A NUMERICAL ELASTIC-PLASTIC APPROACH OF THE CRACK INITIATION PROBLEM

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ABSTRACT

An attempt is presented to describe failure conditions for pre-cracked materials in terms of a pair of classical mechanical quantities independent of specimen geometry and loading conditions. It is concluded that critical values of elastic dilatational and distortional strain energy densities can undertake this role in place of critical stress intensity factors. Experimental results show excellent agreement with theoretical predictions. The variation of critical stress intensity factors from plane stress to plane strain conditions is, also, explained in terms of these geometry-invariant quantities.

INTRODUCTION

Historically most of Fracture Mechanics criteria available for the prediction of the conditions necessary for crack initiation, are based directly or not on the concept of Critical Stress Intensity Factors (SIFs). This concept is attractive in the sense that mathematically advantageous linear elasticity considerations can be introduced but, raises fatal questions on the rationality of fracture criteria. Really:

i) Critical value of SIF for the same material under the same loading conditions (e.g. uniaxial tension) varies from K_{IC} in case of plane strain conditions to $K_C \cong 3K_{IC}$ in case of plane stress. This experimental observation implies that either failure properties of the materials depend on specimen geometry (its thickness) or that K_{IC} is not a failure property. If the first alternative is followed then the development of fracture criteria is impossible, a highly undesired situation. Consequently, the abolition of SIFs from their place in fracture criteria seems to be compulsory. The introduction of elastic-plastic stress and strain fields does not heal the situation since plastic strains and infinite stresses, necessary for the definition of stress intensity, are mutually exclusive concepts.

ii) The peculiar behaviour of SIFs, when they serve as critical failure quantities, still remains even in case of plane stress or plane strain states alone. In plane stress states the classical configuration being used for the study of fracture criteria is that of the thin plate containing a straight crack inclined by an angle β to the axis of the uniaxial tensile stress σ_o. It is theoretically predicted by all fracture criteria, e.g. [1,2], that in order for any function of SIFs to keep a constant value versus crack inlination β, an unrealistically high fracture stress for small β-values is required [3]. Indeed, for $\beta<10^o$, the plate can undertake higher stresses when precracked rather than uncracked. Rationally, one can conclude that materials show better performance when they contain cracks slightly inclined to the load axis!

In the present study an attempt is presented to overcome the above described difficulties in a simple and consise way not based on SIFs. For that the T-criterion of failure [4,5], is applied in the case of a typical aluminium alloy to see whether or not a mechanical quantity keeps constant value versus crack inclination β without necessitating unrealistically high fracture stresses or changing from plane stress to plane strain conditions.

THEORETICAL CONSIDERATIONS

General aspects

It is commonly accepted that stress state of a material affect its behaviour at failure. The fact that fracture in the most cases is preceded by varying degrees of plastic deformation, can be considered as the result of the competition between the failure by flow (rupture, shear band deformation) and "tensile" separation of atomic bonds (creation of new surfaces). In what follows, when we refer to material failure we mean the failure of an elementary volume and not the failure of a structure (even in the simple case of a single bar tensioned uniaxially), which is a sequence of an infinite number of failures. It is well established experimentally that there is a great number of modes of failure expanding from brittle fracture (cleavage or intergranular) without macroscopic yielding and rupture with a 100 percent reduction of the area as a result of plastic deformation processes (which may be considered as the extreme case of ductile fracture). Any criterion that is used to predict failure must take into account these extreme modes and describe them with the minimum number of suitable parameters.

In Continuum Mechanics we accept that mechanical work is stored into the material in two separate components of strain energy vis. to change the volume (dilatational strain energy) and to change the shape of the specimen (distortional strain energy). It is apparent that the ratio between these two energies depends on the mechanical properties of the material, the specimen geometry and the loading system. An energy failure criterion presupposes that there is enough available energy to support a failure condition and this failure condition takes place when the material exausts its capability to absorb further energy. Based on this rationale the T-criterion [4] states that a material:

i) fails by fracture when the dilatational strain energy density T_V takes a critical value $T_{V,O}$ and,

ii) fails by yielding (rupture) when the distortional strain energy density T_D takes a critical value $T_{V,O}$.

It is apparent that these two critical values $T_{V,O}$ and $T_{D,O}$ are material properties and also that only the elastic parts of the abovementioned strain energy quantities must be compared with these critical values since plastic work, being irreversibly consumed is not available to cause failure.

Mathematical Formulation

In the case of isotropically hardening materials obeying the Mises yield condition and the associated flow rule the total increment of strain energy dT_O, in the plastic region, is given by:

$$dT_O=pd\theta+\bar{\sigma}d\bar{\varepsilon}+dW_p=dT_V+dT_D+dW_p \tag{1}$$

where:

$$dT_V=pd\theta=1/3\ (\sigma_{ii})(d\varepsilon_{kk}) \tag{2}$$

is the increment of the energy for volume changes due to hydrostatic pressure (dilatational strain energy) which is elastic, by the assumption of the equivoluminal changes in strains

$$dT_D=\bar{\sigma}d\bar{\varepsilon}=s_{ij}de^e_{ij} \tag{3}$$

is the elastic increment of the energy for shape changes (distortional strain energy), s_{ij} being the deviatoric stress tensor and de^e_{ij} the deviatoric elastic strain increment tensor and

$$dW_p=\sigma_{ij}d\varepsilon^p_{ij} \tag{4}$$

is the plastic work dissipated for permanent shape changes as the material is loaded to subsequent yield surfaces in the plastic region and vanishes in the elastic region or in the case of unloading.

The elastic quantities dT_V and dT_D can be integrated over the load-path and the integrals:

$$T_V=\int pd\theta=(1/2K)p^2 \qquad T_D=\int\bar{\sigma}d\bar{\varepsilon}=(1/6G)\bar{\sigma}^2 \tag{5}$$

are the dilatational and the distortional strain energy densities respectively (K being the bulk modulus and G the shear modulus). It is apparent that the curves p-θ and $\bar{\sigma}$-$\bar{\varepsilon}$ are fundamental for the description of the behaviour of the material and that the geometry of the specimen and the external load system only affect the rate which these curves are traced with. The terminal points of these curves are, according to the T-criterion [5], the failure points and the mode of failure depends on which of these two points is reached first. When the dilatational strain energy density T_V gets equal to the critical value $T_{V,O}$, which is a material property and expresses the ability of the material to bear volume changes, we have failure by fracture and analogously failure by yielding happens when the distortional strain energy density T_D reaches the respective capacity of the material $T_{D,O}$.

In the simple case of linear elastic isotropic hardening materials the abovementioned two conditions represent, in the stress space, a closed failure surface which consists of a cylinder with radius $2\sqrt{GT_{D,O}}$ and its axis coinciding with the hydrostatic pressure axis (representing rupture events) and two planes cutting normally the hydrostatic axis at a distance $\sqrt{6KT_{V,O}}$ from the origin of the axis (representing fracture events).

APPLICATION AND RESULTS

In order to check the validity of the previously established theoretical remarks and see whether a mechanical quantity, (necessary for the description of a failure condition), exists and remains unchanged for various geometries we apply the T-criterion to a well known aluminium alloy 7075/T6, whose mechanical properties are: modulus of elasticity E=75000 Mpa, Poisson's ratio $v=0.32$ and yield stress $\sigma_y= 549$ Mpa. In Fracture Mechanics it is common to obtain different geometries using precracked plates with various angles of inclination β with respect to the external load axis (Fig. 1). Any failure criterion has to predict the critical external load and the angle of crack initiation θ_o.

There is a series of experiments [6] which give the critical external load for crack initiation versus the inclination angle β for aluminium 7075/T6, and the results are shown in Fig 2. The specimens used were precracked plates with dimensions 90X200 mm, with thickness 2 mm and length of initial crack a=15 mm.

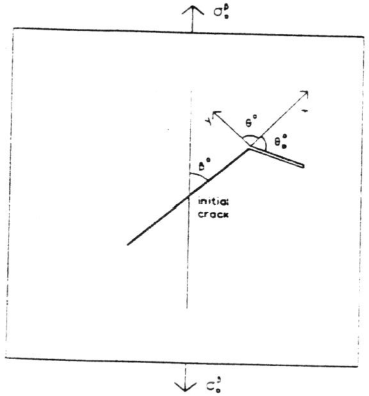

Fig.1 Geometry of the problem.

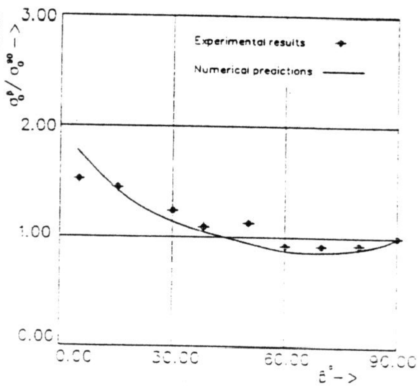

Fig.2 Critical external load for fracture vs. crack iclination.

In order to find the stress and strain fields in the plastic region we use the well-known finite elements programs PAPST and ADINA. The mesh had the same dimensions with the specimens in the experiments and consisted of 34 12-node quadratical isoparametric elements with 306 nodes for the case of $\beta=90^\circ$ (taking into account the symmetries of the geometry of the specimen and loading) and of 48 elements with 418 nodes in the cases of the rest angles of inclination β. Knowing from the experiments the critical load for crack initiation in the case of $\beta=90^\circ$ we can calculate everywhere in the mesh the dilatational and distortional strain energy densities T_V and T_D. In Figs. 3 and 4 the distribution of these energies is drawn around the crack tip.The curves of T_D are the well-known ones attaining a maximum value at $\theta \cong 70^\circ$. We notice that T_V reaches a maximum in the direction $\theta=0$ (where the crack

is expected to initiate) and this value is according to the T-criterion the possible critical value $T_{V,0}$, for brittle fracture to be observed. We know from the experiments the critical external load for the geometry of $\beta=90^\circ$,so we can define $T_{V,0}$.In the sequence we work with various angles β, increasing the external load until at some point in the mesh the quantity T_V reaches this critical value ,where, by definition, this external load σ_o^β is the critical one for crack initiation and the specific point, if joined with the crack tip, defines the angle of crack initiation θ_o. The theoretical predictions are drawn in the same Fig. 2 and the coincidence with experimental results is satisfactory.

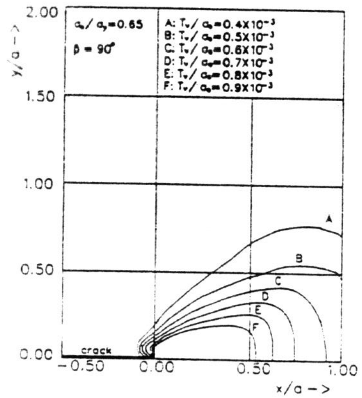

Fig.3 Distribution of dilatational strain energy density around the crack tip

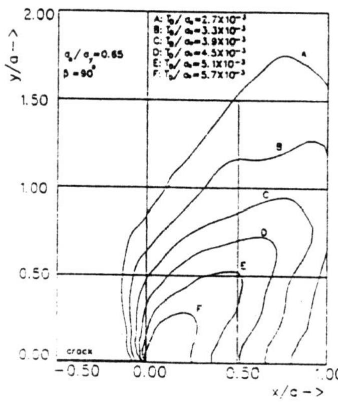

Fig.4 Distribution of distortional strain energy density around the crack tip

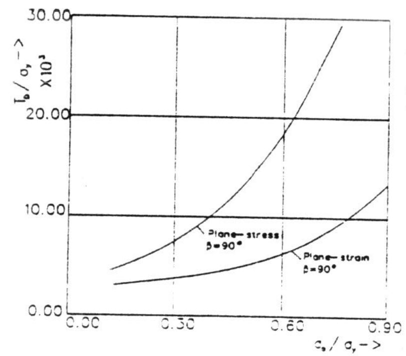

Fig.5 Distortional strain energy as a function of external load in plane stress and plane strain conditions

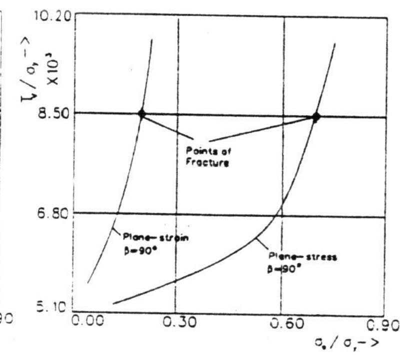

Fig.6 Dilatational strain energy as a function of external load in plane stress and plane strain conditions

At this verse two points must be emphasized. First, a global minimum in the theoretical curve exists at $\beta \cong 60^{\circ}$ in accordance with some of the "elastic" fracture criteria and the experimental data presented here. Second, the expected values of σ_0^{β} for $\beta<15^{\circ}$ are much lower than those predicted by other criteria and they fit well with the experimental results.

Next, we examine the influence of the thickness of the specimen in the distribution of strain energy densities around the crack tip. In a thin specimen the plastic zone extends a distance which is comparable to its thickness, the deformation around the tip is accompanied by a latteral thinning and the type of fracture is of the shear type. As the thickness is increased the type of fracture resembles the tensile one and the fracture surface is flat in the center of the specimen, where plane-strain deformation exists. The plastic-zone size is smaller because the stress condition is triaxial rather than biaxial (as it is in the plane-stress condition) and the critical external load is lower than that in the case of the thin specimen.

All the abovementioned remarks can be easily and simply explained if we apply the T-criterion to plane stress and plane strain conditions for the same material (keeping the same geometry except of the thickness of the specimen). In Fig. 5 the distribution of the maximum distortional strain energy density in the direction $\theta=0$ is plotted. It is obvious that in the case of plane stress conditions this maximum attains greater values, for the same externally applied load, than in the case of plane strain and it explains the larger extension of the plastic zone as well as the failure by rupture since T_D gets easier equal to $T_{D,0}$. On the contrary, as it is shown in Fig. 6 the maximum dilatational strain energy density is greater in the case of plane strain conditions and the critical external load for fracture in this case is approximately the one third of that in the plane stress.

This observation drives to the most important conclusion for Fig. 6. Namely it explains the variation of critical SIF from plane stress to plane strain conditions. It is generally accepted experimentally that this quantity increases from a base value K_{IC} measured at plane strain to $K_C \cong 3K_{IC}$ or more at plane stress [7]. This increase in critical SIF is observed because the crack initiates when and only when T_V takes a value independent from specimen thickness. For example assuming that $T_{V,0}/\sigma_y=8.50$ in Fig. 6, the necessary external load for crack to initiate is $\sigma_0/\sigma_y \cong 0.26$ in case of plane strain and $\sigma_0/\sigma_y \cong 0.72$ for plane stress, corresponding to $K_{IC}=0.26\sigma_y\sqrt{\pi a}$ and $K_C \cong 0.72\sigma_y\sqrt{\pi a}$ and $K_C/K_{IC} \cong 2.77$ although for $T_{V,0}/\sigma_y=6.80$ it is obtained that $K_C/K_{IC} \cong 4.40$. The same is concluded when crack inclination β takes various values. Consequently, **in case of fracture there exists a mechanical quantity ($T_{V,0}$) remaining constant and independent of specimen geometry.**

The role of the dilatational strain energy density in fracture is further supported by the curves in Fig. 7 where the ratio T_V/T_D of the **elastic** components of the total strain energy density is plotted versus reduced load at infinity σ_0/σ_y, at various distances x/a ahead the crack. For large distances (x/a=5) it remains constant not affected by the presence of the crack. For smaller distances it keeps a roughly constant value independent of the type of the material which varies from purely linear elastic to elastic-perfectly plastic, implying that only elastic quantities are equally active for fracture regardless the type of the material.

It is experimentally verified that the ratio of the critical external loads $\sigma_0^{\beta}/\sigma_0^{90}$ for small values of crack inclination β, ($\beta<15^{\circ}$), is much greater in the case of brittle materials than ductile ones. If we take as a measure of ductility of a material the ratio $\varepsilon_f/\varepsilon_y$ (where ε_f is the equivalent strain at fracture and ε_y the first yield strain) the variation of the critical external loads for fracture $\sigma_0^{5}/\sigma_0^{90}$ vs. the ductility of an hypothetical material, is shown in Fig. 8, assuming various values of $T_{V,0}$. It is concluded that for more ductile materials the ratio for of critical load for $\beta=5^{\circ}$ to the respective quantitty for $\beta=90^{\circ}$ reduces considerably in accordance with experimental evidence.

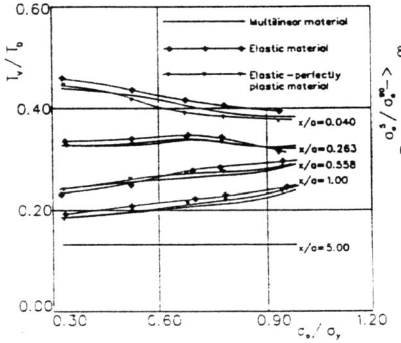

Fig.7 The ratio T_V/T_D as a function of the external load

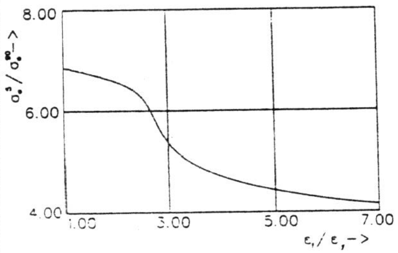

Fig.8 The ratio of critical external loads $\sigma_0^{5}/\sigma_0^{90}$ as a function of the ductility of the materials

CONCLUSIONS

It was shown in the present work that there exist a mechanical quantity, the dilatational strain energy density , which at the moment of initiation of a crack reaches a maximum value $T_{V,0}$ independent of geometrical factors like specimen thickness and crack inclination. Consequently, it can serve as the critical quantity for the development of a rational fracture criterion. Such a criterion cannot be based on the concept of stress intensity factors to the degree they depend on specimen geometry which by no means can be regarded as a fracture property of materials.

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