# Experimental Study of Mixed Mode Stress Intensity Factors in Rotating Disk Having the Cracks by Photoelasticity and Caustic Method.

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ABSTRACT

In a single crack, stress intensity factors are calculated as a function of the inclined crack of length 2a and the position of the crack at different angular velocities 1300rpm, 1800rpm, 2100rpm and at different values of the inclination crack angle $\phi$ and those values are also measured in models of rotating disks using the photoelastic stress freezing method. The ratio $K_{II}/K_{I}$ of the stress intensity factors was determined by using the isochromatic fringe loops angle $\theta_m$ near the crack tip. Espetially, stress intensity factors $K_I$ and $K_{II}$ obtained separately from the isochromatic fringes of the mixed mode, were used to invesigate further the influence of $K_I$ and $K_{II}$ on fracture in rotating disks. The results of this experiment coincided favorably with the theoretical results of Ishida and Terada (1).

On the other hand, branched cracks are frequently encountered in unstable fracture of brittle materials. Furthermore, the behaviors of branched cracks have been observed for the mixed mode with complicated geometry. Stress intensity factors $K_I$ and $K_{II}$ which are determined by using the photoelastic and caustics method, are function of the branched crack of length ratio $a_{II}/a_I$ and of the branched crack angles $2\theta$ at two velocities. The interaction in branched cracks was also discussed, together with the comparison of the results by these experimental methods with those of single crack.

Key Words: Experimental Stress Analysis, Fracture, Photoelasticity, Stress Intensity Factors, Rotating Disks

## INTRODUCTION

There are various kinds of rotary machines such as turbine rotor, compressor rotor, flywheels, etc.. In the recent progress of high-velocity and high -efficiency, fracture of blades or disks due to centrifugal force in rotary machines often caused serious accidents. Therefore, great emphasis should be placed in the studies of the strength for all kinds of fracture, the findings obtained from which are well applicable to safety design. Stress intensity factor has been effectively applied as the quantity in terms of the strength on the stress field around the crack tip to various problems of fracture so far. However, few experiments were available on stress intensity factor against rotators, and Ishida and Terada have given a theoretical expression of the mixed mode of $K_I$ and $K_{II}$. On the other hand, about the analysis of stress intensity factor of a rotating disk employing the photoelastic method, Blauel, Beinert and Wenk (2) (3) and Lai Zhengmei and Sun Ping (4) have reported the values of stress intensity factor K. However, there were few experiments concerning $K_I$ and $K_{II}$ against fracture among various experiments and few studied on rotating disks. In actual structural materials, a crack especially in the mixed mode of $K_I$ and $K_{II}$ concomitantly works in most cases.

In this study, the mechanical behavior around a crack tip in the three kinds of rotating speed was analyzed by means of the stress freezing method using the rotating disks of epoxy resin which have several artificial cracks with different angles and length at some places. This was done in order to experimentally investigate the influences of the inclination of crack angle and initial crack length on the values of the stress intensity factor of a rotating object in which mixed mode was obtaied by determining photoelastic fringe loop angle $\theta m$ and the correlation between the initial crack and the inclination angle was obtained (5) (6). An experimental trial was also made on separation of $K_I$ from $K_{II}$ and the influence of $K_I$ and $K_{II}$ on the fracture of a rotating disk was also studied by several researchers.

In relation to rotating disk having branched cracks, in particular, there seems to be no reports. It is very important obtain stress intensity factors at the tips of cracks under the mixed mode of branched cracks and to utilize those resultant informations obtained therefrom in making safety design.

Attention was given to branching phenomenon of crack frequently encountered in fracture mechanics problems. Stress intensity factors $K_I$ and $K_{II}$ of rotating disk models having a forked crack of two branches of equal or unequal length were obtained using both photoelastic and caustics methods (7) (8) (9). From result obtained, experimental comparison was conducted to determine the effects of branch length of crack and angle between branches on the stress intensity factors. Mutual interference between branches and their relation to a single crack were also examined (10) (11) (12) (13) (14).

MODEL of ROTATING DISK to be TEST

As seen in figure 1 the model disk was prepared by a usual casting method in which araldite B and HT901 as a hardening agent were first mixed at the weight ratio of 100:30 and were once cast in a cylinder with the diameter of 240mm. After that, the model rotating disk was finished by using lathe. The dimension of the rotating disk model were 220mm in outside diameter and 6mm in thickness. Round holes with the diameter of 2mm were made at 4 places at an interval of 90° on the circumference with 150mm in diameter, from which cracks were made using a fine hacksaw blade with the thickness of 0.15mm. Nine rotating disks were prepared. 7 kinds of the inclination angles to the radius direction and 5 kinds of crack length 2a were employed.

Photoelastic sensitivities in the frozen stress state were $\alpha$=3.67 ~ 3.93mm/N. In order to obtain the stress intensity factor, the photograph of photoelastic fringe was enlarged to about 20 time. The correlation the distance $r_m$ of the fringe nunber Nm and the angle $\theta_m$ from crack tip was investigated.



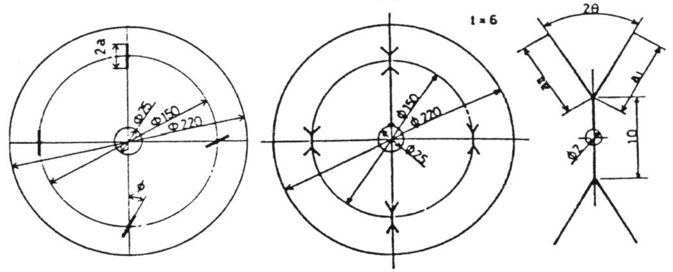


Fig. 1 Model of the rotating disks ( Single cracks and Branched cracks )

In caustics method, transmitted type was used by employing the divergent beam from a laser source ( out put 5mmW, He-Ne gas laser ). One branch was maintained at a constant length of a=10mm and another branch was changed in length to a=0,3,5,10mm, moveover the angle between branches was changed to 2θ =30°,60°, 90°,120°. In all 32 different branched cracks were photographed and enlarged to 20 times to obtain stress intensity factors therefrom.

CALCULATION OF STRESS INTENSITY FACTOR

Calculation methods of stress intensity factor were used expressions (1) and (2) for the photoelastic and caustics methods.

$$\left.\begin{aligned} K_I &= \frac{N_m(2\pi r_m)^{1/2}}{at\{(\sin\theta_m + 2A\cos\theta_m)^2 + A^2\sin^2\theta_m\}^{1/2}} \\ K_{II} &= \frac{AN_m(2\pi r_m)^{1/2}}{at\{(\sin\theta_m + 2A\cos\theta_m)^2 + A^2\sin^2\theta_m\}^{1/2}} \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} K_I &= \frac{1.671}{z_0 t|C_0|} \times \frac{1}{\lambda^{3/2}} \times \left(\frac{D}{\delta}\right)^{5/2} \times \frac{1}{\sqrt{1+\mu^2}} \\ K_{II} &= \frac{1.671}{z_0 t|C_0|} \times \frac{1}{\lambda^{3/2}} \times \left(\frac{D}{\delta}\right)^{5/2} \times \frac{\mu}{\sqrt{1+\mu^2}} \end{aligned}\right\} \quad (2)$$

$$F_I = \frac{K_I}{\sigma_0\sqrt{\pi a}}, \quad F_{II} = \frac{K_{II}}{\sigma_0\sqrt{\pi a}} \quad (3)$$

$$\sigma_0 = \frac{(3+\nu)}{8}\frac{W}{g}\omega^2 R^2 \quad (4)$$

Photoelastic fringe loops in mixed mode of $K_I$ and $K_{II}$ and the basic patterns for caustics image are presented in Figure 2.

It can be seen that photoelastic fringe loops are generated in vertical direction when $K_I$ only works, and those are generated in a nonsymmetric declined direction in the case of mixed mode. For caustics images, a pattern of circle is indicated when only $K_I$ exists and a spiral form of demolished circle is observed in the case of mixed mode.

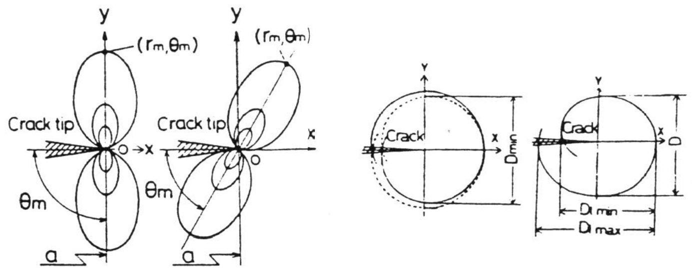


Fig. 2 Basic patterns of model and mixed mode ( Photoelastic fringe loop and Caustics image )

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 3 shows typical photographs of the photoelastic fringe at 1,300rpm, 1,800rpm and 2,100rpm obtained in this experiment. The fringe pattern proves that the isochromatic fringe loop at the crack tip with $\phi$ =0° occurred toward almost vertical direction ( $\theta$m=90°) to the crack direction and therefore only $K_I$ ($K_{II}$=0) worked. It is also found that is isochromatic fringe loop with $\phi$ =45° showed an asymmetrically inclined shape to the crack direction ($\theta$m=69.3) and stress intensity factors of mixed modes, $K_I$ and $K_{II}$ worked concomitantly.

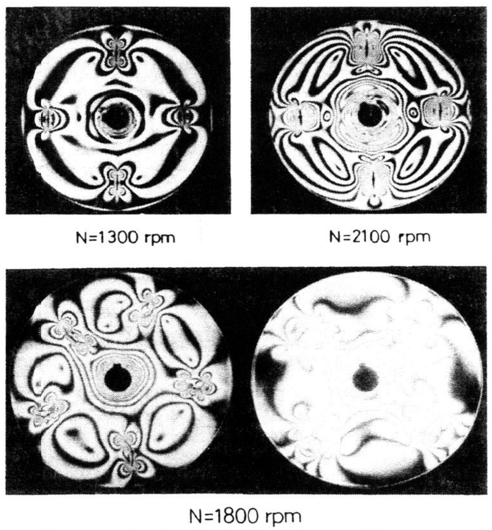


Fig. 3 Whole field pattern (Single crack and Branched crack)

Figures 4 and 5 shows the photographs of the isochromatic fringe loops at the inner crack tips for the inclination of crack angle $\phi$ and the caustic patterns.

Photoelastic fringe loops in mixed mode and the basic patterns for caustics image are presented in figures. When carefully examined, it can be seen that photoelastic fringe loops are generated in vertical direction ( $\theta$m=90°) when only K exists ($\phi$ =0°), and those are generated in a nonsymmetric declined direction in the case of mixed mode, whereas with caustics images, a pattern of circle is indicated in the case of $K_I$ only and a spiral form



of deformed circle, in the case of mixed mode.

The value of $F_I$ and $F_{II}$ in the photoelastic and caustics method corresponded well. It was decreased as $\phi$ was increased, whereas $F_I$ much greater than $F_{II}$ , and even at $\phi$ =90° at which $F_I$ was minimum, $F_I$ was about seven times larger than F (Figure 6). Therefore, as for as the fracture of rotating disk is concerned it is considered that the influence due to $K_{II}$ would be of small order, and that due to $K_I$ is large.

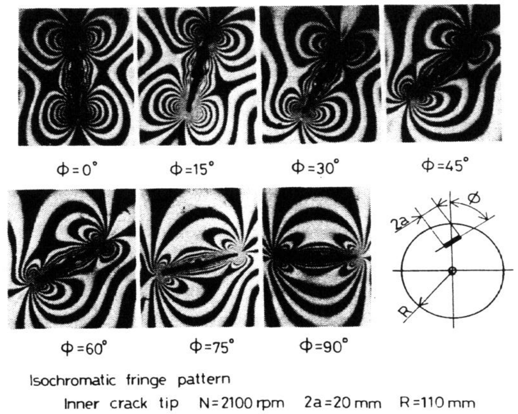


Fig. 4 Isochromatic fringe patterns for the different values of $\phi$

As seen in figures 9, isochromatic fringe loops are formed in nonsymmetric direction inclined relative to the direction of crack, and caustics image shows a spiral formed from an incomplete circle, clearly indicating that $K_I$ and $K_{II}$ are simultaneously acting.

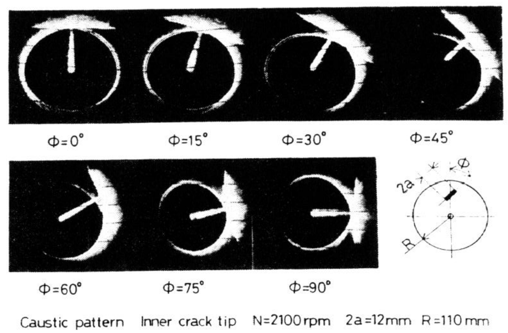


Fig. 5 Caustic patterns for the different value of $\phi$

As 2$\theta$ gets larger, the value of F increases while the value of F is reduced, and at $\theta$ =45° value of $F_I$ is 8 times as large as value of $F_{II}$ for the inner crack tip of a branched crack in circumferential direction (Fig. 7). In the case of a branched crack in radial direction (figure 8), values of dimensionless stress intensity factors $F_I$ and $F_{II}$ grow together as the ratio of branch length $a_{II}/a_I$ increases.

Increases in $F_I$ value is particularly remarkable in the range of small $a_{II}/a_I$ ratio, and the rate of increase gets slower as this ratio approaches to $a_{II}/a_I$=1.0, but the situation of $F_I$ much greater than $F_{II}$ is maintained for the whole range of $a_{II}/a_I$ or for any value of $\theta$. It was confirmed that the rupture by centrifugal force of a rotating disk was effected far greater by $F_I$ value than $F_{II}$ value. In the range of small $a_{II}/a_I$ ratio, because of uneven length of branches, or in the range of small 2$\theta$, the caustics image is not found completely due to the interference between the branches of crack. This results in making image over-

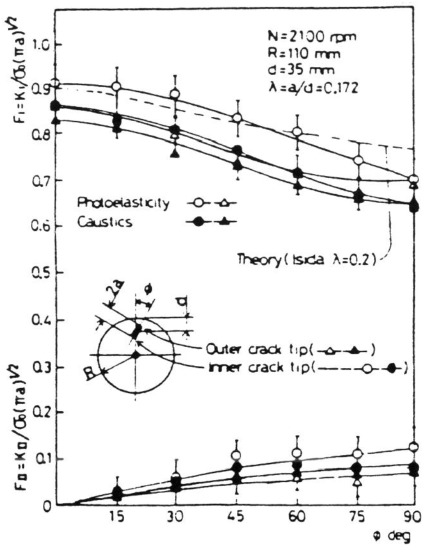


Fig. 6 Correlation between $F_I$, $F_{II}$ and $\phi$

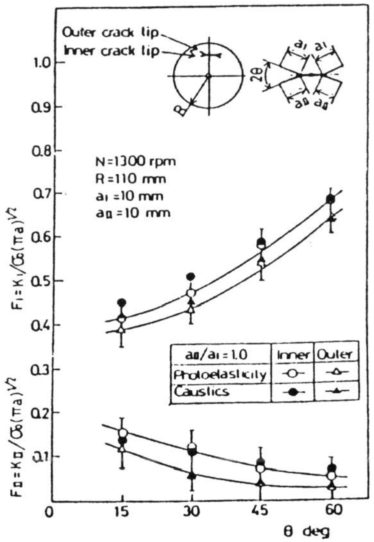


Fig. 7 Correlation between $F_I$, $F_{II}$ and θ

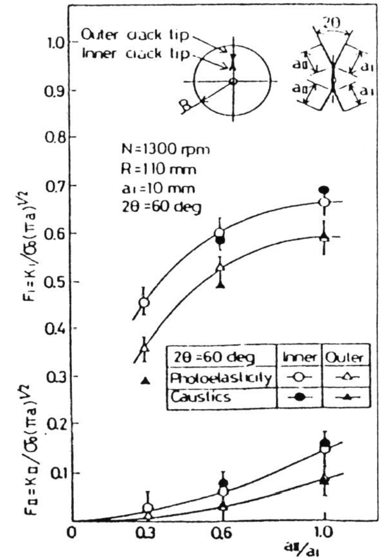


Fig. 8 Correlation between $F_I$, $F_{II}$ and $a_{II}/a_I$

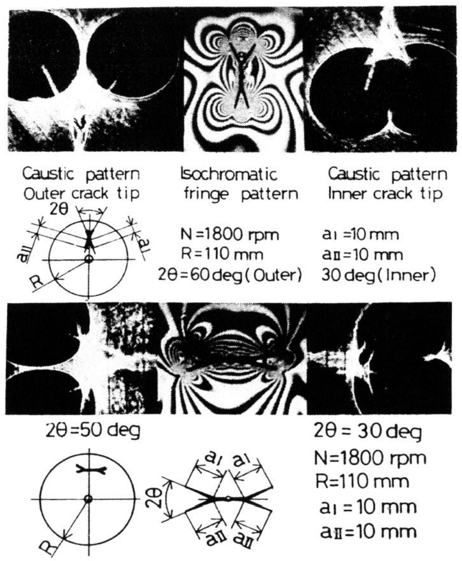


Fig. 9 Caustic and Isochromatic fringe pattern ( radial and circumferential crack)



lapped and unclear and this render the measurements impossible results (figure 9).

SUMMARY

Employing the mixed mode crack model (single crack and branched crack) in which $K_I$ and $K_{II}$ worked concomitantly in the cracks on the random circumference of rotating disks, $K_{II}/K_I$ value was obtained from the photoelastic fringe loop angle by means of the stress freezing method, and $K_I$ and $K_{II}$ were also obtained from the distance rm from the crack tip and number fringe N. In consequence, the following results were obtained.

(1) The stress intensity factor ratio $K_{II}/K_I$ at each crack tip could be obtained by measuring the angle $\theta m$ of the projection direction of the photoelastic fringe loop occurring on the crack tip. It was found that as $\theta m$ became smaller, the value $K_{II}/K_I$ was gradually increased. The photoelastic fringe loop angle $\theta m$ was found to range between 50° and 90°. For measurement of $\theta m$ error factor was considered, but relatively favorable results were obtained since the value of $K_{II}/K_I$ was changed by as small as approximately 0.012 against 1° of $\theta m$, within the above range of $\theta m$.

(2) The value of $K_{II}/K_I$ ratio of the stress intensity factor was almost linearly increased with the increase of the inclination angle of the crack. This was considered due to the fact that the value of $K_I$ was decreased due to the influence of $K_{II}$ occurring with the increase of $\phi$. Furthermore, it was proved that the higher the rotating speed, the smaller the $K_{II}/K_I$ value. This was due to the fact that the $K_I$ value was remarkably increased compared with the $K_{II}$ value when the revolutions per minute was higher.

(3) The $F_I$ showed the maximum value at $\phi$=0° of the inclination angle of the crack, and with the increase of $\phi$ the $F_I$ value was gradually decreased, whereas the $F_{II}$ value was increased. However, at any $\phi$ $F_I$ was much greater than $F_{II}$, and at $\phi$ =90° $K_{II}$ was not 0 but it was far smaller against the $K_I$ value , and at the maximum value of $F_{II}$ the $K_I$ value was approximately 2 times at 1,300rpm and 3 times at 2,100rpm. Accordingly, it was found that the fracture of the rotating disk was influenced much by $K_I$ and less by $K_{II}$ , corresponding well with the theoretically analyzed results reported by Ishida and Terada.

(4) The greater the rotating speed and the non-dimensional length a/d of the crack become the more $K_I$ and $K_{II}$ were increased. Especially, this tendency was remarkably clear in the $K_I$ value.

(5) As $2\theta$ gets larger, the value of $F_I$ increases while the value of $F_{II}$ is reduced, and at θ=45°, value $F_I$ is 8 times as large as value $F_{II}$ for the inner crack tip of a branched crack in circumferential direction.

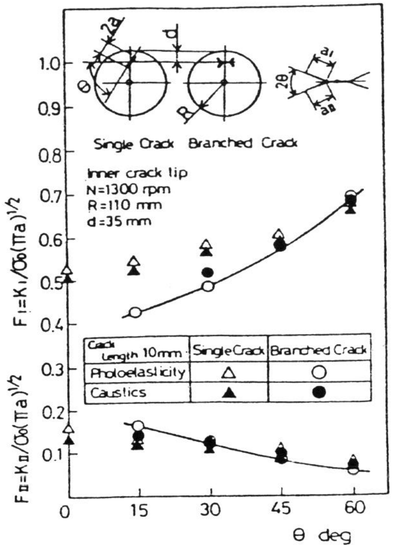


Fig. 10 Correlation between $F_I$, $F_{II}$ and θ (branched and single cracks)

In the case of a branched crack in radial direction, values of dimensionless stress intensity factors $F_I$ and $F_{II}$ grow together as the ratio of branch lengths $a_{II}/a_I$ increases. Increase in $F_I$ value is particularly remarkable in the range of small $a_{II}/a_I$ ratio, and the rate of increase gets slower as this ratio approaches to $a_{II}/a_I=1.0$, but the situation of $F_I$ much greater than $F_{II}$ is maintained for the whole range of $a_{II}/a_I$, or for any value of $\theta$. It was confirmed that the rupture by centrifugal force of a rotating disk was effected far greater by $F_I$ value than $F_{II}$ value.

(6) In the range of small $a_{II}/a_I$ ratio, because of uneven length of branches, or in the range of small $2\theta$, the caustics image is not formed completely due to interference between branches of crack. This results in making image overlapped and unclear and this render the measurements impossible or erroneous. From experimental results, $2\theta$ must be limited to 60° minimum.

(7) When a branched crack having the same length and the same eccentricity with a single crack are compared with this single crack, trends are similar to each other and $F_{II}$ values, in particular, are found in perfect agreement. In the range of small $a_{II}/a_I$ and $2\theta$, measured values are irregularly scattered because of interference or unbalance between branches, whereas in the range of large $a_{II}/a_I$ and $2\theta$, stress intensity factors bear a close resemblance to those of a single crack. Accordingly, in the range where $a_{II}/a_I$ and $2\theta$ have comparatively large values, approximation can be made to a single crack of the same length and eccentricity.

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