Mathematical Morphology and Quantitative Fractography

J. L. CHERMANT and M. COSTER
Laboratoire d'Etudes et de Recherches sur les Matériaux, Lermat
ISMRa-Université, Bd Maréchal Juin – 14032 CAEN Cedex, France.

ABSTRACT

In this paper we present methods of quantitative fractography - morphological quantification of the fracture, profilometric analysis, simulation of fractures - using the mathematical morphology concepts and which can be used with automatic image analyzers. The results obtained bring fracture microstructural information to mechanical engineers investigating fracture mechanics.

KEYWORDS

Quantitative fractography, mathematical morphology.

INTRODUCTION

With the development of automatic image analyzers, quantitative image analysis is becoming increasingly apparent in various scientific fields and in particular for quantitative studies of fracture of materials, known as quantitative fractography. Moreover mathematical morphology is a privileged tool for image processing and analysis. The resources of mathematical morphology can be used for investigations in quantitative fractography. The scope of this paper is to present what mathematical morphology can bring to quantify and to investigate non-planar surfaces.

DIFFERENT TYPES OF ANALYSIS IN QUANTITATIVE FRACTOGRAPHY

In quantitative fractography there are two centres of interest.

The first one is related either to the fracture feature or to the nature of the phases encountered on the fracture, from which it is necessary to estimate their size and importance. The features characterization is obtained from a projected image and not on the surface itself. To have access to parameters on the surface, we must use stereometric relationships, (El Soudani, 1974; Coster et al., 1983 ; Underwood, 1987). These stereometric relationships use parameters characteristic of the fracture morphology. The second one corresponds to the morphological investigation or pattern recognition of fracture. In these conditions, we have to quantitatively characterize the fractured surface irrespective of the nature of the fracture features. In this paper only this second point will be taken into consideration.

MORPHOLOGICAL QUANTIFICATION OF THE FRACTURE

As for shape investigations of objects, morphology of the fractured surface can be described either from dimensionless parameters, as the linear or surface roughness (Coster et al., 1983 ; Underwood, 1987 ; Coster et al., 1985), or from functions obtained by harmonic analysis (Passoja et al., 1978 ; 1981), by fractal analysis (Mandelbrot et al.,1984) or by mathematical morphology which then becomes only a support for the fractal analysis (Coster et al., 1978). Beucher and Hersant (1978) have developed an automatic method allowing to reconstruct the fracture surface of steels, but it is very time consuming and can be applied only with brittle fractures with facets. So morphological analysis of fractured surfaces can be investigated generally either from stereoscopic views (Bauer et al., 1981) or from fracture profiles, obtained by intersecting the surface of fracture by a perpendicular plane. In practice this last method is the most often used. Although profilometric analysis has firstly been developed with morphological parameters similar to shape factors, our investigation will be restricted to the profilometric analysis using mathematical morphology concepts. Then we will discuss the simulation methods using mathematical morphology.

PROFILOMETRIC ANALYSIS BY MATHEMATICAL MORPHOLOGY

Profilometric analysis by mathematical morphology can be divided in two classes : methods of fractal or of granulometry type.

Methods of fractal type

Two methods will be presented, the Minkowski method and the increasing size boxes method.

a. Minkowski method

The fractal methods have been the first to be developed and used (Coster et al., 1978). The convergence between fractal methods and mathematical morphology was unavoidable as the method to determine the fractal dimension of a line by disk covering is nothing else than the use of the dilation of this line by disks of increasing size (Fig. 1), followed by the calculation of the approximated perimeter from the measure of the dilated surface. This method is called Minkowski method (Mandelbrot, 1977). The approximated perimeter, $L_2(\partial X,\lambda)$, is given by (λ being the size of the dilation D for the fracture line ∂X) :

$$L_2(\partial X,\lambda) = \frac{A(D^{\lambda}(\partial X))}{2\lambda}$$

This equation, similar to that of Santalo (1953), necessitates no hypothesis on the nature of the line (overlapping or not). It can be replaced by image transformations allowing to construct, geometrically, the line corresponding to this perimeter. This method has been devised previously by Coster and Deschanvres (1978). To construct this approximated line, a closure, F, of size λ on the fracture profile ∂X, followed by a complementation of the image and by the construction of the skeleton by influence zone, Sz, are used, (Fig. 2) :

$$Y = Sz\,[(F^{\lambda}(\partial X))^{c}]$$

Considering the size of the frame of measurements, Z, and of the fracture profile length, this last one is only partially known : the length of the line ($\partial X \cap Z$) has then no meaning. So the length $L_2(\lambda,\partial Y \cap Z)$ is replaced by a parameter which has then a local meaning : the linear roughness, R_L, obtained from the ratio of the measured length to its projection, $L'_2(\lambda,\partial Y)$. It is given by:

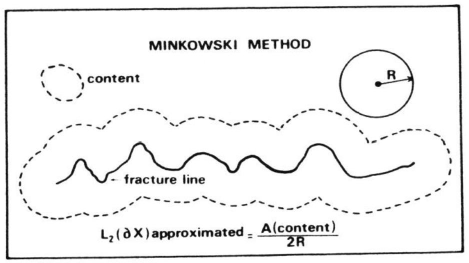

Fig. 1 : Covering method of Minkowski.

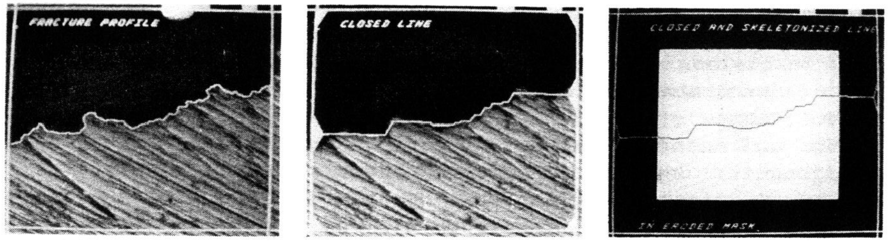

Fig. 2 : Example of different stages of the Minkowski method adapted by mathematical morphology : a) profile of steel rupture surface ; b) closed line, using closure step H of 20 pixels, c) closed and skeletonized line in the eroded frame.

$$R_L(\lambda,\partial\lambda) = \frac{L_2(\lambda,\partial Y)}{L'_2(\lambda,\partial Y)}$$

With this method $L_2(\lambda,\partial Y)$ is obtained by numbering the pixels of the closed, skeletonized and clipped line. This method has been used many times (see for example Lavolé, 1981 ; Coster et al., 1980). Its allows to calculate the fractal dimension, $\mathfrak{D}$, either from the derivative of the bilogarithmic curve, according to :

$$\mathfrak{D} = 1 - \frac{\partial \log R_L(\lambda,\partial Y)}{\partial \log \lambda}$$

or by adjustment according to the square least method to obtain the mean fractal dimension. This last method has been utilized to investigate ruptures of steels under stress corrosion or ruptures of cold-worked brasses (Chermant et al., 1987a & b).

Recently this method has been improved. In these papers, all the morphological transformations were Euclidean type and it is the reason why measurements on the transformed profile are undertaken in an eroded frame (theorem of the frame of measurements, (Coster et al., 1985 ; Serra, 1982)). The use of geodesic transformations allows to preserve the initial size of the frame of measurements (Lantuejoul et al., 1984). The set Y is then written :

$$Y = Sg_z\left[(F_z^\lambda(\partial X))^c\right]$$

This improvement has been also used for the investigation of the rupture of brass specimens in correlation with their cold-worked state (Chermant et al., 1987b), (Fig. 3).

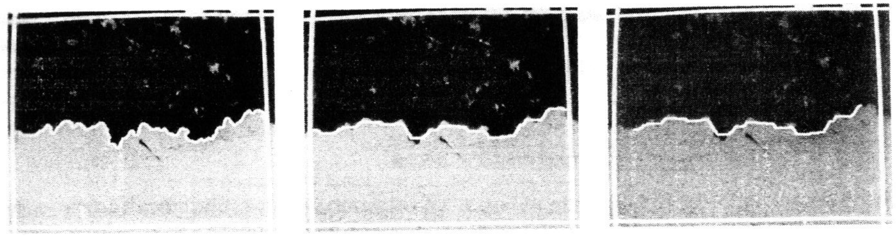

Fig. 3 : a) Video image with the digitized line profile for a cold worked brass; video image with the digitized and skeletonized line profile obtained after closing by an hexagon of size 15 pixels according to the Euclidean (b) and geodesic space (c).

b. Increasing box method

The same authors have proposed to use an other method, called method of increasing boxes, which is also of fractal type. Consider $y \in \partial X$ and $B_y(\lambda)$ a box centered on y and X the fractured material. The principle of increasing box method is to follow the perimeter and area evolution of $X \cap B_y(\lambda)$ when λ increases. The algorithm, illustrated on figure 4, is the following :

- box construction by dilation : $B_y(\lambda) = D^{\lambda H}\{y \in \partial X\}$
- intersection : $Y(\lambda) = X \cap B_y(\lambda)$; $\partial Y(\lambda) = \partial X \cap B_y(\lambda)$
- measurements : $L_2(Y,\lambda) = \text{meas } \partial Y(\lambda)$; $A(Y,\lambda) = \text{meas } Y(\lambda)$

The evolution of these parameters is analyzed by fractal plot. Like for roughness investigations, the slope gives information on the morphology of the fracture (Fig. 5). In Chermant et al.

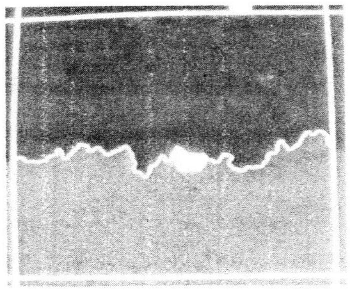

Fig. 4 : Video image with a box (size 15) centered on the fracture line and intersected by the nickel phase, deposited on the fracture on the cold-worked brass specimen.

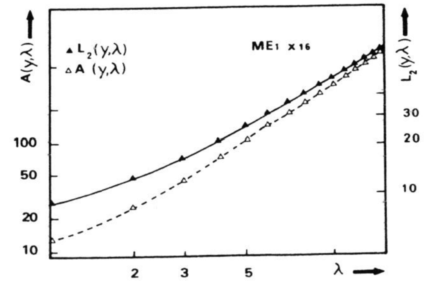

Fig. 5 : Fractal plot using increasing box sizes, λ, with $A(Y,\lambda)$: surfaces area of the boxes ; $L_2(Y,\lambda)$: perimeter of the boxes.

(1987b) the method of Minkowski and of increasing boxes have been compared : the influence of the method on the results has been shown and the differences interpreted.

Methods of granulometry type

Mathematical morphology allows to characterize the fracture profiles by methods of granulometry type. Because linear roughness, $R_L(\partial X)$, is insufficient to describe profiles, Serra (1984) has proposed another parameter : the microroughness index $R_{mL}(\partial X)$. The microroughnesses lead on the profile to low curvature radii. To access to these microroughnesses, we can consider the complementary set (∂X^c) to the fracture line. A line of analysis will form small segments at a level with these microroughnesses. The knowledge of the density in number of these segments, $f(\partial X^c, \ell)$, will inform on these microroughnesses. This granulometric function is obtained from the linear eroded of size ℓ by a classic way (Coster et al., 1985), (Serra, 1982) (Fig. 6). In so far as ℓ is small and assuming that ∂X possesses curvature radii finite at any point, there exists the relation (with du the elementary arc on ∂X, R the curvature radii and $\bar{H}$ the mean curvature) :

$$f(X, X^c, \ell) = \frac{1}{8} \frac{\ell}{L_2(\partial X)} \int_{\partial X} \frac{du}{R^2} = \frac{\ell}{8\bar{H}^2}$$

This result is interesting : it shows that the histogram $f(\ell)$ is proportional to ℓ for small values of ℓ. Now if we suppose that the fracture profile possesses angulous points, the histogram will not go through the origin (Fig. 7) and we will have :

$$f(X, X^c, \ell) = k + \frac{\ell}{8} \bar{H}^2 = k + R_{mL}(\partial X)$$

Serra proposed to take $\bar{H}^2$ as parameter to describe the micro-

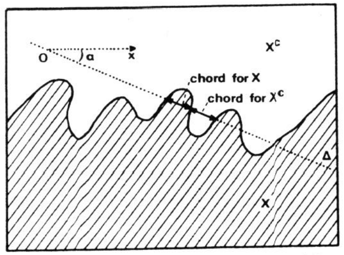

Fig. 6 : Fracture profile and analysis line Δ : determination of the chord in X and in X^c.

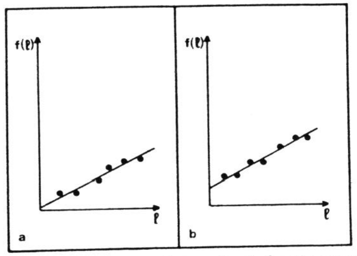

Fig. 7: Theoretical histograms of $f(\ell)$, with absences (a) or presence (b) of angulous points.

roughness. The value of k can in fact be biased by the noice which is added up to the angularities.This method has not presently been applied to fracture profile investigations. Nevertheless it gives nice results to investigate microroughness of particles (Gougeon, 1988).

QUANTITATIVE FRACTOGRAPHY AND SIMULATION

It is not always easy to interpret results obtained from quantitative fractography, in particular for the morphology of fracture line. There are guide lines which enable prediction of the morphology as a function of the mechanical behaviour of the material before and during fracture. In attempting to describe results one solution consists of constructing theoretical models, for which one has fixed a priori the deformation laws and which can be simulated on automatic apparatus. In this section we will focus only with simulations which use mathematical morphology.

Chermant et al. (1983) proposed a model of simulation for brittle fracture of polycrystalline mosaic, simulated by a Voronoï partition constructed from a Poisson point process (i.e. a random set of points according to a given density), using the skeleton by influence zone (SKIZ) of this set of points. To simula-

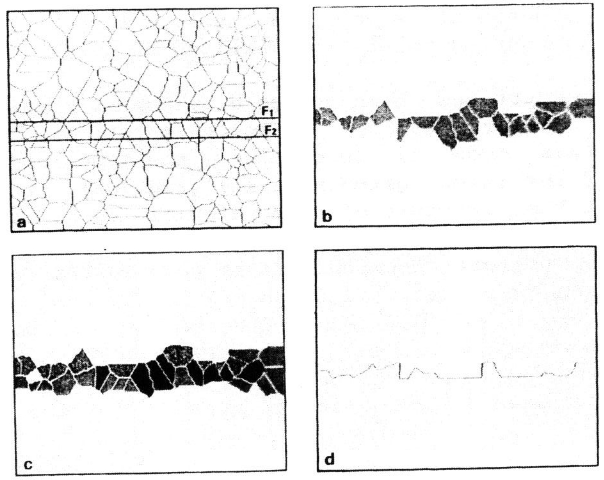

Fig.8 : Simulation of brittle fracture by Voronoï partition : a) setting path of crack propagation (lines F_1 and F_2) ; b) reconstruction of grains intersected by line F_1 ; c) reconstruction of grains intersected by line F_1 and/or line F_2 (black grains are common to both lines); d) final profile of brittle fracture.

te a fracture path (Fig. 8) on a Voronoï partition (in memory M1), the position of the mean direction of crack is defined by tracing a line, F_1, (in M0) parallel to its X axis. The intersection between M0 and M1 provides segments from which the grains intersected by this line can be reconstructed. According to the initial hypothesis the grains will be either circumvented or traversed by the fracture. As selection criterium between trans- or intergranular fracture for a particular grain, we have chosen a parameter Δ, related to the grain size in the direction perpendicular to the fracture and to its position with respect to the line in the same direction, such that all the grains intersected by the line F_1 with points of height $F_1+\Delta$ are considered to be fractured transgranularly. These missing grains are replaced by a transgranular fracture line. After dilation the upper contour represents the fracture profile. The influence of the frame of measurements on the profilometric analysis has thus been investigated using this type of simulation, as well as linear roughness, R_L, and fractal dimension, $\mathfrak{D}$, as a function of the size, D, of the Voronoï partition (Fig. 9).

Ductile fracture has also been simulated in setting, on a width, Δ, of the materials, Poisson points which represent the cavities of plastic flow (Lavolé, 1981). Starting from the neighbouring point from one of the side of the measurement frame, one joints step by step the points which are located the closest with each other. This process stops when the opposite side of the frame is reached. Neighbouring points are obtained by dilation operation.

Similar procedure has been proposed by Osmont et al. (1987) to simulate crack propagation in porous materials. The starting point for this model is to consider the true and un-damaged structure of the investigated material (Fig.10a). These authors assume that the fracture of the material is brittle and that crack propagation results from the principle of minimum energy consumption. As pore crossing by the crack front requires no

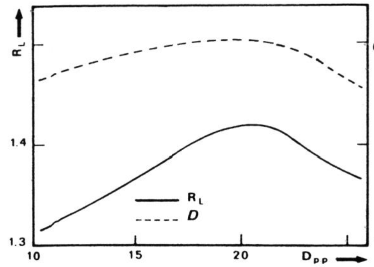

Fig. 9 : Change in R_L and $\mathfrak{D}$ as a function of the size D of the Voronoï partition (in picture point).

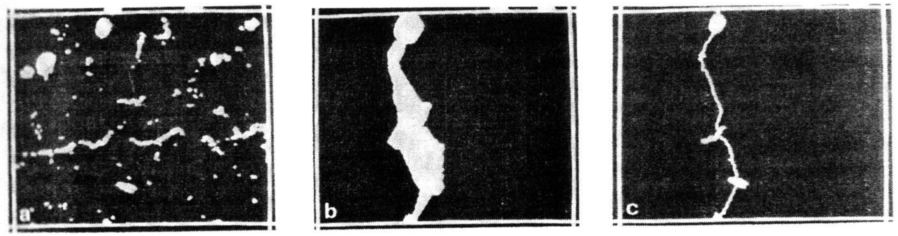

Fig. 10 : a) Video image of polycrystalline graphite where the pores appear in white ; b) set of the geodesic paths ; c) simulated crack with respect to pores, after a clipping conditional skeleton of the image b (from Osmont et al., 1987).

energy, it is expected that the crack path will preferably go through the pores. This hypothesis can be simulated by mathematical morphology in using the geodesic propagation (Lantuejoul et al., 1984) with a crack velocity of finite value in the material and infinite in the pores. The result can be interpreted in terms of geodesic distance, $d_X(x,y)$, between the initial point, x, and the finishing point in the measurement window, y. This result is compared to the case of a bulk material for which the distance between these two points is nothing else than the Euclidean distance, d(x,y). To obtain this geodesic distance, it suffices to iterate the elementary operations of mathematical morphology : dilation, union, intersection. This method has been improved by using dodecagonal dilations instead of hexagonal dilations (Fig. 10b and c). Such simulation has been compared to real crack propagations as that on figure 11. This confirms the validity of the model proposed.

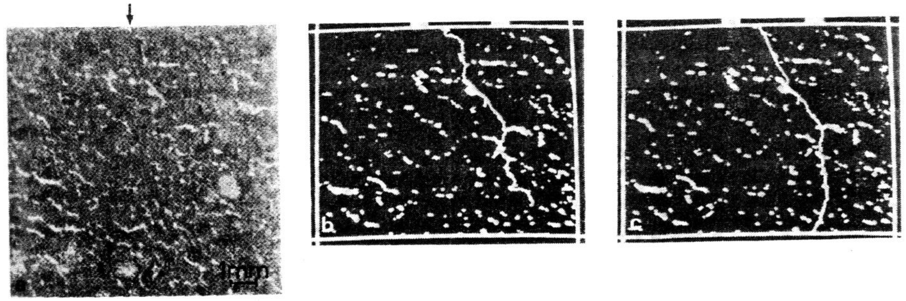

Fig. 11 : Comparison between true crack, from three point bending test, and simulated crack in polycrystalline graphite, from Osmont et al. (1987) : a) initial image ; b) true crack ; c) simulated crack.

CONCLUSION

These different methods, based on mathematical morphology, show the possibilities that quantitative fractography can bring to the mechanical investigations. With the technological progresses and the enhanced capabilities of new automatic image analyzer systems, quantitative fractography is promising for the future.

REFERENCES

Bauer, B. and A. Haller (1981). Pract. Met., 18, 327.

Beucher, S. and T. Hersant (1978). Inter. Symp. on Quantitative Metallography, (Associazione Italiana di Metallurgia, Ed.), Firenze, Italia, 257.

Coster, M. and A. Deschanvres (1978). Pract. Met., 8S, 61.

Coster, M., J.L. Chermant and P. Dujardin (1980). Mikroscopie (Wien), 37, 364.

Coster, M. and J.L. Chermant (1983). Int. Met. Rev., 28, 228.

Coster, M. and J.L. Chermant (1985). Précis d'Analyse d'Images, (CNRS, Paris, Ed.).

Chermant, L., J.L. Chermant and M. Coster (1983). Acta. Stereol., 2/SI, 381.

Chermant, L., J.L. Chermant and M. Coster (1987a). J. Microsc. Spectro. Electr., 12, 173.

Chermant, L., J.L. Chermant and M. Coster (1987b). (Acta. Stereol., 6/III, 845.

El Soudani, S.M. (1974), Metallography, 7, 271.

Gougeon, G. (1988). Thèse de Doctorat, Université de Caen.

Lavolé, J. (1981). Thèse CNAM, Université de Caen.

Lantuejoul, C. and F. Maisonneuve (1984). Pattern Recognition, 17, 177.

Mandelbrot, B. (1977). Fractal Form Chance and Dimension, (W. Freiman, San Francisco, Ed.).

Mandelbrot, B., D.E. Passoja and A.J. Paullay (1984). Nature, 721.

Osmont, P., D. Jeulin and G. Larquetoux (1987). Acta. Stereol., 6/III, 381.

Passoja, D.E. and D.J. Amborski (1978). Microstructural Science, (J.E. Benet, L.R. Cornwell and J.L. Mc Call, Ed.), 143.

Passoja, D.E. and J.A. Psioda (1981). ASTM-STP n° 739, 355.

Santalo, L.A. (1953). Introduction to integral geometry, (Hermann, Ed.), Paris.

Serra, J. (1982). Image Analysis and Mathematical Morphology, (Academic Press, Ed.).

Serra, J. (1984). J. Microscopy, 134, 227.

Underwood, E.E. (1987). Acta. Stereol., 6/III, 855.