A UNIFIED MODEL OF BRITTLE MODE I FRACTURE IN TENSION AND IN COMPRESSION

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ABSTRACT

The so-called ρ-model of brittle mode I fracture is elaborated and extended. It takes into account the influence of the crack transverse dimensions and residual stresses near the crack front in an unloaded body on its fracture. New interpretations have been offered of such basic notions as a crack parameter and a fracture parameter. Through the use of a central crack in a polymethylmethacrylate plate as an example, it is shown what experimental data and in what way enable one to evaluate those parameters. The gap between ρ-model and classical fracture models has been bridged by incorporations of the following notions: an effective cut $l(k)$, an effective stress intensity factor $K_I(k)$ and the initial value K_{Iu} of the $K_I(k)$ parameter. Unlike their traditional analogues, the $l(k)$ and $K_I(k)$ parameters depend on the state of the material tested and on the ratio of loads applied along and across the crack plane.

KEY WORDS

Brittle fracture, tension, compression, biaxial loading, crack parameters, fracture parameters.

INTRODUCTION

Failures of brittle materials in compression and in tension are usually investigated from different standpoints and independently of each other. In the first case the continuum mechanics approaches are mostly used, while in the second the fracture mechanics ones are extensively applied. This unnatural gap originates from the fundamental Griffith (1920, 1924) publications. The need for some general theory had long been felt and was continuously becoming more urgent. To judge by the situation to date, no coherent and complete theory of such type has yet emerged. The main obstacle here is the absence of a unified fracture model which would make it feasible to apply the same type of analysis to mode I crack behaviour in tension and in compression under uniaxial and multiaxial loading.

In classical fracture models a real crack with mismatched surfaces in an unloaded body is usually replaced by a volumeless mathematical cut. Thus possible influence of the crack transverse dimensions and residual stresses at its front on the fracture are eliminated from the subsequent analysis. Naumenko (1987, 1991a) advanced a unified model of brittle fracture (ρ-model) according to which a neglect of the initial crack state parameters can be justified only in the case of macrocrack behaviour evaluation in uniaxial tension or bending. Yet in the case of multiaxial loading by tension and/or compression the above replacement may prove to be unacceptable at all. Recent evidence has focused attention on the initial crack state and the results presented below point out that corresponding parameters must be incorporated in general theory of brittle fracture.

THE BASIC NOTIONS OF THE UNIFIED FRACTURE MODEL

Characteristic Crack States. For a nonpropogating central crack in a biaxially loaded linearly-elastic plate (Fig.1) they are

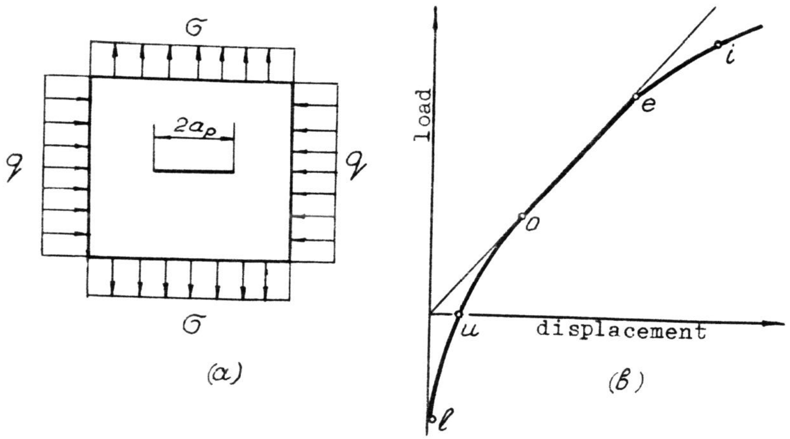

Fig. 1. A plate with a through crack (a) and typical diagram "load vs crack face displacement at its end" (b).

designated by the following indices: "u" is the initial state at σ=q=o; "o" is the instant when the residual stress effect near the crack front is vanished; "e" is the moment of transition from elastic to elasto-plastic deformation of the

plate; "i" is the crack growth onset instant and "l" is the state of the complete crack faces contact in a compressed body. Of key importance are the data referring to the states "u" and "o" which will be considered from the following standpoint. The crack propagating in a brittle material is instantly arrested by the complete plate unloading when the crack attained prescribed size 2 a_u (Fig.2). Prior to and after arrest the crack is a cavity formed by two surfaces symmetrical with respect to

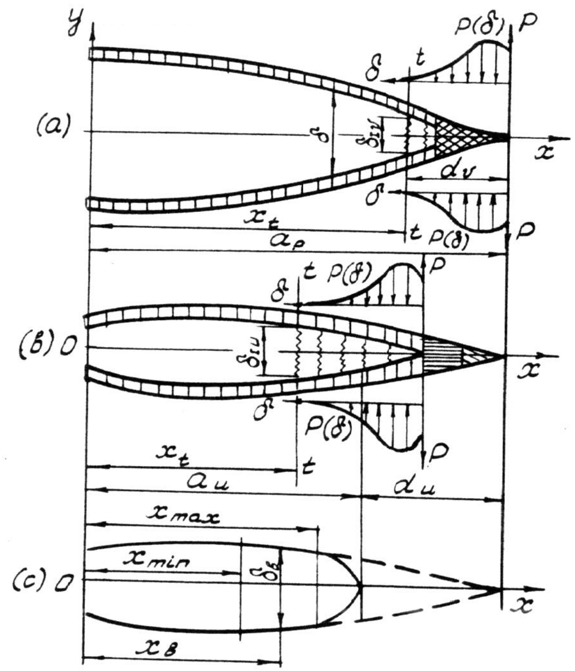

Fig. 2. Supposed behaviour of an isolated through mode I crack at the stage of its growth with a velocity v (a), after unloading (b) and the procedure of experimental determination of the physical crack length (c): - zone of inelastic deformation and fracture; - a layer of damaged material; - zone of partially remained bonds; - compression zone; - elastically deformed material.

the xoz plane. In the state "u" the crack surfaces cannot come into a complete contact due to physico-chemical material transformations in the fracture process zones. Residual stresses arising after unloading in combination with the crack dimensions and type predetermine its initial profile. For instance, a sufficiently long central through crack can have dumbbell like shape (Naumenko 1987, 1991a).

Crack Parameters. For the ρ-model these are (Fig.2): a_u - half length of a visually identified central crack in the unloaded plate; a_p - the crack physical half length; ρ - the minimum curvature radius of the equivalent elliptical hole vertices; d - the hole end region length; and K_{1u} - initial stress intensity factor value. For brittle materials a visible site of the crack end may be situated both behind ($a_u > x_t$) and in front ($a_u < x_t$) of the crack tip. The position of the latter is shown by the tt line which bounds the unloaded part of the crack surfaces. The rest of the crack surface belongs to the end region where the cohesive forces p(δ) are acting. Its upper bound ($x=a_p$) corresponds to the crack physical end. A ralatively simple procedure of an unambiguous determination of the end region boundaries can be developed if the crack tip position is assumed to coincide with the visible crack end (Fig.2c).

Let us consider the procedure of the determination of parameters K_{Iu} and ρ through the use as an example of a central crack in a plate of optically transparent model material. This is polymethylmethacrylate (PMMA) which has the following characteristics in the laboratory conditions: elasticity modulus E=2.88 GPa; Poisson's ratio ν=0.41; ultimate strength in tension σ_u=71.4 MPa. Using the technique of combined loading of a specimen by tension, transverse compression and impact wedge indentation a short precrack was grown (a_u=0.19W, where 2W is the plate width). After optical interferometric determination of the crack initial profile, δ(x), in the specimen midsection, the crack was extended. The cycle "crack opening measurement - crack extension" had been repeated ten times until the crack length a_u=0.69W was reached. (The experiments were performed together with A.E.Elagin). Then by the extrapolation of the profile elliptical portion ($x_{min} \leqslant x \leqslant x_{max}$), as is shown in Fig.2c, the values a_u, a_p were determined and using the known relationships for a strip with a central cut the equivalent stresses σ_u and the corresponding magnitude of $K_{Iu} \simeq 3.83 \times 10^{-2}$ MPa $\sqrt{m}$ were calculated. Here σ_u are the tensile stresses which ensure the similarity of the cut profile and the crack profile at $x_{min} \leqslant x \leqslant x_{max}$. Let us replace an isolated crack by some elliptical hole of equal length. The radius ρ of its vertices is defined so that at any biaxial stress ratio $k=q/\sigma$ for the plate considered (Fig.1) the corresponding ρ_0 value at the instant "o" is constant. Characteristic stresses σ_0 and q_0 (Fig.1b) for a uniaxially loaded PMMA plate with a short crack were defined experimentally. The corresponding value of ρ equal to 12.5 μm must be regarded as tentative until more extensive

data for an isolated crack (a_p<0.1W) are available.

Fracture Parameters. These are: $\mathcal{J}_I$ is the potential energy release rate at an infinitesimal increment of the equivalent elliptical hole dimension $a = a_p$; δ_I is the opening displacement and α_I is the opening displacement angle of the hole tips. For some characteristic state "h" of the plate with an isolated crack at plane strain (Fig.1) the fracture parameters are:

$$\mathcal{J}_{Ih}(k)=\pi[\sigma_h(k)]^2[a+0.5\rho\, k^2+0.375\sqrt{\rho a}(1-k)^2](1-\nu^2)/E; \quad (1)$$

$$\delta_{Ih}(k)=2(b_h-b)[1-(x_t/a_h)^2]^{1/2}; \quad (2)$$

$$\alpha_{Ih}(k)=2\ \mathrm{arctg}[x_t(b_h-b)/a_h(a_h^2-x_t^2)^{1/2}], \quad (3)$$

where a and $b = \rho\, a$ are the major and minor semi-axes of the equivalent ellipse; a_h and b_h are the deformed ellipse semi-axes. In the problem considered the biaxiality ratio k varies within the limits $k^- \leqslant k \leqslant 0$ were $k^- = q_h/\sigma_u$. The lower bound k^- corresponds to the uniaxial compression case and depends on the crack length since

$$\sigma_u \approx K_{Iu}/\left[\pi\left(a_p+0.375\sqrt{\rho\, a_p}\right)\right]^{1/2}.$$

The values δ_I, α_I and $d = a-x_t$, where d is the length of the ellipse end region, have a definite physical meaning only when one of them is determined experimentally. Naumenko and Mitchenko (1991) provided conclusive evidence that for PMMA fracture the angle α_I can be employed as a fracture parameter, whose characteristic values are independent of the k ratio. The corresponding material characteristics $\alpha_{IO}=2.5\cdot10^{-3}$, $\alpha_{Ii}=11.5\cdot10^{-3}$ and formula (3) were used for the calculation of the hole tips positions x_t and the following fracture parameters values: $\delta_{IO}(0)$=0.6 μm; $\delta_{IO}(k^-)$=0.26 μm; $\delta_{Ii}(0)$=3.6 μm; $\delta_{Ii}(k^-)$=3.3 μm; $\mathcal{J}_{IO}(0)$=2 J/m^2; $\mathcal{J}_{IO}(k^-)$=78.6 J/m^2; $\mathcal{J}_{Ii}(0)$= 111.9 J/m^2; $\mathcal{J}_{Ii}(k^-)$=4.3 kJ/m^2.

THE GRIFFITH-IRWIN (GI) MODEL

Relation (1) and the equivalence of Griffith and Irwin approaches allow to incorporate the notions of an effective mathematical cut

$$l(k)=[a+0.5\ \rho\ k^2+0.375\sqrt{\rho\ a}(1-k)^2]\ E/[E-(1-2\ \nu^2)\ \sigma_u] \quad (4)$$

and an effective stress intensity factor

$$K_I(k)=\sigma[\pi l(k)]^{1/2}=[\vartheta_I(k)\cdot E/(1-\nu^2)]^{1/2}. \qquad (5)$$

The nonequality of the $\vartheta_{Ih}(k^-)$ and $\vartheta_{Ih}(0)$ magnitudes indicates that characteristic values of parameter $K_I(k)$ are dependent on the k ratio. This dependence can be defined if we use the experimental results of the PMMA fracture investigation under biaxial tension-compression loading Pisarenko et al. (1981). They allow to write as the first approximation:

$$\sigma_h(k)=\sigma_h(0)/\{1-k[\sigma_u-\sigma_h(0)]/k^-\cdot\sigma_u\}, \qquad (6)$$

which is valid at $k^- \leqslant k \leqslant 0$. On the curve obtained using (5) and (6) (Fig.3) two portions of the relative constancy of the

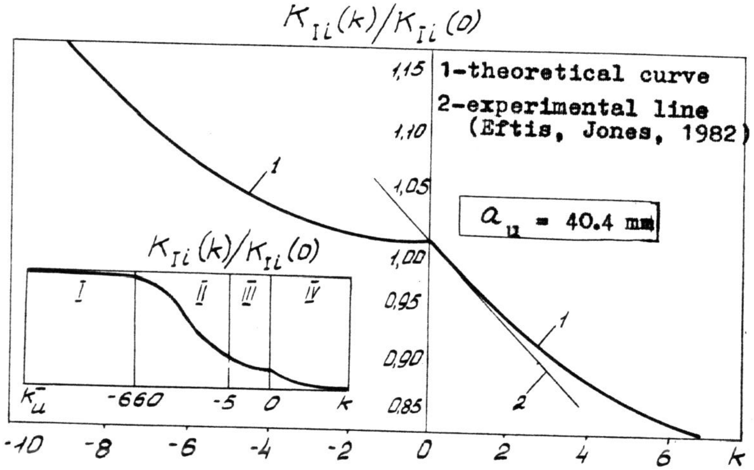

Fig. 3. The crack onset $K_I(k)$ value variation with load biaxiality ratio (general view is shown separately).

$K_I(k)$ values at the crack growth onset are shown: I$(-1224 \leqslant k < -660)$ and III$(-5 < k \leqslant 0)$. Within them the deviation from the nearly constant values of $K_{Ii}(k^-)=3.84$ MPa$\sqrt{m}$ and $K_{Ii}(0)= 0.62$ MPa$\sqrt{m}$ does not exceed 5 percent. Portion IV of the curve shows the experimental data from the paper of Eftis, Jones (1982) and the results of calculations by eq.(5) based on those data.

THE BARENBLATT-LEONOV-PANASYUK-DUGDALE (BLPD) MODEL

To extend the limits of applicability of the classical fracture

mechanics approaches the parameters of the BLPD-model, as well as those of the GI-model are defined on the basis of the unified fracture model. The crack is replaced by an effective cut $2l(k)$ and the positions of its tips x_t are determined so that the characteristic magnitudes of the crack tip opening displacement δ_{Ih} are equal to the value obtained earlier with the ρ-model. Therefore, the length of the end region $d_h(k)$ and the cohesion forces $p_h(k)$ must satisfy the expressions:

$$d_h(k) = \pi K_{Ih}^2(k)/8\, p_h^{\,2}(k); \qquad p_h(k) = (1-\nu^2)K_{Ih}^2(k)/E\,\delta_{Ih}(k).$$

Assuming that at $k \to k^-$ the $\delta_i(k)$ value decreases linearly, and at $k \geqslant 0$: $\delta_i(k) = \delta_i(0)$, for a PMMA plate we shall get dependences shown in Fig.4. For the case of uniaxial loading we

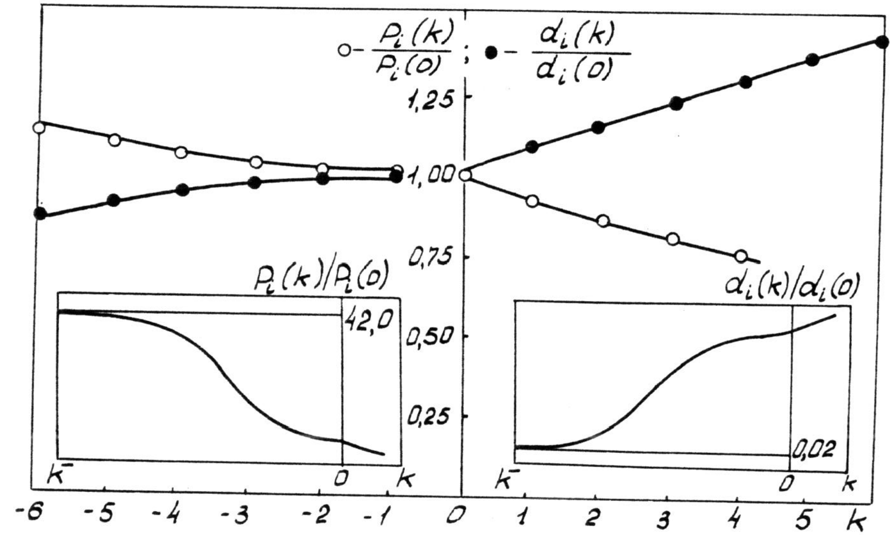

Fig.4. Dependences of the crack onset end region parameters $p_i(k)$ and $d_i(k)$ on the load biaxiality for a plate in Fig.1a (a general view is shown separately).

have: $d_i(0)= 157.9$ μm; $p_i(0)= 31$ MPa; $d_i(k^-)= 3.4$ μm and $p_i(k^-)= 1303$ MPa. The order of the $p_i(k)$ magnitudes agrees with the hypothesis of Williams (1980) on the existence of three levels of critical fracture stress for polymers. For PMMA at 20°C they are: $\sigma_I = 2500$, $\sigma_{II} = 550$ and $\sigma_{III} = 130$ MPa. At the same time, the scale of the $d_i(k)$ value changes from the magnitude comparable to the craze length to the size of the area of single fracture act in a craze. Apparently, with a decrease in the k ratio the following fracture mechanisms dominate in succession:

craze formation (σ_{III}), local failure of molecular bonds of van der Waals bonds type (σ_{II}) and breaking of the main bonds (σ_I).

CONCLUSION

The unified fracture model enables one to solve the problems of mode I crack propagation in tension and in compression under uniaxial and biaxial loading on the basis of the same initial hypothesis. This fact allows to eliminate the unnatural gap between the available approaches to the investigation of those phenomena and extends the limits of efficient applicability of the classic fracture mechanics theories. However, interrelated usage of the ρ, GI and BLPD-models is connected with the necessity to overcome a number of obstacles. One of the most complicated of them is the correct determination of the compressive load q_h and the corresponding profiles of an isolated crack. In a laboratory experiment, considerable deviations from the crack isolation conditions and the prescribed uniform field of compressive stresses are almost unavoidable. On the other hand, the experience reveals that those deviations are often accompanied by unacceptably great quantitative and sometimes qualitative alterations in the crack profile. According to Naumenko (1991b) in the case of compression the extension of the region, where a free surface of the through hole influences appreciably the crack fracture process zone, can be an order of magnitude larger than that in tension.

REFERENCES

Eftis,J. and Jones,D.L. (1982) Influence of load biaxiality on the fracture load of center cracked sheets. Int.J.Fract., 20 1, 17-26.
Griffith,A.A. (1920) The phenomena of rupture and flow in solids. Phil.Trans.Roy.Soc., London, Ser.A, 221, 163-198.
Griffith,A.A. (1924) The theory of rupture, in Proc. 1-st Int. Congr.Appl.Mech. Delft, pp. 55-63.
Naumenko,V.P., Mitchenko,O.V. (1991) Deformation parameters of mode I fracture in tension and in compression. (in Russion) Problem Prochn. 10, 26-31.
Naumenko,V.P. (1987) Brittle fracture and strength of materials in compression and in tension. (in Russion) Preprint.Inst. for Problems of Strength, Ukr.Ac.Sci., Kiev, Ukraine, 38p.
Naumenko,V.P. (1991a) Modelling of brittle fracture in tension and compression. In: Fracture Processes in Concrete, Rock and Ceramics (J.G.M. van Mier, J.G.Rots and A.Bakker), 1, 183-192. E & F.N.Spon, London.
Naumenko,V.P. (1991b) Nucleation and growth of a macrocrack in compression and in tension. (in Russion) Fiziko-khimiches-kaya mekhanika materialov, 27, 5, 62-67.
Pisarenko,G.S., Naumenko,V.P., Koval,V.I. (1981) Method of the experimental investigation of fracture under biaxial loading. (in Russion) Probl. Prochnosti 12, 3, 5-9.
Williams,J.G. (1980) Modelling crack tip failure mechanisms in polymers. Metal Science, 14, 8 and 9, 344-350.