CRACK IN DAMAGED BODY AT SHEAR STRESS STATE

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ABSTRACT

The problem of a crack interaction with an arbitrary set of small cracks at transverse shear state has been solved using singular integral equations and the small parameter method. Attention has been paid to analyse the possible contacting of crack edges.

KEY WORDS: shear crack, damage, stress intensity factors, crack closure, singular integrals, small parameter.

The crack interaction with an arbitrary set of small cracks under tensile loading has been investigated by Romalis and Tamužs since (1984) in a number of papers, some of them generalized by Romalis and Tamužs (1989). In the present paper the same methodology (singular integral equations and the small parameter method) is applied to analysing the interaction of large and small cracks at shear loading. The event of cracks closing up and contacting of edges is an essential feature of this problem. This phenomenon for the shear loading of dissimilar materials with cracks has been revealed as early as in the sixties comninou and references can be found in paper of Dundurs and Comninou (1979).

1. STATEMENT OF THE PROBLEM AND A SOLUTION DISREGARDING THE CONTACT ZONE.

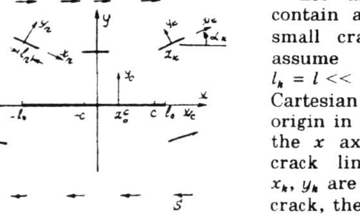

Fig. 1. A location scheme of the main crack (with contact zone) and an array of small cracks.

Let an elastic isotropic plane contain a crack of size $2l_0$ and N small cracks of size $2l_k$. Let us assume that $l_k = l << l_0$ ($k = 1, 2, \ldots, N$). The Cartesian coordinates x, y have the origin in the main crack middle and the x axis direction is along the crack line. The local coordinates x_k, y_k are associated with each small crack, the middle point of which has the coordinate $z_k^0 = x_k^0 + iy_k^0$, with the slope angle α_k to the x-axis (Fig. 1). The cracks edges are free of tractions, while the shear stress $\tau^\infty = S$ is applied to infinity. The problem can be changed into solving the plane stress/strain state with boundary conditions on the crack lines:

$$\sigma_n^\pm - i\tau_n^\pm = p_n(x_n) = iSe^{-2i\alpha_n}, \quad |x_n| < l_n \;\; (n = 0, 1, \ldots, N) \qquad (1.1)$$

Singular integral equations for the system of cracks with self-equilibrated stresses (1.1) on their lines have been derived by Panasyuk, et al (1976)

$$\int_{-l_n}^{l_n} \frac{g'_n(t)}{t-x}\,dt + \sum_{\substack{k=0\\k\neq n}}^{N} \int_{-l_k}^{l_k} [g'_k(t)K_{nk}(t,x) + \overline{g'_k(t)}L_{nk}(t,x)]\,dt = \pi p_n(x)\ ; \quad n = 0, 1, \ldots, N \tag{1.2}$$

The expressions for regular kernels K_{nk} and L_{nk} have been given by Panasyuk, et al (1976) :

$$K_{nk}(t,x) = \frac{e^{i\alpha_k}}{2}\left(\frac{1}{T_k - X_n} + \frac{e^{-2i\alpha_n}}{\overline{T}_k - \overline{X}_n}\right);$$

$$L_{nk}(t,x) = \frac{e^{-i\alpha_k}}{2}\left(\frac{1}{\overline{T}_k - \overline{X}_n} - \frac{T_k - X_n}{(\overline{T}_k - \overline{X}_n)^2}\,e^{-2i\alpha_n}\right);$$

$$T_k = te^{i\alpha_k} + z_k^0,\quad X_n = xe^{i\alpha_n} + z_n^0 \tag{1.2'}$$

The $g'_n(x)$ represent the derivatives of displacement jumps on the crack lines:

$$g_n(x) = v_n(x) - iu_n(x)\ ;\quad v_n = \frac{2\mu}{\kappa+1}[v_n]\ ;\quad u_n = \frac{2\mu}{\kappa+1}[u_n]\ ;$$

where μ - shear modulus, ν- Poisson's ratio, $\kappa = 3 - 4\nu$ for plane strain state, $\kappa = (3-\nu)/(1+\nu)$ for plane stress state, $[u_n]$ and $[v_n]$ - displacement jumps.

Solution of system (1.2) should satisfy the condition

$$\int_{-l_n}^{l_n} g'_n(t)\,dt = 0\ ;\quad n = 0, 1, \ldots, N \tag{1.4}$$

After regularization of (1.2) with the Karleman-Vekua method as given by Muskhelishvili (1968), let us single out an equation for the main crack ($n = 0$) and substitute the variables by their dimensionless ones $t = l_n\tau$; $x = l_n\chi$

It results to

$$g'_0(\chi) = \frac{1}{\pi\sqrt{1-\chi^2}}\left\{-\int_{-1}^{1}\frac{\sqrt{1-\tau^2}}{\tau-\chi}p_0\,d\tau + \sum_{k=1}^{N}\lambda\int_{-1}^{1}[g'_k(\tau)M_{0k} + \overline{g'_k(\tau)}N_{0k}]\,d\tau\right\},$$

$$g'_n(\chi) = \frac{1}{\pi\sqrt{1-\chi^2}}\left\{-\int_{-1}^{1}\frac{\sqrt{1-\tau^2}}{\tau-\chi}\lambda p_n\,d\tau + \right.$$

$$\left. + \int_{-1}^{1}[g'_0(\tau)M_{n0} + \overline{g'_0(\tau)}N_{n0}]\,d\tau + \sum_{\substack{k=1\\k\neq n}}^{N}\int_{-1}^{1}[g'_k(\tau)M_{nk} + \overline{g'_k(\tau)}N_{nk}]\,d\tau\right\}. \tag{1.5}$$

where

$$M_{nk}(\tau,\chi) = \frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\xi^2}}{\xi-\chi}K_{nk}(\tau,\xi)\,d\xi\ ;$$

$$N_{nk}(\tau,\chi) = \frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\xi^2}}{\xi-\chi}L_{nk}(\tau,\xi)\,d\xi\ . \tag{1.6}$$

Parameter $\lambda = l/l_0 << 1$, and the solution of (1.5) can be expressed as a sequence of λ

$$g'_n(\chi) = \sum_{p=0}^{\infty} g'_{np}(\chi)\lambda^p,\quad n = 0, 1, \ldots, N. \tag{1.7}$$

After some calculations, similarly to the paper of Romalis, Tamužs (1984), the following expressions for the first coefficients g'_{np} are obtained:

$$g'_{00}(\chi) = \frac{iSl_0\chi}{\sqrt{1-\chi^2}}$$

$$g'_{02}(\chi) = \frac{iSl_0}{2\sqrt{1-\chi^2}}\sum_{k=1}^{N}[J_k(u_k,\alpha_k)m_{0k1}(\chi) - \overline{J_k(u_k,\alpha_k)}n_{0k1}(\chi)] \tag{1.8}$$

$$J_k = \frac{1}{2}\left[(2e^{-2i\alpha_k} - 1)\frac{\overline{u}_k}{\sqrt{\overline{u}_k^2 - 1}} + \frac{u_k}{\sqrt{u_k^2-1}} - \frac{e^{-2i\alpha_k}(\overline{u}_k - u_k)}{(\overline{u}_k^2 - 1)^{3/2}}\right], \tag{1.9}$$

$$m_{ok1}(\chi) = e^{i\alpha_k}\mathrm{Re}\left(\frac{e^{i\alpha_k}(u_k\chi - 1)}{(\chi-u_k)^2\sqrt{u_k^2-1}}\right);$$

$$n_{ok1}(\chi) = \frac{e^{-i\alpha_k}}{2}\left[(u_k - \overline{u}_k)e^{-i\alpha_k}\frac{-\chi^2 + 2\overline{u}_k^3\chi - 3\overline{u}_k^2 + 2}{(\chi-\overline{u}_k)^3(\overline{u}_k^2-1)^{3/2}} + \right.$$

$$\left. + (e^{i\alpha_k} - e^{-i\alpha_k})\frac{\overline{u}_k\chi - 1}{(\chi-\overline{u}_k)^2\sqrt{\overline{u}_k^2-1}}\right]$$

$$u_k = z_k/l_0$$

For microcrack

$$g'_{k1} = \frac{iSl_0\chi}{\sqrt{1-\chi^2}}J_k(u_k,\alpha_k)\,. \tag{1.10}$$

A set of equations (1.5) is derived disregarding the contact of crack edges, but their solution indicates the presence of contact zone, if $[v_0] < 0$, $[v_n] < 0$ are obtained.

The displacement jumps can be found by direct integration.

$$g_n(\chi) = \int g'_n(\chi)\,d\chi + C$$

According to condition (1.4) the constant $C = 0$.

Using Eqs. (1.8) - (1.10), (1.3) the following expressions are obtained:

$$[\,v_0\,] = \lambda^2\frac{\kappa+1}{2\mu}Sl_0\sqrt{1-\chi^2}\,\frac{1}{2}\sum_{k=1}^{N}\mathrm{Im}\left\{J_ke^{i\alpha_k}\mathrm{Re}\left(\frac{e^{i\alpha_k}}{(u_k-\chi)\sqrt{u_k^2-1}}\right) + \right. \tag{1.11}$$

$$\left. + \overline{J}_k\frac{e^{-i\alpha_k}}{2(\overline{u}_k-\chi)\sqrt{\overline{u}_k^2-1}}\left[2(u_k-\overline{u}_k)e^{-i\alpha_k}\frac{2\overline{u}_k^2 - \overline{u}_k\chi - 1}{(\overline{u}_k-\chi)(\overline{u}_k^2-1)} - e^{i\alpha_k} + e^{-i\alpha_k}\right]\right\}, \tag{1.11}$$

$$[\,v_k\,] = \lambda\frac{\kappa+1}{2\mu}Sl_0\sqrt{1-\chi^2}\,\mathrm{Im}\{J_k\}\,. \tag{1.12}$$

where J_k is defined by (1.9).

Equation (1.12) shows that $[v_n]$ maintains always the shape $\sim (1-\chi^2)^{1/2}$ with coefficient, which can be either positive or negative, depending on the defect location and orientation. It means that microcracks can be of completely open or closed modes. Formula (1.11) reveals that the main crack can be of open mode and fully or partially closed. In the last case point c, where $[v_0] = 0$ can be taken as a limit of the contact region.

2. CONSIDERATION OF THE CRACK CLOSURE.

To solve the problem by taking into account the closure it is necessary to formulate new boundary conditions. For open cracks, the previous boundary conditions (1.1) are valid, for closed cracks the conditions are as follows:

$$\tau_n(x) = -S\cos 2\alpha_n \,, \quad [\, \upsilon_n \,] = 0 \,, \quad |\, x \,| < l_n \,. \tag{2.1}$$

For open cracks the unknown functions are $[u_n]$, $[\upsilon_n]$; for closed cracks the shear jump $[u_n]$, contact pressure σ_n and the contact zone boundary c should be found.

The initial system of Eq.(1.2) has to be divided into two sets of real and imaginary parts according R.Goldshtein et al (1992). Let us assume that only one open region with size $2c$ will arise on the main crack (look Fig. 1):

$$\int_{-c}^{c} \frac{\upsilon'_0(t)}{t - x}\, dt + \sum_{k=1}^{N} \int_{-l}^{l} [u'_k(t)\mathrm{Im}(K^c_{0k} - L^c_{0k}) + \upsilon'_k(t)\mathrm{Re}(K^c_{0k} + L^c_{0k})]dt = \pi\sigma^c_0 \,, \quad |\, x \,| < c \,,$$

$$\int_{-l}^{l} \frac{\upsilon'_n(t)}{t - x}\, dt + \int_{-l_0}^{l_0} u'_0(t)\mathrm{Im}(K_{n0} - L_{n0})\, dt + \int_{-c}^{c} \upsilon'_0(t)\mathrm{Re}(K^c_{n0} + L^c_{n0})\, dt +$$

$$+ \sum_{\substack{k=1 \\ k \neq n}}^{N} \int_{-l}^{l} [u'_k(t)\mathrm{Im}(K_{nk} - L_{nk}) + \upsilon'_k(t)\mathrm{Re}(K_{nk} + L_{nk})]dt = \pi\sigma_n \,,$$

$$n = 1, 2, \ldots, N \quad |\, x \,| < l \,. \tag{2.2}$$

$$\int_{-l_0}^{l_0} \frac{u'_0(t)}{t - x}\, dt + \sum_{k=1}^{N} \int_{-l}^{l} [u'_k(t)\mathrm{Re}(K_{0k} - L_{0k}) - \upsilon'_k(t)\mathrm{Im}(K_{0k} + L_{0k})]dt = \pi\tau_0 \,, \quad |\, x \,| < l_0 \,,$$

$$\int_{-l}^{l} \frac{u'_n(t)}{t - x}\, dt + \int_{-l_0}^{l_0} u'_0(t)\mathrm{Re}(K_{n0} - L_{n0})\, dt - \int_{-c}^{c} \upsilon'_0(t)\mathrm{Im}(K^c_{n0} + L^c_{n0})\, dt +$$

$$+ \sum_{\substack{k=1 \\ k \neq n}}^{N} \int_{-l}^{l} [u'_k(t)\mathrm{Re}(K_{nk} - L_{nk}) - \upsilon'_k(t)\mathrm{Im}(K_{nk} + L_{nk})]dt = \pi\tau_n \,,$$

$$n = 1, 2, \ldots, N \quad |\, x \,| < l \,. \tag{2.3}$$

Conditions (1.4) for (2.3) and (2.4) have the following expressions:

$$\int_{-l_n}^{l_n} u'_n(t)dt = 0 \,, \qquad \int_{-c}^{c} u'_0(t)dt = 0 \,, \qquad \int_{-l_n}^{l_n} \upsilon'_n(t)dt = 0 \tag{2.4}$$

where $n = 0, 1, \ldots, N$

The kernels K_{nk}, L_{nk} are defined by (1.2'), but in expressions for K^c_{0k}, L^c_{0k}, K^c_{no}, L^c_{no} the following variables are used $T^c_0 = t + z^c_0$, $X^c_o = x + z^c_0$, where z^c_o is the centre of the open region in the main crack (Fig. 1).

Using variables $t = l_n \tau$; $\kappa = l_n \chi$ the system (2.2), (2.3) can be expressed in a dimensionless form and the solution is sought as a sequence of λ

$$u'_n(\chi) = \sum_{p=0}^{\infty} u'_{np}(\chi)\lambda^p \,, \quad \upsilon'_n(\chi) = \sum_{p=0}^{\infty} \upsilon'_{np}(\chi)\lambda^p \,. \tag{2.5}$$

The kernels K^c_{nk}, L^c_{nk} are expanded in the series of λ

$$K^c_{0k} = -\frac{e^{i\alpha_k}}{2}\sum_{p=0}^{\infty}(\lambda\tau)^p 2\mathrm{Re}\left[\frac{e^{ip\alpha_k}}{(\varepsilon\chi + d_0 - u_k)^{p+1}}\right] = \sum_{p=0}^{\infty} K^c_{0kp}\lambda^p \,;$$

$$L^c_{0k} = \frac{1}{2}[\lambda\tau(\, e^{-i\alpha_k} - \, e^{i\alpha_k}) + \bar{u}_k - u_k]\sum_{p=0}^{\infty}(p + 1)\frac{(\lambda\tau)^p e^{-i(p+1)\alpha_n}}{(\varepsilon\chi + d_0 - \bar{u}_k)^{p+2}} = \sum_{p=0}^{\infty} L^c_{0kp}\lambda^p \,;$$

$$K^c_{n0} = \frac{e^{-i\alpha_n}}{2}\sum_{p=0}^{\infty}(\lambda\chi)^p 2\mathrm{Re}\left[\frac{e^{ip\alpha_n}}{(\varepsilon\tau + d_0 - u_n)^{p+1}}\right] = \sum_{p=0}^{\infty} K^c_{n0p}\lambda^p \,;$$

$$L^c_{n0} = \frac{e^{-i\alpha_n}}{2}[(\varepsilon\tau + d_0)\,(e^{i\alpha_n} - e^{-i\alpha_n}) + u_n e^{-i\alpha_n} - \bar{u}_n e^{i\alpha_n}] \times$$

$$\times \sum_{p=0}^{\infty}(p + 1)\,(\lambda\chi)^p \frac{e^{-ip\alpha_n}}{(\varepsilon\tau + d_0 - \bar{u}_n)^{p+2}} = \sum_{p=0}^{\infty} L^c_{n0p}\lambda^p \,;$$

$$\varepsilon = \frac{c}{l_0}\,, \quad u_k = \frac{z_k}{l_0}\,, \quad d_0 = \frac{z^c_0}{l_0} \tag{2.6}$$

Expansion of kernels K_{nk}, L_{nk} looks similar, only $\varepsilon = 1$, $d_0 = 0$. Placing (2.5), (2.6) into systems (2.2), (2.3) and equating the expressions at equal λ—powers the recurrent procedure for determination of coefficients (2.5) is established.

It has been found that the closing of cracks affect neither the shear displacements u_{00}, u_{02}, u_k nor the transverse jump υ_{k1}. Their derivatives u'_{00}, u'_{02}, u'_{k1}, υ'_{k1} are determined by formulae (1.8) - (1.10), (1.3). The transverse displacement jump on the main crack is determined by υ'_{02}, including in the formula the unknown size of the open zone

$$\upsilon'_{02}(\chi) = \frac{\varepsilon S l_0}{2\sqrt{1 - \chi^2}}\sum_{k=1}^{N}[\mathrm{Re}J_k\mathrm{Im}(m^c_{0k1}(\chi) - n^c_{0k1}(\chi)) + \mathrm{Im}J_k\mathrm{Re}(m^c_{0k1}(\chi) + n^c_{0k1}(\chi))] \,, \tag{2.7}$$

where

$$m^c_{0k1} = \frac{e^{i\alpha_k}}{\varepsilon^2}\mathrm{Re}\left[\frac{(\delta_k\chi - 1)e^{i\alpha_k}}{(\chi - \delta_k)^2(\delta_k^2 - 1)^{1/2}}\right], \quad \delta_k = \frac{u_k - d_0}{\varepsilon} \,,$$

$$n^c_{0k1} = \frac{e^{-i\alpha_k}}{2\varepsilon^2}\left[(u_k - \bar{u}_k)e^{-i\alpha_k}\frac{-\chi^2 + 2\bar{\delta}_k^3\chi - 3\bar{\delta}_k^2 + 2}{\varepsilon\,(\chi - \bar{\delta}_k)^3\,(\bar{\delta}_k^2 - 1)^{3/2}} + \frac{(e^{i\alpha_k} - e^{-i\alpha_k})\,(\bar{\delta}_k\chi - 1)}{(\chi - \bar{\delta}_k)^2\,(\bar{\delta}_k^2 - 1)^{1/2}}\right] \tag{2.8}$$

J_k, m_{0k1}, n_{0k1} are determined by (1.9)

An expression for υ_{02} is obtained by direct integration of (2.7), taking into account (2.4).

$$\upsilon_{02}(\chi) = S l_0\sqrt{1 - \chi^2}\frac{1}{2}\sum_{k=1}^{N}[\mathrm{Re}J_k\mathrm{Im}(m^*_{0k1}(\chi) - n^*_{0k1}(\chi)) + \mathrm{Im}J_k\mathrm{Re}(m^*_{0k1}(\chi) + n^*_{0k1}(\chi))] \,, \tag{2.9}$$

$$m^{*}_{0k1} = \frac{e^{i\alpha_k}}{\varepsilon^2}\,\mathrm{Re}\left(\frac{e^{i\alpha_k}}{(\delta_k - \chi)\,(\delta_k^2 - 1)^{1/2}}\right),$$

$$n^{*}_{0k1} = \frac{e^{-i\alpha_k}}{2\varepsilon^2}\,\frac{1}{(\bar{\delta}_k - \chi)\sqrt{\bar{\delta}_k^2 - 1}}\left((u_k - \bar{u}_k)e^{-i\alpha_k}\frac{2\bar{\delta}_k^2 - \bar{\delta}_k\chi - 1}{\varepsilon(\bar{\delta}_k - \chi)\,(\bar{\delta}_k^2 - 1)} + e^{-i\alpha_k} - e^{i\alpha_k}\right).$$

In the case of closure of microcracks ($v_{k1} = 0$) Eq.(2.2) is used and $v_{k1} = 0$. Stress intensity factors are defined by the formula:

$$K^{\pm}_{I0} - iK^{\pm}_{II0} = \mp \lim_{\chi \to \pm 1}\left(\frac{1 - \chi^2}{l_0}\right)^{1/2}\left[v'_0(\chi) - iu'_0(\chi)\right] \qquad (2.10)$$

The value of K_{II0} is determined by u_0 and, therefore, it is not affected by the closure phenomenon, while the expression K_{I0} reveals its dependence on c and the contacting small cracks

$$K^{\pm}_{I0} = \lambda^2 S L_0 \frac{1}{4\varepsilon}\sum_{k=1}^{N}\Bigg\{-\mathrm{Re}(J_k)\mathrm{Im}\left[2e^{i\alpha_k}\mathrm{Re}\left(\frac{e^{i\alpha_k}}{(\delta_k \mp 1)\sqrt{\delta_k^2 - 1}}\right) + \frac{e^{-i\alpha_k}}{\varepsilon(\bar{\delta}_k \mp 1)\sqrt{\bar{\delta}_k^2 - 1}}\left((u_k - \bar{u}_k)e^{-i\alpha_k}\frac{2\bar{\delta}_k \pm 1}{\bar{\delta}_k^2 - 1} + e^{-i\alpha_k} - e^{i\alpha_k}\right)\right] +$$

$$+ \mathrm{Im}(J_k)\mathrm{Re}\left[-2e^{i\alpha_k}\mathrm{Re}\left(\frac{e^{i\alpha_k}}{(\delta_k \mp 1)\sqrt{\delta_k^2 - 1}}\right) + \frac{e^{-i\alpha_k}}{\varepsilon(\bar{\delta}_k \mp 1)\sqrt{\bar{\delta}_k^2 - 1}}\left((u_k - \bar{u}_k)e^{-i\alpha_k}\frac{2\bar{\delta}_k \pm 1}{\bar{\delta}_k^2 - 1} + e^{-i\alpha_k} - e^{i\alpha_k}\right)\right]\Bigg\}. \qquad (2.11)$$

The unknown parameter c is determined by the condition of the singularity absence in this point (Goldshtein et al 1992)

$$K_I(\pm c) = 0\,. \qquad (2.12)$$

Practically c is found by an iterative process using for the first approach the domain where no "overlapping" of crack edges is observed and then expanding it until (2.12) is fulfilled.

3. NUMERICAL RESULTS.

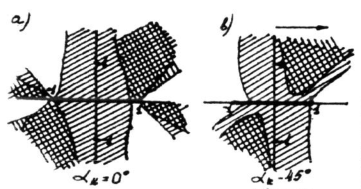

Fig. 2. Domains where the located microcrack evokes a partial (single shadow) or full (double shadow) closure of the main crack. a) — when slope angle $\alpha_k = 0°$, b) $\alpha_k = -45°$).

The domains where defects with different orientation cause a full or partial closure of the main crack is defined by (1.11) and presented in Fig. 2.

To estimate the effect of microcrack closure on the stress intensity factor, a numerical analysis is carried out for a single defect located on the y–axis with a distance of $0{,}6l_0$, $\alpha_k = 0°$, $\lambda = 0{,}1$. The value of K^{+}_{I} , disregarding the closure zone on the main crack, is $K^{+}_{I}/Sl_0^{1/2} = 0{,}13 \cdot 10^{-3}$, but taking into account the closure zone we obtain $K^{+}_{I}/Sl_0^{1/2} = 0{,}193 \cdot 10^{-3}$. (On the left side the closure zone expands up to the crack tip and $K^{-}_{I} = 0$.)

CONCLUSIONS

1. The boundary conditions are formulated and a solution method is elaborated for the macro- and microcrack interaction analysis by taking into account the possible crack closure.
2. It has been revealed that small cracks (microcracks) can have two modes - fully open and fully closed. Microcracks can cause full or partial closure of the main crack.
3. The length of contact zone on the main crack does not affect the domains where microcracks are closed.
4. The value of K_{II} at the main crack tip can be calculated disregarding the contact zone, i.e., by admission of the "overlapping" of crack edges. For a correct determination of K_I the presence of contact zone should be taken into account.

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