ANALYSIS OF A MULTIPLE CRACK BRANCHING IN NON-ISOTROPIC MATERIALS

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ABSTRACT

A non-isotropic fracture criterion, based on the Griffith law, has been proposed by Sanchez-Palencia and the author and applied to the following situation: even with very symmetric external loadings, a preexisting notch orthogonal to the fibers (or the cleavage direction) tends to kink and generates a crack propagating along this privileged fracture direction. However, in some specific cases of loadings, it can be observed that the kink stops and the crack restarts to propagate in the original notch direction. Unfortunately, the above mentioned analysis of the kink process cannot predict this restart phenomenon, then it is necessary to imagine new mechanisms. This paper is devoted to the description of such a process: the onset and growth of a 3-branch crack at the tip of a straight one.

KEYWORDS

Non-isotropic elastic materials, composite materials, brittle fracture.

INTRODUCTION

In classical brittle fracture mechanics [Bui, 1978], isotropy is invoked twice, first for the elastic behaviour and second for the fracture criterion. It seems rather natural to consider that non-isotropic elastic materials enjoy non-isotropic fractures modes [Leguillon & Sanchez, 1991]. A fibrous material such as wood is a good example, but this applies also to 1D fiber reinforced composites or cleavable materials. In a first step, it is necessary to recall some brief results of a previous paper [Leguillon, 1992]. It deals with a 1D fiber reinforced carbon/epoxy (T300/914) with the following non-zero 2D elastic moduli:

$$a_{1111} = 12.7 \ , \quad a_{1122} = 6.72 \ , \quad a_{2222} = 145. \ , \quad a_{1212} = 4.85 \quad \text{(Gpa)}$$

The fracture energy per unit length (in 2D elasticity) is denoted γ and takes 3 distinct values: γ_2 for fractures occuring along the fibers, $\gamma_1 > \gamma_2$ for fractures orthogonal to the fibers and $\gamma_3 >> \gamma_1 > \gamma_2$ in the other directions (to inhibit these remaining directions for simplicity, but it does not play an important role). Then, it is proved (see eqn.(6) below) that if $\gamma_2 < 0.09\ \gamma_1$, a classical singular mode 1 acting at the original notch tip triggers a kink along the fibers instead of a straight propagation. Depending on the non-singular part of the stress field around the initial notch tip, this kink can be stable (*i.e.* can grow only with increasing external loads) or unstable (at least at the microscopic level that is the scale where asymptotics are performed).

Nevertheless, whatever the kink length, if it stops, the above analysis is unable to predict the onset of a new crack in front of the original notch as observed [Goldstein, 1991]. It is the classical deficiency of the Griffith criterion: the energy release rate vanishes. A mechanism such as a 3-branch crack growth instead of a simple kink is a trial to offer an answer to this problem. Two cases are analyzed depending on the respective lengths of the branches. They are compared to the straight propagation and to the kinked one. We emphasize on the following point: in the next we discuss the conditions of existence of such processes, we do not presume of their real occurence, the 3 mechanisms are competing.

THE 3-BRANCHES MECHANISM

This study takes place within the matched asymptotic expansions method ([Van Dyke, 1964, Maz'ya & Nazarov, 1988]). Two states of a structure are examined: the unperturbed one corresponding to a body with a single straight notch (Fig.1) and the perturbed one including in addition a small defect with diameter ℓ at the tip of the original notch. The parameter ε is chosen as the ratio between ℓ and a characteristic length of the structure $L : \varepsilon = \ell/L << 1$ (L can be for instance the width of the specimen). Two symmetric mechanisms have been analyzed. In the first one the straight branch is by $\lambda < 1$ smaller than the kinked ones (whose respective lengths both equal ℓ) and in the second one it is the kinked branches which are by $\mu < 1$ smaller than the straight one (with length ℓ) (Fig.1).

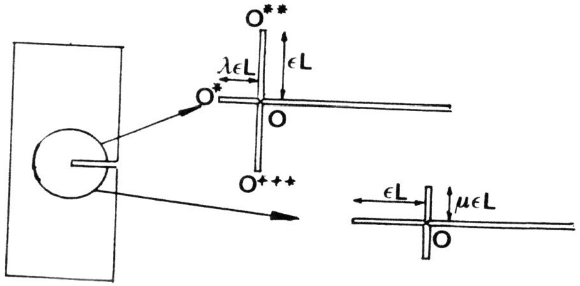

Fig.1. Geometry of the 3 – branch crack

The change in potential energy between these two states can be expanded as:

$$\delta W^{3b} = W^0 - W^\varepsilon = \varepsilon A_{pq} k_p k_q + O(\varepsilon\sqrt{\varepsilon}) \tag{1}$$

where the A_{pq}'s are constants depending on λ (or μ), computed by contour integrals in the inner problem of the matched asymptotic process. The k_p's are the stress intensity factors of the two classical fracture modes (more precisely the non-isotropic counterparts), they deal with the unperturbed state. In the case of symmetric external loadings implying a single mode 1 acting at the notch tip, (1) reduces to:

$$\delta W^{3b} = \varepsilon A_{11} k_1^2 + O(\varepsilon\sqrt{\varepsilon}) \tag{2}$$

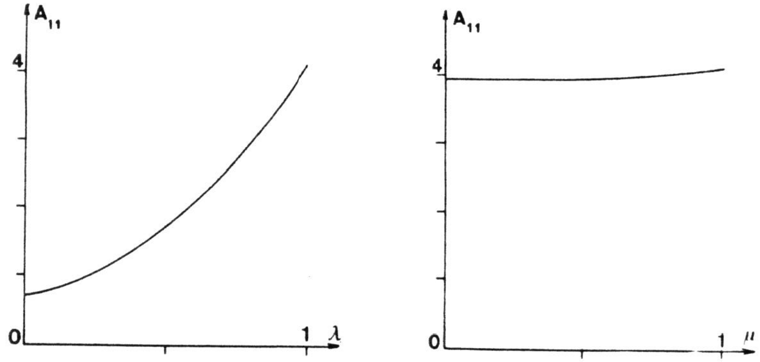

Fig.2. A_{11} vs λ and μ

In the following, the analysis will be restricted to the situation where the double-kink propagation is easier than the straight one . In these cases, the change in potential energy (2) can be written respectively as (3) and (4) for branches with length ℓ:

$$\delta W^{2b} = \varepsilon\ A_{11}^{\pi/2}\ k_1^2\ +\ O(\varepsilon\sqrt{\varepsilon}) \tag{3}$$

$$\delta W^{1b}\ =\ \varepsilon\ A_{11}^{\pi}\ k_1^2\ +\ O(\varepsilon^2) \tag{4}$$

$A_{11}^{\pi/2}$ and A_{11}^{π} are, as A_{11}, positive constants but independent of λ (or μ), computed by contour integrals in the inner problems corresponding to the respective geometries. The upper index denotes the angle between the new crack and the original notch. Moreover: $\delta W^{3b}(\lambda = 1)\ \neq\ \delta W^{1b}\ +\ \delta W^{2b}$. The Griffith thresholds which allow respectively the kinked propagation and the straight one are defined by:

$$k_1^2 \geq L\ \frac{2\ \gamma_2}{A_{11}^{\pi/2}}\ \ ;\ \ k_1^2 \geq L\ \frac{\gamma_1}{A_{11}^{\pi}} \tag{5}$$

and the condition to the kinked propagation to be easier than the straight one is therefore:

$$\gamma_2 \leq\ \frac{A_{11}^{\pi/2}}{2\ A_{11}^{\pi}}\ \gamma_1 \tag{6}$$

which brings to $\gamma_2 \leq 0.09\ \gamma_1$ for the composite material described in the introduction.

THE GRIFFITH CRITERION

First, let us study the mechanism involving λ instead of μ: the straight branch is shorter than the colateral ones. The energy dissipated during the 3-branch fracture process is $(\lambda\gamma_1 + 2\gamma_2)\ell$ and it is tempting to write down the Griffith criterion as:

$$\delta W^{3b} \ \geq\ (\lambda\ \gamma_1 + 2\ \gamma_2)\ell \ \Leftrightarrow\ k_1^2 \geq\ L\ \frac{\lambda\ \gamma_1 + 2\ \gamma_2}{A_{11}(\lambda)} \tag{7}$$

but this arises to be insufficient, the criterion must contain in addition the conditions allowing propagations in the 3 directions involved by the process. This can be done by considering the straight propagation of the 3 branches separately. For this purpose, let us consider a small increment $\delta\ell$ of ℓ and $\delta\varepsilon = \delta\ell/L << \varepsilon$, the lateral branches both change by $\delta\ell$ and the straight one only by $\lambda\delta\ell$. It leads to the two relations (8), by symmetry, the second one must be invoked twice at the tips of the two lateral branches:

$$\begin{cases} A_{11}^{*\pi}\ k_1^{*2} \geq\ L\ \gamma_1 \\ A_{11}^{**\pi/2}\ k_1^{**2} +\ A_{22}^{**\pi/2}\ k_2^{**2} \geq\ L\ \gamma_2 \end{cases} \tag{8}$$

$A_{11}^{*\pi} = A_{11}^{\pi}$, $A_{11}^{**\pi/2}$ and $A_{22}^{**\pi/2}$ are positive constants independent of λ and the coupling term $\left(A_{12}^{**\pi/2} +\ A_{21}^{**\pi/2}\right)$ is zero for straight propagations (as above all these constants can be computed by contour integrals). k_1^* and the k_p^{**}'s are the stress intensity factors of the two singular modes at O^* (by symmetry $k_2^* =\ 0$) and at O^{**} and O^{***} (Fig.1) (k_2^{**} takes opposite values on the two lateral branches but this does not play any role). These stress intensity factors can be expanded as [Leblond, 1989, Leguillon, 1992]:

$$\begin{cases} k_1^* &= F_{11}^{\pi} k_1 +\ O(\sqrt{\varepsilon}) \\ k_p^{**} &= F_{1p}^{\pi/2}\ k_1 +\ O(\sqrt{\varepsilon}) \end{cases} \tag{9}$$

where F_{11}^{π} and the $F_{1p}^{\pi/2}$'s are constants depending on λ and computed by contour integrals, they are the stress intensity factors of the modes 1 and 2 in the 3-branch inner problem at the tips of the branches. Replacing (9) into (8) leads to:

$$k_1^2 \geq\ L\ \frac{\gamma_1}{A_{11}^{\pi}\ \ (F_{11}^{\pi}(\lambda))^2} \tag{10}$$

$$k_1^2 \geq\ L\ \frac{\gamma_2}{A_{11}^{**\pi/2}\ \left(F_{11}^{\pi/2}(\lambda)\right)^2 +\ A_{22}^{**\pi/2}\ \left(F_{12}^{\pi/2}(\lambda)\right)^2} \tag{11}$$

On the other hand, assuming that for a small $\delta\varepsilon$ and $\varepsilon \geq \varepsilon_0 > 0$ the 3 processes are nearly independent, the corresponding change in potential energy (2) must equal the sum of the changes due to the 3 branches, by identification, it leads to:

$$A_{11}(\lambda)\ =\ \lambda\ A_{11}^{\pi}\,(F_{11}^{\pi}(\lambda))^2 +\ 2\ \left[A_{11}^{**\pi/2}\left(F_{11}^{\pi/2}(\lambda)\right)^2 +\ A_{22}^{**\pi/2}\left(F_{12}^{\pi/2}(\lambda)\right)^2\right] \tag{12}$$

As a particular case: $A_{11}(0) = A_{11}^{\pi/2}$. Using (12), (11) can be rewritten :

$$k_1^2 \geq\ L\ \frac{2\ \gamma_2}{A_{11}(\lambda)\ -\ \lambda\ A_{11}^{\pi}\,(F_{11}^{\pi}(\lambda))^2} \tag{13}$$

Finally, the Griffith criterion gathers the 3 inequalities (7), (10) and (13). For small λ, $F_{11}^{\pi}(\lambda)$ behaves like $\sqrt{\lambda}$ as $\lambda \to 0$, this is the behaviour of the stress intensity factors of a crack initiating on the straight edge of a homogeneous body. Thus (10) excludes solutions corresponding to small values of λ.

Some attention can be paid now to the second mechanism involving lateral branches shorter by $\mu < 1$ than the straight one. The equivalent to (12) reads:

$$A_{11}(\mu)\ =\ A_{11}^{\pi}\,(F_{11}^{\pi}(\mu))^2 +\ 2\ \mu\ \left[A_{11}^{**\pi/2}\left(F_{11}^{\pi/2}(\mu)\right)^2 +\ A_{22}^{**\pi/2}\left(F_{12}^{\pi/2}(\mu)\right)^2\right] \tag{14}$$

Then the 3 inequalities of the Griffith criterion become (7) and:

$$k_1^2 \geq\ L\ \frac{\gamma_1}{A_{11}^{\pi}\,(F_{11}^{\pi}(\mu))^2} \tag{15}$$

$$k_1^2 \geq\ L\ \frac{2\ \mu\ \gamma_2}{A_{11}(\mu)\ -\ A_{11}^{\pi}\,(F_{11}^{\pi}(\mu))^2} \tag{16}$$

In (14), $F_{11}^{\pi/2}(\mu)$ and $F_{12}^{\pi/2}(\mu)$ are the stress intensity factors of the fracture modes 1 and 2 at the tip of the lateral branches. As already mentioned for $F_{11}^{\pi}(\lambda)$ and for similar reasons, these terms tend to zero like $\sqrt{\mu}$ as $\mu \to 0$. Thus, the right hand side of the second inequality (16) tends to infinity like $1/\mu$, inhibiting any mechanism involving small values of μ.

As a consequence, the 3-branch mechanism is meaningful only for branch lengths of the same order of magnitude.

STABILITY

Stability means here that a system of cracks can propagate but must stop for lengths remaining small and inside the validity framework of asymptotic expansions. A new growth of these cracks requires an additional external load. On the other hand, unstability means that the Griffith criterion is more and more violated as the system of cracks grows and then that a stop is unpredictable at the microscopic level.

Considering only the first mechanism (the second one leads to similar conclusions), the analysis requires an additional term in the expansions:

$$\delta W^{3b} =\ \varepsilon\ A_{11} k_1^2 +\ \varepsilon\sqrt{\varepsilon}\ B_1 k_1 T\ +\ O(\varepsilon^2) \tag{17}$$

B_1, as A_{11}, is a positive constant depending on λ and computed by a contour integral. T is the stress intensity factor of the non-singular part of the stresses, that is the tension parallel to the original notch. From (17), it is easy to derive that, if $T\ >\ 0$ then the

Griffith criterion is more and more violated once it has been reached. On the other hand, if $T < 0$, then the criterion holds true untill:

$$\sqrt{\varepsilon} \leq \frac{A_{11}(\lambda)\, k_1^2 - L\,(\lambda\, \gamma_1 + 2\, \gamma_2)}{B_1(\lambda)\, k_1 \mid T \mid} \tag{18}$$

But, one must not omit that the criterion has now 3 parts, two of them expressing the ability for the cracks to propagate separately. The change in potential energy due to a small increment $\delta\varepsilon$ is:

$$\delta W^{\varepsilon} = \left(A_{11}(\lambda)\, k_1^2 + \frac{3}{2}\sqrt{\varepsilon}\, B_1(\lambda)\, k_1 T\right) \delta\varepsilon + O(\varepsilon)\, \delta\varepsilon \tag{19}$$

It involves the energy release rate: the derivative of δW^{3b} with respect to ε (or to ℓ). The same change in potential energy due to this increment but computed in the 3 branches separately is:

$$\delta W^{\varepsilon} = \left[\lambda\, A_{11}^{\pi} k_1^{*2} + 2\left(A_{11}^{**\pi/2}\, k_1^{**2} + A_{22}^{**\pi/2}\, k_2^{**2}\right)\right] \delta\varepsilon + O(\delta\varepsilon^2) \tag{20}$$

This result is partly due to the fact that the equivalent terms to B_1 vanish in the straight propagations [Leguillon, 1992], otherwise the remainder would be $O(\delta\varepsilon\sqrt{\delta\varepsilon})$. Considering an additional term in the expansions of the stress intensity factors gives:

$$\begin{cases} k_1^{*} = F_{11}^{\pi} k_1 + \sqrt{\varepsilon}\, G_1^{\pi}\, T + O(\varepsilon) \\ k_p^{**} = F_{1p}^{\pi/2}\, k_1 + \sqrt{\varepsilon}\, G_p^{\pi/2}\, T + O(\varepsilon) \end{cases} \tag{21}$$

where G_1^{π} and the $G_p^{\pi/2}$'s are constants depending on λ and computed by contour integrals. Replacing (21) into (20) and identifying in (19) yield (12) and:

$$\begin{aligned} \frac{3}{2}\, B_1(\lambda) = {} & 2\,\lambda\, A_{11}^{\pi} F_{11}^{\pi} G_1^{\pi}(\lambda) + 4\,\Big(A_{11}^{**\pi/2} F_{11}^{\pi/2}(\lambda)\, G_1^{\pi/2}(\lambda) + \\ & A_{22}^{**\pi/2} F_{12}^{\pi/2}(\lambda)\, G_2^{\pi/2}(\lambda)\Big) \end{aligned} \tag{22}$$

The important point to derive from above is that, since it is numerically checked that $F_{11}^{\pi}(\lambda) \geq 0$ and $G_1^{\pi}(\lambda) \leq 0$:

$$\begin{cases} A_{11}^{\pi}\, F_{11}^{\pi}(\lambda)\, G_1^{\pi}(\lambda) \leq 0 \\ 4\,\left(A_{11}^{**\pi/2} F_{11}^{\pi/2}(\lambda)\, G_1^{\pi/2}(\lambda) + A_{22}^{**\pi/2} F_{12}^{\pi/2}(\lambda)\, G_2^{\pi/2}(\lambda)\right) = \\ \frac{3}{2}\, B_1(\lambda) - 2\,\lambda\, A_{11}^{\pi} F_{11}^{\pi} G_1^{\pi}(\lambda) \geq 0 \end{cases} \tag{23}$$

The additional terms to the two mechanisms takes opposite signs, one of the two processes is necessarily unstable. Moreover, depending on the sign of T, if the propagation is more and more activated in one system, then it decreases in the other. As a consequence, the 3-branch mechanism can develop only with unstable conditions.

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