# On Measuring the Instantaneous Stress Intensity Factor For Propagating Cracks

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ABSTRACT

A new method for measuring $K_I$ associated with a propagating crack is presented. A row of strain gages is placed at a constant distance above the crack line and each gage is oriented to eliminate the $B_o$ term in the multi-parameter representation of the strains. The method of analysis described here provides a solution with three parameter accuracy. Both static and dynamic analyses were performed to show the relation between the strain and the stress intensity factor. The difference between the static and the dynamic results was small for crack velocities less than $0.3c_R$. The method was demonstrated with an experiment on 4340 alloy steel.

KEY WORDS

Stress intensity factor, strain gage, dynamic, 4340 steel.

## INTRODUCTION

Until recently the determination of the instantaneous stress intensity factor associated with a propagating crack was limited to optical methods including photoelasticity and caustics. With transparent polymers, Dally (1979) and A. S. Kobayashi (1978) have made extensive use of transmission photoelasticity to determine the instantaneous stress intensity factor associated with a propagating crack. They also examined characteristics of the fracture process such as arrest and branching toughness. T. Kobayashi and Dally (1980a) used birefringent coatings and reflection photoelasticity to determine the stress intensity factor and crack velocity relationship for a 4340 steel. Theocaris (1981), Kalthoff (1980b), Rosakis (1982) and Ravichandar and Knauss (1984) have employed the method of caustics for similar investigations with both transparent polymers and opaque metals. These experimental methods have provided significant information giving insight to many characteristics of the fracture process. However, optical methods are relatively difficult to employ and require expensive high speed photographic recording systems available to relatively few investigators world wide. Here, a new method based on strain gage is presented which is more easily employed by industrial laboratories.

Irwin (1957) first suggested the use of strain gages to determine $K_I$, but nearly 30 years had passed before Dally and Sanford (1987a) demonstrated the approach. Berger and Dally (1988) have extended this approach making use of multi-element strain gages permitting an over-deterministic solution which improves the accuracy in the $K_I$ measurement.

More recently Shukla, Chona and Agarwal (1987b) have indicated the use of strain gages to determine $K_I$ for a propagating crack. The method advanced by these authors is similar to the approach described in this paper. However, there are several differences regarding the orientation of the gages, the use of a higher order solution and feature extraction from the strain-time record to improve accuracy.

## A STRAIN GAGE EXPERIMENT WITH BRITTLE STEEL

The experiment to demonstrate this method was conducted with a 12.5 mm thick modified compact tension specimen with W = 254 mm. The specimen was fabricated from 4340 steel heat treated to a hardness of $R_C$ = 51, obtained with an oil quench without subsequent drawing. A slit was machined in the specimen with a cut off wheel. The tip of this slit was finished to a chevron which reduced the load required for initiation of the artificial crack. Shallow face grooves 0.63 mm deep with a 45 degree included angle were machined on both sides of the specimen along the crack line to guide the crack during propagation.

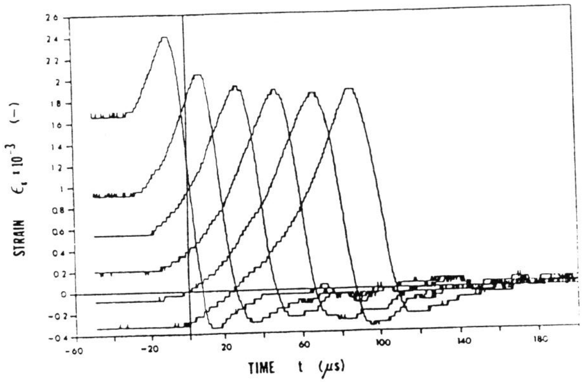


Fig. 1 Strain-time traces from the six strain gages.



Six strain gages (Micro-Measurement type CEA-06-125UW-350) were mounted along a line parallel to the face grooves at a position $y_0$ = 10.6 mm. Each gage was oriented at an angle $\alpha$ = 61.3 degree to eliminate the gage response to the $B_0$ term. The gages were placed on 12.7 mm centers. The specimen was loaded in a hydraulic, servo-controlled testing machine operated under displacement control. The load increased until the specimen failed by high speed crack propagation. Records of the strain gage output were obtained by using a bridge and amplifier unit with a band pass frequency of DC to 120 kHz and digital oscilloscopes operating at a sampling rate of 200 ns/pt. A graph showing the strain-time traces referenced to a common time base and strain amplitude is shown in Fig. 1.

## FIRST ANALYSIS---STATIC

Consider the static solution for the strain on a gage oriented at some angle $\alpha$ relative to the line of the running crack. This expression is given by Dally (1987a) as:

$$2\mu\epsilon_g = A_0 r^{-1/2}[k\cos(\theta/2) - (1/2)\sin\theta\sin(3\theta/2)\cos(2\alpha) + (1/2)\sin\theta\cos(3\theta/2)\sin 2\alpha] + B_0[k + \cos 2\alpha] + A_1 r^{1/2}\cos(\theta/2)[k + \sin^2(\theta/2)\cos 2\alpha - (1/2)\sin\theta\sin 2\alpha] + B_1 r[(k + \cos 2\alpha)\cos\theta - 2\sin\theta\sin 2\alpha] \quad (1)$$

$$\text{where } A_0 = K_I/[2\pi]^{1/2} \quad (2)$$

$$\text{and } k = -(1 - v)/(1 + v) \quad (3)$$

This is a four parameter representation of the strain sensed by a gage located at a position defined by $r,\theta$. With a running crack, the angle $\theta$ is unknown, in addition to the coefficients $A_0, B_0, A_1$ and $B_1$ in eqn (1). Note, that r and $\theta$ are related since the crack propagates along the x-axis and the gages are all positioned at a constant distance $y_0$ from the line of the crack.

To reduce the complexity of eqn (1), let $B_1$ = 0 and limit the solution for $K_I$ to three parameter accuracy. $B_0$ term is eliminated by selecting the gage orientation angle $\alpha = 61.3^o$ consistent with Poisson's ratio v = 0.3 for steel. These changes reduce eqn (1) to:

$$2\mu\epsilon_g = A_0 r^{-1/2} f(\theta) + A_1 r^{1/2} g(\theta) \quad (4)$$

where $f(\theta)$ and $g(\theta)$ are defined by the corresponding bracketed terms in eqn (1).

Next, observe that the position co-ordinates r and $\theta$ are related by:

$$r = y_0/\sin\theta \quad (5)$$

Since $y_0$ is a constant in this measurement r depends only on $\theta$. Therefore, one can rewrite eqn (4) as:

$$2\mu\epsilon_g = A_0 f^*(\theta) + A_1 g^*(\theta) \quad (6)$$

For constant velocity propagation, simplifications in the analysis become evident since the horizontal distance along the crack line relative to the gage, $x_1$, is changing at a constant rate given by:

$$dx_1/dt = c \quad (7)$$

where c is the velocity of crack propagation.

The velocity of the crack propagation was determined from the time of occurrence of either the peak strain or the zero crossing and the strain gage positions. A nearly linear relationship between gage location and the time of the peak or the zero strain was observed with a slope giving the velocity c = 656 m/s. With constant velocity crack extension, a relation exists between the time of crack propagation, the angle $\theta$ and the crack tip position $x_1$.

The final step in the analysis was to develop a series of master curves to show the influence of the higher order term $A_1 r^{1/2} g^*(\theta)$ on the strain gage signal $\epsilon_g$. Note that:

$$2\mu\epsilon_g/a_0 = f^*(\theta) + [A_1/A_0] g^*(\theta) \quad (8)$$

Values of $A_1/A_0$ ranging from 0 to -2 were considered and curves showing $2\mu\epsilon_g/A_0$ as a function of time $t_1$ are presented in Fig. 2. These are master curves for this experiment; however, the curves are not general since they depend upon the gage position $y_0$ and the crack velocity. The curves contain the information necessary to determine $A_1$ and $A_0$ or $K_I$ for each of the six gages during the passage of the crack by the gage.

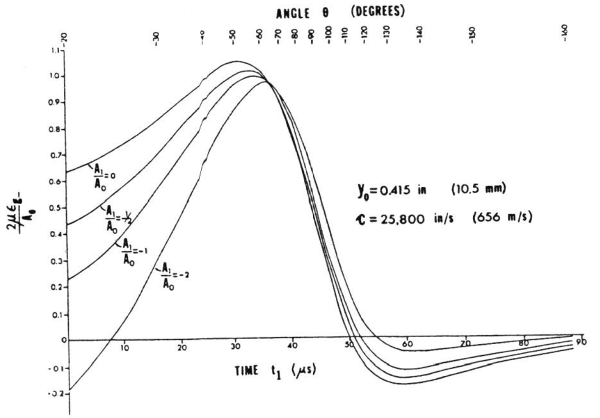


Fig. 2. Gage parameter $(2\mu\epsilon_g/A_0)$ as a function of $t_1$ for different ratios of $A_1/A_0$.



Examination of these results shows: (1) A profound influence of the $A_1$ term as the crack approaches the gage with $\theta<40^\circ$ and $t_1<25$ $\mu$s. (2) A relatively small dependence of $A_1/A_0$ (± 5 per cent on the peak value of $2\mu\epsilon_g/A_0$. (3) A very small effect of $A_1/A_0$ on $2\mu\epsilon_g/A_0$ on the downside of the master curves with $60^\circ<\theta<100^\circ$. (4) A close grouping of the curves at the zero crossing indicating its potential as a timing index. (5) The negative peak is shallow, poorly defined, low in magnitude and strongly dependent upon $A_1/A_0$.

## DETERMINING THE $A_1/A_0$, $A_0$ AND $K_I$

To extract $K_I$ as a function of time from a typical strain gage trace, it is necessary to match the strain-time record to one of the master curves. This fitting process was facilitated by noting one of the features of the $\epsilon$-$t_1$ curves, namely, the time between the rise and fall at some level of strain associated with the positive peak. The time $(\delta t)_{3/4}$ from rise to fall at a strain level equal to 3/4 of the peak strain was selected to determine the ratio $A_1/A_0$. It was noted that $(\delta t)_{3/4}$ varies from 27 to 16 $\mu$s as $A_1/A_0$ changes from 0 to -2. The same measurements for $(\delta t)_{3/4}$ from the strain-time traces shown in Fig. 1 were made and $(\delta t)_{3/4}$ = 21.3 $\mu$s with a range of only 0.8 $\mu$s.

The consistency in $(\delta t)_{3/4}$ indicates that $A_1/A_0$ is essentially constant as the crack propagates past the six strain gages. The average value of $(\delta t)_{3/4}$ = 21.3 $\mu$s corresponds a curve in Fig. 2 with $A_1/A_0$ = -0.6. A single master curve for $A_1/A_0$ = -0.6, given in Fig. 3, shows the relation between $A_0$ and the strain $\epsilon_g$. The result provides a three parameter determination of $K_I$ from a strain-time trace.

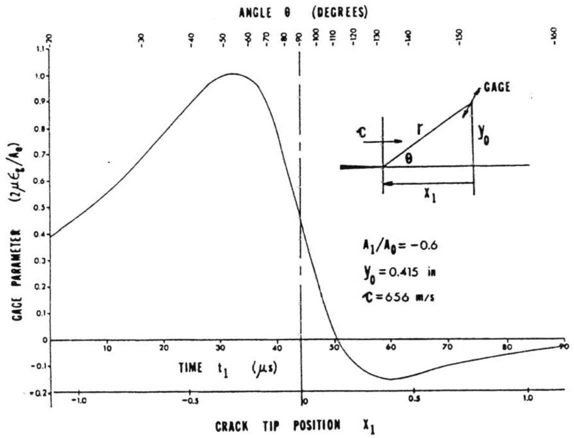


Fig. 3. Parameter $(2\mu\epsilon_g/A_0)$ as a function of $t_1$, $\theta$ and $x_1$.

As an example, consider the peak value in Fig. 3 where the master curve for this experiment gives $2\mu\epsilon_g/A_0 = 1.01$ or $A_0=2\mu\epsilon_g/1.01$. Recall eqn (2) and substitute into this relation to obtain:

$$K_I = 2\mu\sqrt{2\pi}\epsilon_g/1.01 \qquad (9)$$

where the units for $K_I$ are consistent with the choice of units for the shear modulus $\mu$. The peak strains $\epsilon_g$ were determined from Fig. 1 and substituted into eqn (9) to give $K_I$ as shown in Table 1.

Table 1

Peak Strains and Stress Intensity Factor from Each Gage

| Gage No. | Strain ($\mu$m/m) | Stress Intensity Factor ksi$\sqrt{}$in | Stress Intensity Factor MPa$\sqrt{}$m |
|---|---|---|---|
| 1 | 2404 | 137.2 | 150.6 |
| 2 | 2036 | 116.1 | 127.5 |
| 3 | 1923 | 109.8 | 120.6 |
| 4 | 1881 | 107.3 | 117.8 |
| 5 | 1844 | 105.3 | 115.6 |
| 6 | 1867 | 106.6 | 117.0 |

These results show that $K_I$ decreases by about 15 per cent as the crack propagates between gages 1 and 2. A small decrease in $K_I$ (9 per cent) is observed as the crack propagates between gages 2 and 5. A slight increase in $K_I$ occurs as the crack approaches gage 6.

## SECOND ANALYSIS---DYNAMIC

The dynamic equations giving the cartesian stresses as a multi-parameter series in terms of r,θ are given by Chona (1987c). These equations were programmed into Math-Cad and converted into a dynamic expression for $\epsilon_g$. Solutions for the maximum value of $2\mu\epsilon_g/A_0$ were developed with $A_1/A_0 = -0.6$ for crack velocities which increased from 0 to 0.50 of the Rayleigh wave speed $c_R$. The ratio of $2\mu\epsilon_g/A_0$ between the zero velocity and the higher velocities, gives a correction factor $C_v$ which can be used to adjust static results for the effect of crack velocity. This ratio for $C_v$ is shown in Fig. 4 as a function of $c/c_R$. Note that $C_v$ varies from 1.0 to 1.18 as the velocity ratio $c/c_R$ goes from 0 to 0.5. For this experiment, $c/c_R = 0.22$ and $C_v = 1.027$ indicating that the static and dynamic methods of analysis give nearly the same results.

A second correction factor, $C_g$ accounts for the effect of the shallow side grooves and is given by:

$$C_g = \sqrt{(B/B_n)} \qquad (10)$$

where B is the thickness of the specimen 12.7 mm.
$B_n$ is the ligament at the side grooves 11.6 mm.



Using these values in eqn (10) gives $C_g = 1.048$.

The total correction factor C is:

$$C = Cg/Cv = (1.048)/(1.027) = 1.02 \qquad (11)$$

Considering the variability observed in fracture testing to determine initiation and/or arrest toughness this correction factor C is relatively small. However, to improve the accuracy of the determination of $K_I$ for a running crack the correction factor C should be used.

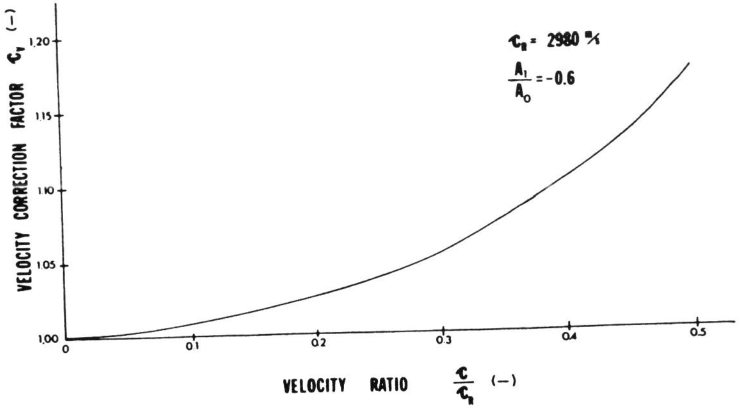


Fig. 4 Correction factor $C_v$ as a function of crack velocity $c/c_R$.

## CONCLUSIONS AND DISCUSSION

A method has been developed which will permit the determination of the stress intensity factor associated with a crack propagating at high speed. This method should aid in the determination of the $K_I$-c curve which is needed to characterize crack propagation in engineering alloys.

The velocity effect is small as long as $c/c_R < 0.3$ and modest corrections are possible to adjust the value of $K_I$ to account for velocity. When $c/c_R > 0.3$ the corrections become large and data analysis based on a complete dynamic solution is recommended.

The strain-time traces give essentially a continuous recording of $K_I$ with time. Here, only one of the many valid data points available from each gage has been used to illustrate the application of this method.

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