in the composite represents the working life.

Single Fracture. Assume that in stage ν with m partially or completely fractured structure elements k elements find oneself abreast The result of such event with probability

$$P=C_m^k / C_n^m, \tag{27}$$

we treat as appearance in material a crack of dimension k. We take into consideration such two-component model of composite material with crack: composite is effectively anisotropic continuum in macrovolume; its elastic property defined in fracture single moment; material rigidity parameters in microvolume determined from formula (6), where accepted that $b_{(\upsilon)}$ - material effective rigidity on single fracture stage. Assume that under conditions $\langle W_t \rangle_{(\upsilon)} = U_{t(\upsilon)}$ crack grows up according to mechanism t on the dimension volume υ and repeate proposed procedure up to spontaneous crack propagation.

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