DEVELOPING IRWIN FORMULA TO IMPROVE THE ACCURACY OF EVALUATION OF STRESSES AND CRACK TIP PLASTIC ZONE SIZE

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ABSTRACT

Irwin formula was improved by introducing three new elements - three functions, accounting for the applied stress, crack length, specimen width and the distance to the crack tip. Based on the improved formula the more exact relation is derived for predicting the plastic zone size (PZS) at the crack tip. The errors in the stress distribution and PZS are analyzed over wide ranges of the relative crack length, net stress and stress intensity factor. The present analysis results are compared with the results of numerical analyses and the actual PZS values, reported in the literature.

KEYWORDS

Crack tip, stress distribution, stress intensity factor, plastic zone size.

When predicting the fatigue crack growth time under the operational loading spectrum, one of the dominant parameters is the plastic zone size (Chang et al., 1981). It is calculated using the relation based on Irwin formula for the description of stress distribution at the crack tip. Koskinen (1963), Kang and Liu (1972) found out that this relation has a significant error.

The present paper objective is to improve Irwin formula to decrease the error in the prediction of stress distribution and PZS at the crack tip. Irwin formula and the appropriate relation for PZS have the form (Irwin, 1957; McClintock and Irwin, 1964)

$$\sigma_x = \sigma_y = K/(2\pi x)^{1/2} \qquad (1)$$

$$x_p = (1/\gamma\pi)(K/\sigma_{02})^2 \qquad (2)$$

where σ_x, σ_y are stresses along the crack propagation and across this direction, respectively; x is the distance from crack tip; γ is the constant (=2 at the plane-stress state, =6 at the plane-strain state). σ_{02} is the material yield limit; x_p is PZS along the crack propagation (Fig. 1); K is the stress intensity factor, calculated by the formula

$$K=\sigma(\pi l)^{1/2}\ f(\lambda)$$

where σ is the applied stress;
$f(\lambda)$ is the function accounting the structural element width effect. In the case of a specimen with a central crack this function is represented as a polynomial (Brown and Strawley, 1967)

$$f(\lambda)=1+0.128\lambda-0.288\lambda^2+1.525\lambda^3$$

$\lambda=2l/B$ is the relative crack length;
$2l$ is the crack length; B is the specimen width.

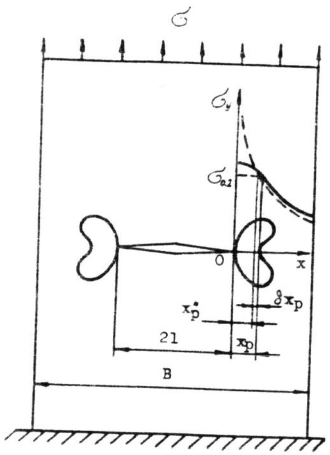

Fig. 1. The PZS analysis scheme for the specimen with a crack.

Irwin formula development contains the following: three functions, depending on the applied stress σ and the specimen geometry parameters (2l, x, B) are included into the right-hand side of equation (1). These functions are structurally similar to those, previously used by the present author to improve Kirsch formula describing the stress distribution in the specimen with a central hole (Dotsenko and Polyakov, 1988). In the works on fracture mechanics (Knott, 1979) it was noted that the initial solution for stresses in a specimen with a central crack was casted as a series however, for practical applications only the first, singular term was left, that stands in the right - hand side of the formula (1), and the remainder is omitted. Therefore the introduced functions really compensate the omitted terms. The improved formula has the form

$$\sigma_x \quad \sigma_y - K/(2\pi x)^{1/2}+\sigma_{net}-\sigma f(\lambda)[(x/B)+8\lambda^2(x/l)^{1/2}] \qquad (3)$$

where

$$\sigma_{net} = \sigma/(1-\lambda)$$

It follows from (3) that in order to calculate the stresses at a specimen point located at a distance x from the crack tip, it is necessary to sum up the singular term with the stress σ_{net} and to subtract two functions depending on σ, 2l, x and B.

Specific forms of these functions were determined so as to minimize the error in the stress distribution around the crack tip.

For simplicity the formula (1) will be symbolized by (I) (Irwin formula), and the formula (3) by (ID) (Irwin formula improved by the author). The error was assessed by comparing the area under the predicted stress distribution curve with the area under the stress distribution curve in the gross section. For the error the following equation was used

$$\Delta = 1 - [2\int_0^{(B-2l)/2}\sigma_y dx\ /\ \sigma B\ ] \qquad (4)$$

Substituting (1) and (3) into (4) we get

$$\Delta\ (I) = 1-2f(\lambda)[(\lambda/2)(1-\lambda)]^{1/2} \qquad (5)$$

$$\Delta(ID)=\{[(\lambda/2)(1-\lambda)]^{1/2}-(1-\lambda)^2/8-(8/3)[\lambda(1-\lambda)]^{3/2}\}f(\lambda) \qquad (6)$$

where $\Delta(I)$ and $\Delta(ID)$ are the errors in the stress distributions by the formulae (I) and (ID), respectively.

The formula (ID) is derived for the plane strain at the crack tip. At the plane-stress state the stresses at the crack tip are less than those under the plane strain (Villareal et al, 1975). This decrease can be taken into account by multiplying the right-hand side of the formula (3) by the function

$$\varphi(\lambda) = 1/(1+0.054\sin 2\pi\lambda).$$

Using Mises criterion for plastic zone boundary definition, we get from formulae (1) and (3):

$$x_p(I) = (1/\gamma\pi)(K/\sigma_{02})^2 \tag{7}$$

$$x_p^*(ID) = (1/\gamma\pi)\{K/[(\sigma_{02}/\varphi(\lambda)) - \sigma_{net}+\sigma f(\lambda)(x_p^*/B+8\lambda^2(x_p^*/l)^{1/2})]\}^2 \tag{8}$$

where $x_p(I)$ and $x_p^*(ID)$ are the PZS values, calculated using the formulae (I) and (ID), respectively.

The relation (8) allows us to get the PZS value at the crack tip when the stress diagram described by the formula (3) maintains its location about the crack tip. At high applied stress levels the plastic deformations around the crack tip cause the diagram to displace (Fig. 1) and PZS to increase. When using the formula (I) the reference PZS value of $2x_p(I)$ is assumed (McClintock and Irwin, 1964; Knott, 1979).

Let`s designate the symbol δx_p for the increment in the PZS. Then according to the formula (ID) the PZS is

$$x_p(ID)=x_p^*(ID)+\delta x_p \tag{9}$$

where $\delta x_p=0$ if $\sigma_{net}<\sigma_{p1}$;

$$\delta x_p=\delta x_{pm}\{1-[1-(\sigma_{net}-\sigma_{p1})^2/(\sigma_{02}-\sigma_{p1})^2]^{1/2}\} \quad \text{if } \sigma_{p1}<\sigma_{net}<\sigma_{02};$$

σ_{p1} = material proportional limit;
δx_{pm} = the limit increment of PZS occuring at $\sigma_{net}=\sigma_{02}$ and defined as a difference:

$$\delta x_{pm} = x_{pm}-x_{pm}^*;$$

x_{pm} = PZS limit corresponding to the equality $\sigma_{net}=\sigma_{02}$, i.e.

$$x_{pm} = (B-2l_m)/2 = B\sigma/2\sigma_{02};$$

$2l_m$ = limit crack length corresponding to the equality $\sigma_{net}=\sigma_{02}$, i.e.

$$2l_m=B(1-\sigma/\sigma_{02})$$

x_{pm}^* = PZS value calculated by (8) at the crack length of $2l_m$.

Dividing both sides of the formulae (7) and (9) by the half of the net section width, $(B - 2l)/2$, we get the formula for the relative PZS values:

$$\left.\begin{aligned} \bar{x}_p(I) &= 2x_p(I)/(B-2l) \\ \bar{x}_p(ID) &= 2x_p(ID)/(B-2l) \end{aligned}\right\} \tag{10}$$

The curves of the error Δ as a function of the relative crack length, the formulae (5) and (6), are shown in Fig. 2. One can see that the calculation error corresponding to the formula (ID) does not exceed 4%, and that corresponding to the formula (I) is much greater, by a factor of (5 - 20).

The PZS was computed for flat specimens of the 2024-T351 aluminium alloy, similar to those studied by Kang and Liu (1972). Material properties and specimen size are: $\sigma_{02}=365$ MPa, σ_{p1} = 231 MPa, B = 100 - 2000 mm, thickness of 2 - 6 mm, γ = 2. The calculation results are shown in Figs. 3 and 4. At the stress $\sigma_{net}=\sigma_{02}$ the plastic zone propagates through the net section and the PZS must be a half of the net section width, i.e. $x_p=(B-2l)/2$. Therefore at the change of the ratio σ_{net}/σ_{02} in the range of 0 to 1 the relative PZ value defined by the formulae (10) should change from 0 to 1. The value $\bar{x}_p(I)$ does not satisfy this condition. Fig.3 shows the dependence of $\bar{x}_p(ID)$ and $\bar{x}_p(I)$ upon σ_{net}/σ_{02} at different λ; it is seen that when σ_{net}/σ_{02} changes from 0 to 1 the value of $\bar{x}_p(ID)$ changes from 0 to 1, and the value of $\bar{x}_p(I)$ lies in the range of 0 to 0.18 (if PZS is assumed to be $2x_p(I)$, then the value of $\bar{x}_p(I)$ changes from 0 to 0.36).

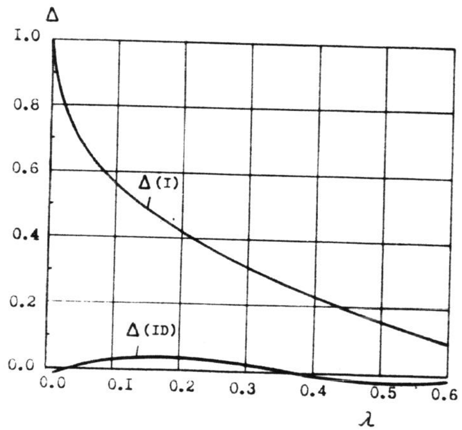

Fig. 2. The relationship between the stress analysis error and the relative crack length.

The family of curves $\bar{x}_p(ID)$ represented in Fig. 3 is universal (as it remains unchanged when the specimen width varies) and seems to be independent of the kind of loading. In this figure the points show the values of $\bar{x}_p$ obtained by Koskinen (1963) through the numerical method for predicting PZS in specimens with cracks under the longitudinal shear. The points coincide with the curves corresponding to the formula (ID) and don`t with the curves plotted according to the formula (I).

The typical curve (for the specimen of width B=2000mm) is shown in Fig. 4 that reflects the relationship between the PZS and the stress intensity factor plotted for different relative stresses $\bar{\sigma}=\sigma/\sigma_{02}$. The PZS by the formula (I) is plotted as a single straight dashed line, and the one by the formula (ID) is plotted as a family of solid straight lines and curves. The straight lines on the plot are common to the specimens of different widths. They present the PZS at stresses $\sigma_{net} < \sigma_{pl}$. The curves correspond to the stress level $\sigma_{net} > \sigma_{pl}$, each is related to a specimen of a specific width. At the fixed values of K and $\sigma_{net} > \sigma_{pl}$, the narrower the specimen, the less the PZS value. The straight lines in the right-hand side of the plot do bend up, and the wider the specimen, the larger is K value, at which the bend begins (for simplicity, the bend in Fig. 4 is shown only for the stress level $\bar{\sigma}$ = 0.1).

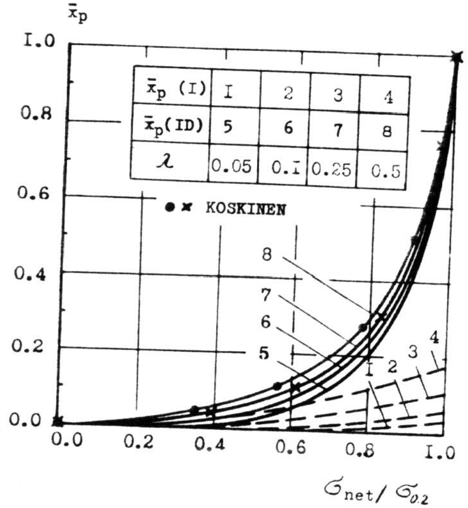

Fig. 3. The relative PZS vs. the ratio σ_{net}/σ_{02}.

The actual PZS values obtained by Kang and Liu (1972) are plotted as the points. They correlate well with the results by the formula (ID) (at $\bar{\sigma}$=0.27) and contradict to results from the formula (I). The PZS values corresponding to two fixed crack lengths (2l=0.01 mm and 2l=1 mm) are shown in the plot by two dash-and-dot lines. One can see that the less the crack length 2l (and the greater the applied stress $\bar{\sigma}$), the larger the difference between x_p(ID) and x_p(I). For small cracks the ratio $x_p(ID) / x_p(I)$ can reach several decimal orders. The equality $x_p(ID) = x_p(I)$ holds at $\bar{\sigma}$ = 0.027, i.e. at very low levels of applied stresses.

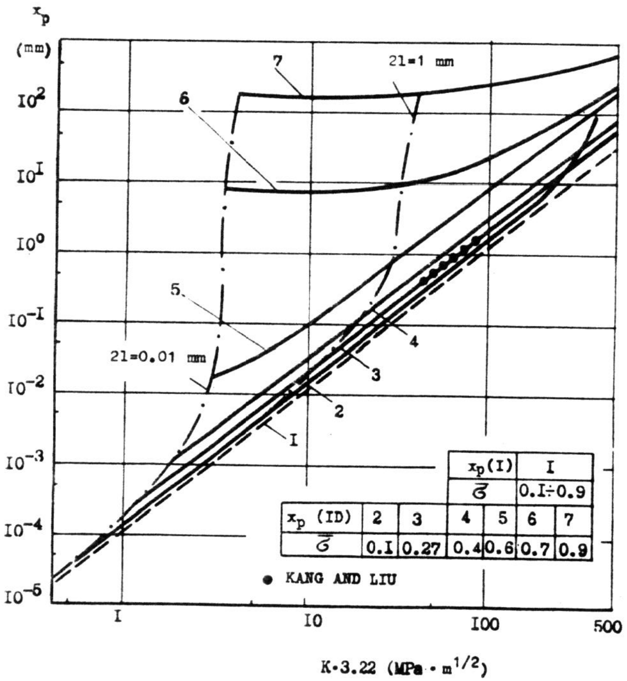

Fig.4 The PZS vs. stress intensity factor.

It can be concluded that the present development to Irwin formula, alongside with the increase in the accuracy of the plastic zone size prediction allows us to extend the range of applied stresses, crack lengths and stress intensity factors where this parameter can be used for the crack, growth rate analysis and the solution of other problems on the crack-resistance of materials and structural elements.

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REFERENCES

Brown W. F., Srawley Y.E. (1967) Plane-strain crack toughness testing of high strength metallic materials.-ASTM STP N410, p. 1-129.