

A CRACK PROPAGATION IN RESIDUAL MICROSTRESS FIELD SIMULATION

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ABSTRACT

Results of studies of the problem of crack propagation in high gradient residual microstress field are given. As a model of a body structure a lattice representation was chosen. In crack propagation examination the influence of following factors was considered: relation of residual stress to the external stress parameters, applied stress orientation towards lattice parameters, the variation of residual stress field. The problem was considered as quasistatic. An infinite system of particles balance equations was solved by means of discrete Fourier transforms and using Green's functions.

KEYWORDS

Crack path, residual microstresses, lattice model, crosspieces formation, crack propagation discrete model

THE PROBLEM STATEMENT

The problem of crack propagation in residual stress field received a widespread discussion. In general, the low gradient residual stresses and crack propagation in such stress field in welded structures are studied for years (Kanazawa, 1973; Mahnenko, 1976; Margolin, 1982). Meanwhile the problem of crack growth in high gradient residual microstress field was not the subject of due attention. As shown in the following, when the values of stress variation scale and crack tip advances are assumed as quantities of the same scale, the residual microstress presence causes the microscopic changes of crack path direction, discrete character of crack growth and crack surfaces roughness.

As a structure model for studies, the lattice was chosen as the most simple one with definite destruction quantum, which does not require a special failure criterion, for which a provisional stresses level was considered. The lattice represents a discrete periodical system, which consists of absolutely rigid masses - the particles disposed in the knots of infinite net. The particles interact with each other by means of linear non-inertial elastic springs of unit length and rigidity.

The following problems were considered:

- the possibility of crosspieces formation and destruction for the case of straight-forward crack front;
- the influence on the crack propagation of following factors in the mode I and mode III cases: relation of the residual stress to the external stress parameters, applied stress orientation towards lattice parameters, the variation scale of the sine-type residual stress field.

For the mode III case a square lattice was considered and for mode I case it was an equilateral triangular lattice.

METHOD OF PROBLEM SOLUTION

For the problem studies, it had been used the mathematical apparatus, proposed and discussed in (Slepyan and Yakovlev, 1980; Slepyan, 1990). The problem was considered as quasistatic, when the crack propagation rate is of a value typical to that in viscous media. The solution of particles displacements evaluation problem in case of couple tensile unit forces application to a spring between two particles, was regarded as a fundamental one. The fundamental solutions have been obtained by examination of mass balance for undamaged deformed lattice. The mass balance equations in vectorial form in Cartesian coordinates system are expressed as follows:

$$\sum_{k=0}^{M-1} Q_k(\bar{X}) \bar{I}_k = -P(\bar{X}) \quad (1)$$

where M is a particle interaction springs number, $P(\bar{X})$ is the vector of external forces, Q is an internal force, applied to a given mass with vector

\bar{X} coordinates along a unit vector \bar{I}_k , directed along the spring with 'k' neighbouring particle.

An infinite system of balance equations for lattice particles can be solved by means of discrete Fourier transforms. For instance, in the mode I case the particle displacement form under Fourier transforms on X and Y is given by:

$$\bar{U}^{ff}(q, s) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \bar{U}(q, \sqrt{3}/2n) e^{i\sqrt{3}/2sn} \quad (2)$$

where $\bar{U}(q, y) = \sum_{m=-\infty}^{\infty} \bar{U}(X) e^{iqm}$; m, n are intact values of lattice particle coordinates: $x=m$, $y=\sqrt{3}/2n$.

In this way, the infinite equation system reduces to only two equations relative to \bar{U} denotions on axes X and Y: \bar{U}_1^{ff} and \bar{U}_2^{ff} correspondingly. For mode I case, one obtains:

$$\left. \begin{aligned} (1+4\text{Sin}^2 q/2 - \text{Cos} q/2 \text{Cos} \sqrt{3}/2s) \bar{U}_1^{ff} + 3\text{Sin} q/2 \text{Sin} \sqrt{3}/2s \bar{U}_2^{ff} &= P_1^{ff} \\ 3\text{Sin} q/2 \text{Sin} \sqrt{3}/2s \bar{U}_1^{ff} + 3(1 - \text{Cos} q/2 \text{Cos} \sqrt{3}/2s) \bar{U}_2^{ff} &= 0 \end{aligned} \right\} \quad (3)$$

where $P_1^{ff} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} P_1 e^{iqm} e^{i\sqrt{3}/2sn} = e^{-1}$.

The fundamental solutions values are obtained by integrating of the system. The standart solution for continuous media was used as the asymptotic for the fundamental solutions.

The displacements of lattice particles in the presence of broken springs are determined by means of superposition principle and Green's functions, for which the obtained fundamental solutions are taken. Spring failure means that couple forces, which completely compensates the forces from this spring side, is applied to the particles. The particle balance equation for one 'k' broken spring is expressed as follows:

$$\sum_{k=0}^{M-1} Q_k(\bar{X}) \bar{I}_k = -R(\bar{X}) \bar{I}_k \quad (4)$$

where $R(\bar{X}) = [-\bar{U}(\bar{X} + \bar{I}_k) + \bar{U}(\bar{X})] \bar{I}_k - S - B$, S and B are residual and external stresses values applied to 'k' spring.

In the case of several broken springs, the equation system relative to displacement differences of particles, between which springs have been broken, was solved; then the particles displacements in lattice knots all around the chosen field, where broken springs are disposed, were determined, and strains

of all springs in this field were determined as well. The crack path was obtained from the supposition, that the most strained springs are broken simultaneously if equally stressed.

RESULTS, SUMMARY AND CONCLUSIONS

The most interesting of obtained studies results are following:
 - the cases of crack propagation under the selfbalanced residual stress field only. Different versions of sine-type residual stress field were investigated: for mode I and mode III cases problems with different values of strain variation scale DL. For the case, when strain variation scale is equal to lattice cell (DL=1), fields of formed microcracks for mode III and I problems are shown in Fig.1 and Fig.2 respectively.

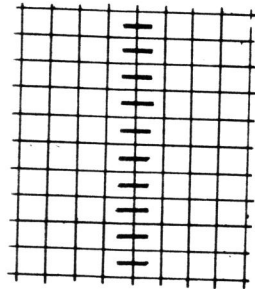


Fig.1 Crack propagation under residual stresses for mode III problem.

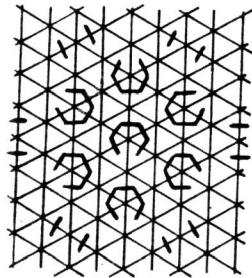


Fig.2 Crack propagation under residual stresses for mode I problem.

For the purpose of comparison with the previous results:
 - the cases of crack growth for modes III and I versions under the external stress field application were studied. The results are given in fig.3 and Fig.4.

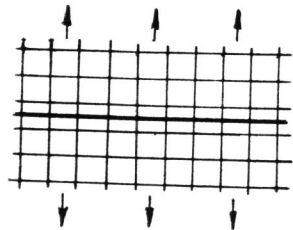


Fig.3 Crack path under external stress field for mode III problem (DL=1)

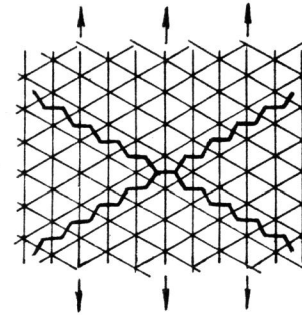


Fig.4 Crack path under external stress field for mode I problem (DL=1)

As shown on Fig.4 lattice geometry can predetermine the preferable direction of the crack path. The case of crack propagation under complete residual and external stress fields was examined also. The kinetics of crack propagation at certain moments for mode III problem is illustrated on Fig.5a,b,c and d, where relation between residual stress and external stress levels $S / S_0 = 0.5$; DL=2.

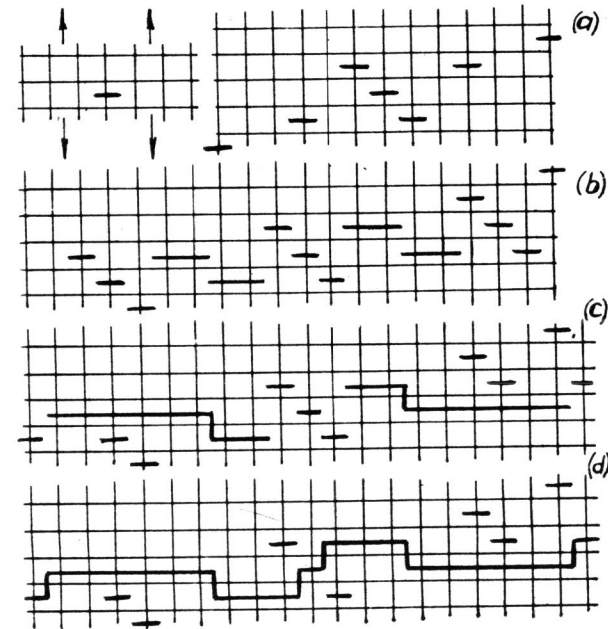


Fig.5. Kinetics of crack propagation for mode III case.

The crosspieces formation and sporadic crack growth are displayed in this figure. The kinetics of crack propagation for mode I case for $S/S = 0.5; DL=1$ is shown in Fig.6a,b,c and d.

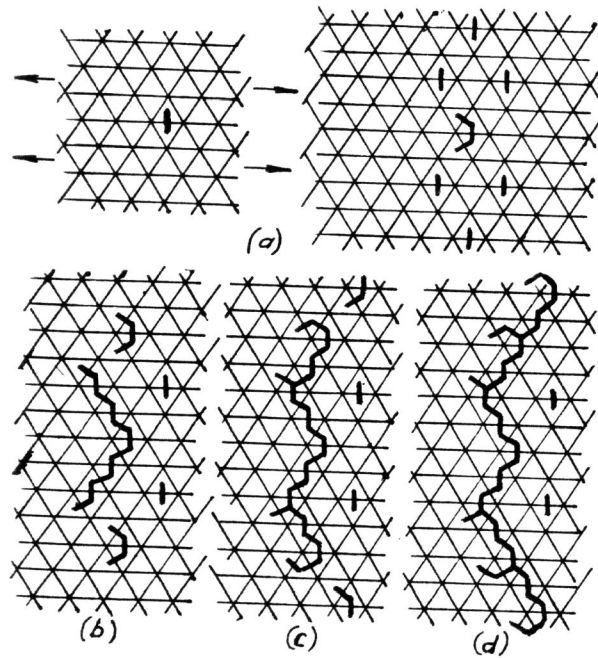


Fig.6 Kinetics of crack propagation for the mode I case.

The increase of the influence of the residual stresses for the crack growth in the same case by increasing the relation value between residual stress and external stress levels S/S is observed. It can be seen in Fig.7.

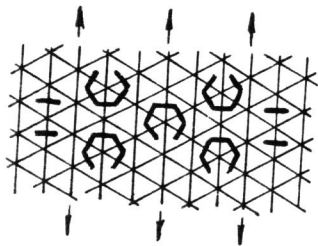


Fig.7. Crack propagation under complete residual and external stresses for mode I case. ($DL=1; S/S=2$).

Finally, the orientation of external stress field and its correlation with geometry of the lattice in mode I case also influences the crack path direction, as it can be seen in Fig.2. for example. Fig.8a and b illustrate the crack propagation under two orientation versions of applied antiplane loading in mode III case, for $S/S = 0.5; DL=2$.

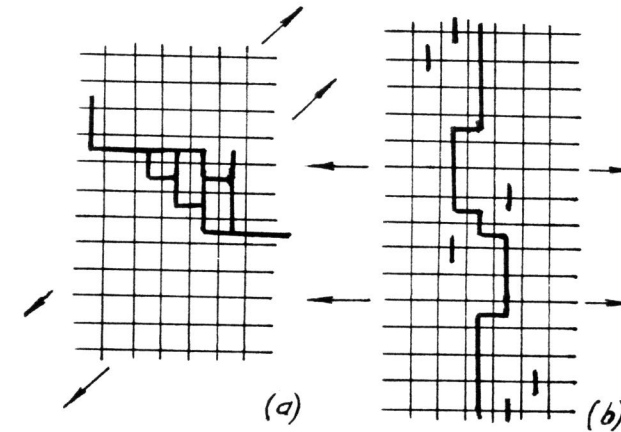


Fig.8. Crack propagation under external stresses of different orientation for mode III case.

In conclusion, the author wishes to express sincerely her appreciation to Prof. Slepyan L.I. for the formulation and discussion of the problem.

REFERENCES

1. Kanazawa, T. (1973). Recent studies on brittle crack propagation in Japan. In: *Dynamic Crack Propagation* (Sih G.C.), Noordhoff Int. Publishing, Leiden.
2. Karzov G.P., Karhin V.A., Leonov V.P., Margolin V.Z. (1982). The calculation of fatigue crack path and fracture parameters at cyclic loading in residual stress field. *Voprosy Sudostroenia, Ser: Svarka*, V.33, p.3-16.
3. Mahnenko V.I. (1976). *Raschetnye metody issledovaniya kinetiki svarochnykh napryazheniy i deformatsiy* (The calculation methods of welding stresses and deformations kinetics examination), Naukova Dumka, Kiev.
4. Slepyan L.I. (1990). *Mechanika treschin* (Crack mechanics), Sudostroenie, Leningrad.
5. Slepyan L.I., Yakovlev Y. S. (1980). *Integralnye preobrazovaniya v nestatsionarnykh zadachah mekhaniki* (Integral transforms in the non-stationary mechanics problems), Sudostroenie, Leningrad.