A PROBABILISTIC CRITERIA OF HETEROGENEOUS AND DEFECT MATERIALS FRACTURE

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ABSTRACT

A probabilistic approach to strength and reliability evaluation of stochastically defect and heterogeneous (composite) materials under the complex stress state is introduced. On the basis of the defects parameters probability distributions and deterministic conditions of fracture the fracture loading probabilistic characteristics were obtained and the boundary state diagrams were built. The calculations were made for the complex, plane and axially symmetric loading and different model material representations with cracks or alien inclusions.

KEYWORDS

Strength, probability of fracture, criteria, crack, inclusion, probability distribution, heterogeneity.

INTRODUCTION

In the process of solid bodies strength loss and fracture the heterogeneities of their structure, the defects of different type and origin presented and developed in structure play the important role. The heterogeneity and presence of defects may be of different structural level and correspondingly may be taken into account obviously or by the averaged material characteristics. In the microlevel the sources of fracture can be the cavities, slots, cracks, scratchs, alien inclusions of different stiffness and shape, which cause the considerable stress concentration near them under bodies loading. The real strength of material depends on the type, size, location and quantity (density) of such heterogeneities. The fracture mechanics study the influence of separate heterogeneities with deterministic characteristics (for example cracks, alien inclusions) on the body limiting endurance. It establishes the deterministic conditions of crack nucleation and propagation in dependence on the physico-chemical (cohesive) and elastic properties, geometry of body, inclusions and defects and also on the external influences on it. But in real bodies and service conditions the indicated structural and service characteristics are changeable, having a certain degree of accidentation (stochastition). Specifically, the stochastition of mechanical and structural parameters, which described the type, sizes, location and quantity of inclusions and defects are very essential. Because of this, the strength of materials and resourse (particularly brittle with high fracture localization) and structures from them are accidental values with definite probability distribution. The probability prediction of such bodies strength properties and their fracture criteria is the actual theoretical problem, which has the great practical significance. Until now, in the most approaches to probability-statistical strength theory creation (from the Weibull's works) heterogeneities and defects in solid body structure obviously were not regarded. The calculation of fracture probability and probability characteristics of a body strength was taken on the basis of the given from logical and experimental consideration of strength limits distributions and (or) body elements stress level, which only satisfactoryly considered the material structural stochasticity. With fracture mechanics development (specifically with cracks theory) the works on the statistical theory strength, which obviously considerated the heterogeneities and defects availability in material structure were published. A review on different approaches can be found in the work (Vitvitskiy and Popina, 1980). In this paper the principles of theory are stated and the examples of strength probability and brittle fracture criteria development, based on the deterministic fracture mechanics and probability-statistical methods are given.

ALGORITHM OF FRACTURE CRITERIA BUILDING

We shall proceed from the material calculation model as the elastic continuum (matrix), in which the accidental by shape, sizes and location defects-cracks and (or) alien inclusions with different from matrix mechanical properties are distributed. Such defects and heterogeneities, which are surrounded by closed matrix, formed the ensemble of heterogeneous elementary particles with one defect or inclusion and formed the material (body) array (Fig.1). We consider the defects and inclusion sizes to be so small, that they don't interact between each other and don't change essentially the stresses at the distance from them, that under the external homogeneous of a body or its part loading, it will be the same for all elements. We shall refer the various defects and inclusions, which differ so, that require for their describing different determined parameters to heterogeneities of different types (for example internal and surface cracks, plate alien inclusions etc.). The heterogeneities parameters of the same type may differ only by its value, which is described by the determined probability distribution . We shall indicate geometrical and

mechanical parameters of determined type heterogeneities by a_i (i=1,2,...n), n - number of the given type of parameters. They can characterize the environment resistance to crack nucleation or propagation, inclusions and cracks configuration, size and orientation. Let us suppose, that for given material the functions of joint probability distribution of parameters $F(a_1,a_2,\ldots,a_n)$ or joint probability density $f(a_1,a_2,\ldots,a_n)$ are connected by the relationship

$$f(a_1,a_2,\ldots,a_n)=\frac{\partial^n F(a_1,a_2,\ldots,a_n)}{\partial a_1 \partial a_2 \ldots \partial a_n}.$$

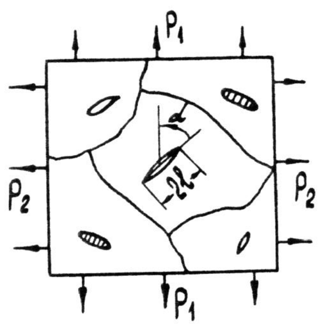

Fig.1 Graphical presentation of loading

In the case of stochastical independence of parameters a_i their joint distribution is equal to the product of each parameters distributions $f(a_1,a_2,\ldots,a_n)=$ $=f_1(a_1)f_2(a_2)\ldots f_n(a_n)$. The form of these functions differ from the structure and material technology manufacturing. The distribution of each parameter separately can be determined on the basis of statistical processing empirical data or from the common phenomenological assumptions. If the material contains the heterogeneities of different types, we shall assume the distributions of determined parameters of each type to be known. The location of heterogeneities of each type density over the volume (or on the surface) of a body we shall take to be uniform. The average number N^0 of each type heterogeneities per unit of a body volume V_0 we consider as known. We regard the common algorithm of described body (material) fracture probability and limit stress probability characteristics determination and probability fracture criteria under the complex homogeneous stress state, characterized by principal stresses p_1, p_2, p_3. At first we regard the material with defects of the same type (for example with identical cracks), and later the approach we shall generalize for the material, which contains the heterogeneities of different type. Let us consider on the basis of deterministic problem solution the condition of boundary state body element which contains one defect

$$p_1 = \varphi(a_1,a_2,\ldots,a_n,\eta,\xi); \quad p_2=\eta p_1, \quad p_3=\xi p_1, \qquad (1)$$

where p_1, p_2, p_3 - the boundary values of principal stresses, η and ξ - the characteristics of stress state complexity. Under the fixed η and ξ the loading is characterized by one parameter p_1. Since the determined parameters a_i are accidental values with known probabilistic distribution $f(a_1,a_2,..,a_n)$ than the values of boundary stresses p_1 characteristics are also the accidental values, which change within limits from p_{1min} to p_{1max}, which by the way depend from the parameters η and ξ. The probability distribution function of boundary stresses p_1 for elements with one defect we find by the formula for distribution function probabilities from the accidental values

$$F_1(p_1,\eta,\xi) = \int\limits_{\varphi(a_1,a_2,\dots,a_n,\eta,\xi)\leqslant p_1} \cdots \int f(a_1,a_2,\dots,a_n)da_1da_2\dots da_n \ . \quad (2)$$

The function $F_1(p_1,\eta,\xi)$ can be interpreted as the distribution of material elements strength limits for the determined stress field. From the formula it is seen that $F_1(p_1)$ depends both on the function $f(a_1,a_2,\dots,a_n)$ which characterized the structure and material properties and the parameters of stress state complexity η and ξ. In most approaches, in which the defects and heterogeneities are not obviously accounted, the distribution $F_1(p_1)$ is assumed and given, in this case its dependence on the type of stress state is not always realized. In considered approach the function $F_1(p_1,\eta,\xi)$ is determined theoretically under the arbitrary stress state. Let us regard the body (its part) with size V (V may indicate volume, area, length). If some unit of volume V_0 contains in average N^0 defected elements, the body with size V contains in average $N=N^0V/V_0$. In brittle materials under the homogeneous stress state the fracture of one defect element may cause the global fracture af all body. Quite secure are such stresses, which don't cause the propagation of any defect in it. More simple solution we obtain in assumption of noninteraction of defects between each other. In this case the boundary loading of body is the same with the boundary loading of its element with the lowest strength. Then the distribution function of boundary stresses for body which consists from N elements is determined by formula (Vytvytskiy and Popina, 1980)

$$F_V(p_1,\eta,\xi) = 1-[1-F_1(p_1,\eta,\xi)]^{N^0V/V_0} \ . \quad (3)$$

Under the fixed values p_1, η and ξ the value of function $F_V(p_1,\eta,\xi)$ gives the probability P of body fracture under the given stress field effect

$$P_V(p_1,\eta,\xi) = F_V(p_1,\eta,\xi). \quad (4)$$

The determination of this probability is one of the main problems of calculation. Under the great $N\to\infty$ the distribution $F_V(p_1,\eta,\xi)$ transform to the distribution of Weibull's type

$$F_V(p_1,\eta,\xi) = 1-\exp\left[-\frac{V}{V_0}N^0C(p_1-p_{1min})^m\right] \ . \quad (5)$$

Contrary to Weibull's approach, which take the magnitudes m and C as constans of material, they are determined from

$$C = \lim_{p_1\to p_{1min}} F_1(p_1,\eta,\xi)(p_1-p_{1min})^{-m} \ , \quad (6)$$

where the relationship of C and m with the structure characteristics and type of stress state is obviously seen. The distribution (5) is the generalization of Weibull's distribution on the complex stress state. If the body contains the heterogeneities of different type, which don't interact between each other, then in this case the generalized distribution function of boundary stresses has the form

$$F_V(p_1,\eta,\xi) = 1 - \prod_{j=1}^{k}(1-F_{1j})^{VN_j^0/V_0} \ . \quad (7)$$

Under the great quantity of heterogeneities

$$F_V(p_1,\eta,\xi) = 1-\exp\left[-\sum_{j=1}^{k}\frac{V}{V_0}N_j^0C_j(p_1-p_{1min})^m\right]. \quad (8)$$

There the index j determines the corresponding values, which belong to the j -type ($j=1,2,\dots,k$) heterogeneity. Having the function F_V you can find the number of boundary loading statistical characteristics for size V body: the mean and the most probability value of boundary loading; the values, which correspond to the given fracture probability; variance and spread in boundary loading ratio etc. For example, the mean value $\langle p_1\rangle$ we obtain according to formula

$$\langle p_1\rangle = p_{1jmin} + \int_{p_{1jmin}}^{p_{1jmax}}\prod_{j=1}^{k}(1-F_{1j})^{VN_j^0/V_0}dp_1 . \quad (9)$$

The equalities

$$\langle p_1\rangle = p_0g_1(\eta,\xi,N_j^0V/V_0), \quad \langle p_2\rangle=\eta\langle p_1\rangle, \quad \langle p_3\rangle=\xi\langle p_1\rangle \quad (10)$$

jointly determine the boundary state body criteria under the complex stress state.

APPLICATION TO DIFFERENT BODY (MATERIAL) MODELS

A Plane Model Body Under the Biaxial Stress State. The isotropical plate with scattering rectilinear cracks of accidental length $2l$ and orientation angle α is under the action of tensile or compressive stresses p_1 and $p_2=\eta p_1$. The crack orientation for isotropical material is equiprobable. Therefore the angle crack distribution $f_1(\alpha)=1/\pi$ ($|\alpha|\leqslant\pi/2$).

For simplification let us suppose that cracks presence probability follows to zero under the nonlimited great length of a crack. The value l is distributed by the extently decreased law $f_2(l)=(r-1)l_1(l+l_1)^{-s}$ $(0\leqslant l\leqslant\infty)$; where l_1 is scale parameter, r characterizes the structure homogeneity (with r increase the probability of great cracks decrease). The joint distribution of defects parameters

$$f(\alpha,l) = (r-1)l_1/[\pi(l+l_1)^r] \qquad (0\leqslant l\leqslant\infty,\ |\alpha|\leqslant\pi/2). \qquad (11)$$

The deterministic criteria of isolate crack propagation under the stress field p_1 and $p_2=\eta p_1$ take into account the change of its propagation direction

$$p_1 = \frac{K_{Ic}}{\sqrt{\pi l}}\varphi_1(\alpha,\eta,\theta_*,\rho), \quad 0\leqslant p\leqslant\infty , \qquad (12)$$

where K_{Ic} is the stress intensity factor, ρ is the ratio of closed cracks faces friction, θ_* is the angle of initial deviation of crack propagation direction. The function φ_1 is obtained in the work (Panasyuk and Berezhnitskiy, 1964). On the basis of relationships (2), (11), and (12) we obtained the boundary state distribution function F_1 for the body element with one crack

$$F_1(p_1,\eta) = (l_1p_1^2)^{r-1}\int_L\frac{1}{\pi}\left[\frac{K_{Ic}^2}{\pi}\varphi_1^2(\alpha,\eta,\theta_*,\rho)+l_1p_1^2\right]^{1-r}d\alpha, \qquad (13)$$

where L is the permissible values area under the given p_1 and η. We obtained the boundary stress distribution function (2) for the element with one crack. On the basis of formula (9) the boundary stress state criteria was built in the mean values of fracture stresses (Fig.2). The solid lines correspond to the obtained solution (Vytvytskiy and Popina,1980) and section lines to the criteria, which don't take into account the chane of crack propagation direction. Under great number of cracks the accounting of crack's propagation direction don't change the type of fracture criteria diagram, with the exception of the field of stress compression advantage.

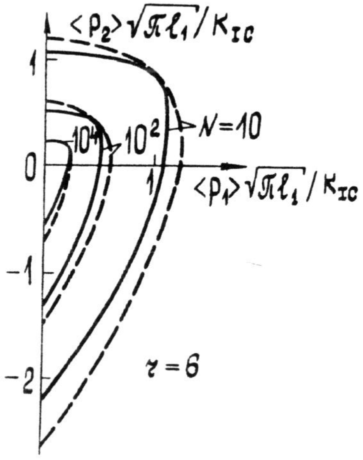

Fig.2 Boundary state diagram $(\rho=0.2)$

Three-dimensional Model. The three-dimensional body is under the action of homogeneous axially symmetric loading. The plane round cracks with accidental radius R and orienation are uniformly distributed over the volume. The crack orientation in such field of stresses is described by one angle α $(0\leqslant\alpha\leqslant\pi/2)$ between the crack normal and axis of loading symmetry. The probability of crack presentation with angle greater than α is $P=1-\cos\alpha$. Then the probability distribution density $f_1(\alpha)=\sin\alpha$. The crack radius is limited $(0\leqslant R\leqslant d)$ and is subjected to the β-distribution $f_2(R) = (r+1)(1-R/d)^r d^{-1}$ $(r\geqslant0)$ (the case of radius distribution by the exponential law was considered by Fisher and Hollomon (1947)). The joint distribution density

$$f(\alpha,r) = \sin\alpha(r+1)(1-R/d)^r/d \qquad (0\leqslant\alpha\leqslant\pi/2, 0\leqslant R\leqslant d) . \qquad (14)$$

The fracture condition we shall take in Sack form (Sack, 1946). The boundary stress has a form $(K_{Ic}\sqrt{d}/2\leqslant p_1<\infty)$

$$p_1 = \varphi(R,\alpha,\eta) = \begin{cases} \dfrac{K_{Ic}\sqrt{\pi R}}{2(\cos^2\alpha+\eta\sin^2\alpha)}, & \sigma_n = p_1(\cos^2\alpha + \eta\sin^2\alpha)\geqslant 0, \\ \infty, & \sigma_n\leqslant 0 ; \end{cases} \qquad (15)$$

The distribution function for body elements with one crack

$$F_1(p_1,\eta) = \iint_{\varphi(R,\alpha,\eta)\leqslant p_1}\sin\alpha\frac{r+1}{d}(1-R/d)^r d\alpha dR \qquad (16)$$

We obtained the function $F_1(p_1,\eta)$ for different stress type. In Fig.3 the fracture criteria are shown. The section-point line corresponds to the minimum fracture loading. For great number of cracks the results coincide with the obtained by the generalized Weibull's distribution (5). The body strength decrease with stress state complexity. The material heterogeneity increasing under the other equal conditions goes to the boundary stresses lowering.

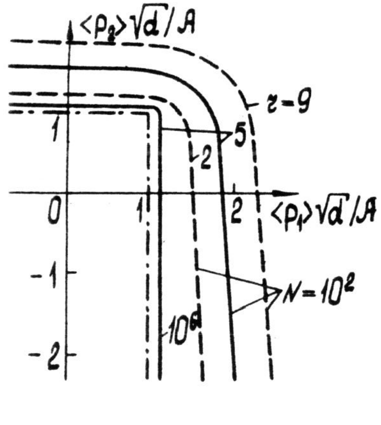

Fig.3 Fracture diagram $(A=\sqrt{\pi}K_{1c}/2)$

A Heterogeneous Plane Model. The elliptical alien inclusions (a and b are their semi-axes) from another material are distributed in the elastic matrix (the elastic constants G_1

and $æ_1$ differ from the constants of matrix G_2 and $æ_2$). We shall concider the inclusions to be soft ($G_1/G_2<<1$) and flatten ($b/a=\delta<<1$). The inclusion length $2a$ and angle of orientation α are accidental values with distribution density $f(\alpha,a)=f_1(\alpha)f_2(a)=(r+1)(1-a/a_m)^r/(a_m\pi)$ $(-\pi/2\leqslant\alpha\leqslant\pi/2,\ 0\leqslant a\leqslant a_m,\ r\geqslant 0)$. We consider, that the fracure began in inclusion. The homogeneous stress state arises in inclusion and the most dangerous is the cracks formation over the whole length. We took The Kulon friction law type as a fracture criteria (Cherepanow, 1983)

$$\tau^1_{xy} = K^1 - \mathrm{tg}\rho^1\sigma^1_y , \tag{17}$$

where K^1 is the engagement ratio, ρ^1 is the friction ratio, τ^1_{xy}, σ^1_y are the tangential and normal stresses in inclusion. The calculations were made by the described algorithm. In Fig.4 the diagrams of inclusion fracture stress mean values for the body with different number of inclusions (for $r=1$, $G_1/G_2=0.1$, $æ_1 = æ_2 = 2$, $\rho^1=0.2$) are given. For the strength determination we should compare the boundary stresses for the matrix with cracks, formed on the inclusions place. The former we obtain as it is aforesaid.

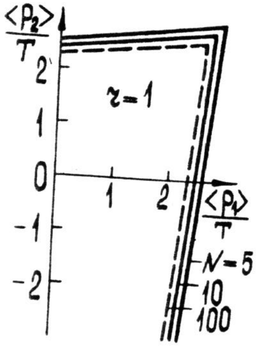

Fig.4 Fracture digram ($T=4K^1/(1+æ_2)$)

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