Advances in J-Integral Estimation Analysis for Circumferentially Flawed Throughwall Pipes in Bending

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ABSTRACT

A J-integral estimation solution is derived for pipes containing a circumferential throughwall crack and subjected to bending moment loading. The solution is applicable to a wide range of crack lengths and include pipe R/t effects. The solution is useful for calculating J directly from single load-displacement curve available from pipe fracture experiment or finite element analysis. As a special case, J solution is presented for a power law hardening material. The approach developed in this paper can be applied to compact tension, bend bar, and center-cracked tension specimens.

KEYWORDS

J-integral; circumferential throughwall cracks; flawed pipes; ductile fracture; fracture toughness; bending moment loading.

INTRODUCTION

Recently, a J-Integral estimation approach was presented by Zahoor (1988). This approach was based on an earlier work by Zahoor (1987a) for the notched round bar specimen, where the J estimation solution was developed for a wide range of crack sizes. In these papers the contained plasticity solution was used to infer the relationship for the plastic component of the displacement due to the crack in terms of the applied load and crack length. For ease of derivation, the work (Zahoor, 1988) neglected certain crack length dependent terms in the derivation of the plastic component of J, J_p. As a consequence, J result for large crack lengths is expected to be approximate for applications involving ductile and tough materials, especially where J_p is a large fraction of the total J.

The objective of this paper is to improve the solution developed in this recent work (Zahoor, 1988) and obtain a more general solution for 1) the plastic component of the displacement due to the crack and 2) the J-integral. As a special case, the J-integral estimation solution is then derived for a material obeying pure power law behavior. Following this, an alternative J-integral solution is derived using the assumption that the crack displacement and crack length terms are separable. Impact of this assumption on the J-integral solution is assessed.

J-INTEGRAL ESTIMATION METHOD

The J-integral estimation methodology for flawed piping commonly used with the load-displacement curve was discussed by Zahoor (1981, 1988). For an elastic-plastic material and within the context of the deformation theory of plasticity, the J-integral can be estimated from the load-displacement curve (Zahoor, 1981, 1988) as

$$J = J_e + J_p. \tag{1}$$

where J_e is the linear elastic component of the J-integral. J_p is the plastic component of J given by

$$J_p = \int_0^M [\partial\varphi_{cp}/\partial(\text{crack area})]\Big|_M \, dM$$

where φ_{cp} is the plastic component of the pipe bending deflection (rotation) due to the crack and M is the applied moment. The φ_{cp} is obtained from total bending rotation (φ) of the cracked pipe as

$$\varphi_{cp} = \varphi - \varphi_{ne} - \varphi_{np} - \varphi_{ce} \tag{2}$$

where the subscripts n and c represent pipe bending rotation in the absence of crack and that contributed by the crack, respectively. The subscripts e and p denote the elastic and plastic components of the bending rotation.

Figure 1 shows a throughwall crack in a pipe. For this geometry, the change in crack area is $2\pi Rt\cdot d(\Theta/\pi)$. This gives the J_p as

$$J_p = (1/2\pi Rt)\cdot \int_0^M [\partial\varphi_{cp}/\partial(\Theta/\pi)]\Big|_M \, dM. \tag{3}$$

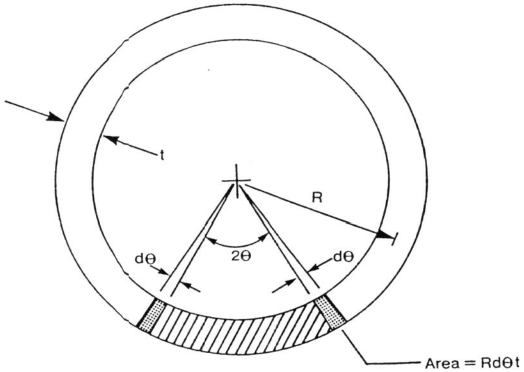

Fig. 1. Geometry of a throughwall crack

It is clear from the foregoing that a solution for J can be obtained if a solution for φ_{cp} is known and its derivative with respect to crack length can be calculated. An approach for obtaining a solution for φ_{cp} is to conduct finite element analyses, but this is costly and time consuming and requires the material stress-strain behavior to be prescribed. In this paper, solutions for φ_{cp} and J_p are developed in a manner that circumvents the problem of prescribing the stress-strain behavior. The J_p solution is obtained in terms of the bending moment and φ_{cp}. The solution for J_e is straightforward (Zahoor, 1988, 1985). This paper develops a solution for J_p.

SOLUTION FOR φ_{cp}

In this section, a solution is developed for the plastic component of the pipe bending rotation due to the crack (φ_{cp}). A solution for φ_{cp} was developed by Zahoor (1988), where for approximate analysis certain crack length dependent terms were neglected. Here, this solution is derived with the objective of retaining all significant crack dependent terms.

The approach used by Zahoor (1988, 1987a) was based on defining the plasticity contribution from the contained yielding behavior for the crack. The motivation for this approach was to identify important crack length parameters that influence the φ_{cp}. Because only the functional form was needed, selection of the contained yielding behavior did not imply a restriction on the resulting solution. It is merely a convenient loading case for which plasticity behavior can be examined accurately and without much difficulty.

The solution for the contained yielding case is developed from the linear elastic solution. The pipe bending rotation due to the crack under linear elastic condition (φ_{ce}) is given by (Zahoor, 1986)

$$\varphi_{ce} = MB_1/(\pi R^2 tE') \tag{4}$$

where

$$B_1 = (\Theta/\pi)^2\cdot[19.739 + 103.7A(\Theta/\pi)^{1.5} + 33.433A(\Theta/\pi)^{4.24} + 166.83A^2(\Theta/\pi)^3 + 123.9A^2(\Theta/\pi)^{5.74} + 26.298A^2(\Theta/\pi)^{8.48}]. \tag{5}$$

$$A = [0.125(R/t) - 0.25]^{0.25} \quad \text{for } 5 \le R/t \le 10$$

$$A = [0.4(R/t) - 3.0]^{0.25} \quad \text{for } 10 \le R/t \le 20.$$

The subscripts c and e refer to the contribution due to crack and linear elastic condition, respectively. The φ_{ce} is directly proportional to the applied bending moment. The function B_1 depends on the crack size and pipe radius to thickness ratio. The above solution is recommended for $5 \le R/t \le 20$ and $0 < \Theta/\pi \le 0.55$ (Zahoor, 1986).

The contained yielding solution is obtained by accounting for the crack tip plasticity. This is done by adjusting the crack length in the linear elastic solution. Using Irwin's suggestion for the plastic zone correction, the effective crack length is given by

$$\Theta_{eff}/\pi = \Theta/\pi + (1/\beta\pi^2 R)(K_I/\sigma_f)^2 \tag{6}$$

where β takes a value of 6 for plane strain and 2 for plane stress, and Θ is the crack-half angle ($a = R\Theta$). σ_f is the flow stress usually defined as the average of yield and ultimate strengths. The definition of σ_f is immaterial since its value is not needed in the derivation that follows.

The Mode I stress intensity factor is (Zahoor, 1985)

$$K_I = (M/\pi R^2 t)\cdot(\pi R\Theta)^{1/2}\cdot F_b(R/t,\Theta/\pi) \tag{7}$$

where

$$F_b = 1 + A[4.5967(\Theta/\pi)^{1.5} + 2.6422(\Theta/\pi)^{4.24}].$$

Substituting (7) into (6), gives

$$\Theta_{eff}/\pi = (\Theta/\pi)\cdot(1 + B_2) \tag{8}$$

where

$$B_2 = (1/\beta)\cdot(MF_b/\pi R^2 t\sigma_f)^2.$$

The φ_{cp} is obtained from the relation

$$\varphi_{cp} = \varphi_{c,eff} - \varphi_{ce}$$

where $\varphi_{c,eff}$ is defined using Θ_{eff}/π in place of Θ/π in B_1. Using Eqs. (4) and (8) in the above, gives

$$\varphi_{cp} = (M/\pi R^2 tE')\cdot B_1\cdot(2B_2 + B_2^2)\cdot[1 + f(B_2, \Theta/\pi)] \tag{9}$$

where the function f in the square bracket is a secondary term and is small compared to 1. This term can be neglected without loosing accuracy since only the functional form is desired. It follows from the above that B_1 and B_2 are separable. B_2 was defined in Eq. (8) and depends on MF_b, pipe size, and material properties. Equation (9) provides a more general result than that in the work by Zahoor (1988) where $(\Theta/\pi)^2$ appeared in place of B_1. The difference between the two solutions can be appreciated by referring to B_1 and noting that the earlier work ignored the crack length dependent terms appearing in the square bracket for B_1. For small Θ/π, the two solutions give approximately the same result.

For a given material and pipe size, B_2 depends only on MF_b. This simplifies Eq. (9), giving

$$\varphi_{cp} = (M/\pi R^2 tE')\cdot B_1\cdot f(M\cdot F_b).$$

Multiplying and dividing the right hand side of the above equation by F_b and rearranging terms, gives the solution for φ_{cp}.

$$\varphi_{cp} = F_1(\Theta/\pi,\ R/t)\cdot g(MF_b) \tag{10}$$

where

$$F_1(\Theta/\pi,\ R/t) = B_1(\Theta/\pi,\ R/t)\ /\ F_b(\Theta/\pi,\ R/t).$$

In this result, the crack length term appears with the bending moment in the g function and also as a separate term in F_1. In general, Eq. (10) also contains R, t, R/t, σ_f, and E' terms which are constant for specific pipe size and material. Consequently, these terms have not been shown explicitly

but could be included in the g function. This function can be defined exactly from Eq. (9) if all terms containing B_2 are retained. The resulting expression is very complex and need not be presented here because, as shown by Zahoor (1988, 1987a, 1981), knowledge of the functional form is sufficient for derivation of the plastic component of J.

It should be observed that the function $g(MF_b)$ resulted from B_2 which defines the crack associated plasticity. Further, the exact solution for B_2 is immaterial to the final result in Eq. (10). Consequently, the solution for φ_{cp} is independent of the plasticity model as long as the plastic solution depends on the MF_b term. The solution in Eq. (10) can be applied to other crack geometries and loading when B_1 and F_b are used for that crack geometry. An additional feature of the result in Eq. (10) is that it can be used for scaling load-displacement curves from different pipe size and crack lengths when the term MF_b in the g function is replaced by B_2.

SOLUTION FOR J_P

The plastic component of J is derived using Eq. (10) in Eq. (3). In this derivation, a solution must be obtained first for the partial derivative of φ_{cp} with respect to Θ/π with the bending moment held constant. Using Eq. (10), carrying out the partial derivative, and following the steps outlined in Zahoor (1988), gives

$$\partial\varphi_{cp}/\partial(\Theta/\pi)\Big|_M = [\partial\varphi_{cp}/\partial M]_\Theta \cdot M(F_b'/F_b) + \varphi_{cp}\cdot(F_1'/F_1) \tag{11}$$

where

$$F_1' = dF_1/d(\Theta/\pi)$$

$$F_b' = dF_b/d(\Theta/\pi).$$

Substituting (11) in (3) and noting that

$$\varphi_{cp}\cdot dM = M\cdot\varphi_{cp} - M\cdot d\varphi_{cp}$$

and rearranging, gives the desired result for the plastic component of J.

$$J_p = (1/2\pi Rt)\cdot[(\beta - \beta_c)\int_0^{\varphi_{cp}} Md\varphi_{cp} + \beta_c\cdot M\varphi_{cp}] \tag{12}$$

where

$$\beta = F_b'/F_b$$

$$\beta_c = B_1'/B_1 - F_b'/F_b$$

$$F_b' = A[6.8951(\Theta/\pi)^{0.5} + 11.2029(\Theta/\pi)^{3.24}]$$

$$B_1'/B_1 = 2/(\Theta/\pi) + (\Theta/\pi)^2\cdot B_3/B_1$$

$$B_3 = [155.55A(\Theta/\pi)^{0.5} + 141.756A(\Theta/\pi)^{3.24} + 500.49A^2(\Theta/\pi)^2 + 711.186A^2(\Theta/\pi)^{4.74} + 223.007A^2(\Theta/\pi)^{7.48}].$$

and F_b and B_1 are defined in Eqs. (7) and (5).

The function g does not appear in the J solution. This shows that the exact solution for g was not needed and only the functional form was sufficient for derivation of the J-integral. In the above J solution, the integral term can be interpreted as the area under the moment (M) versus φ_{cp} curve. This solution is more general than that developed by Zahoor (1988). If only the first term in the sum for B_1'/B_1 is retained, then the two solutions become identical. Table 1 gives β and β_c values for Θ/π values ranging from 0.1 to 0.5 and pipe R/t values of 5, 10, and 20.

Table 1. β and β_c values as a function of Θ/π and R/t

Θ/π	R/t = 5		R/t = 10		R/t = 20	
	β	β_c	β	β_c	β	β_c
0.10	1.54	20.32	1.91	20.43	2.69	20.71
0.15	1.74	13.78	2.13	13.94	2.88	14.32
0.20	1.86	10.57	2.22	10.76	2.90	11.21
0.25	1.92	8.67	2.26	8.89	2.86	9.37
0.30	1.95	7.43	2.26	7.66	2.78	8.14
0.35	1.97	6.55	2.25	6.80	2.70	7.26
0.40	1.98	5.90	2.23	6.14	2.62	6.58
0.45	1.99	5.40	2.21	5.63	2.55	6.04
0.50	1.99	5.00	2.19	5.22	2.49	5.60

A major feature of this solution is that it requires only one load-displacement curve (i.e., one pipe test or one finite element analysis run) for computation of the J_p. This J_p when added to the J_e calculated using Eq. (7) gives the J-Integral. This solution can be applied to a wide range of crack lengths of practical interest. In contrast to the solution by Zahoor and Kanninen (1981), the present J solution incorporates the pipe R/t effects in β and β_c. However, as in the previous work, the effects of pipe R/t are implicit in φ_{cp}. Equation (12) is valid for portion of the load-displacement curve that does not have crack growth. This restriction is obvious from Eq. (11). Consequently, this solution is valid for calculating J_p up to the initiation of crack growth.

ANALYSIS OF PIPE FRACTURE DATA

The J-integral analyses for initiation of crack growth were carried out using flawed pipe experimental data. The load-displacement records for circumferentially cracked throughwall pipes in bending were available from Kanninen, et al (1982). The tests were conducted on Type-304 stainless steel pipes at 24C. Experimental data from four pipe tests were analyzed; this included 51, 102, and 406-mm diameter pipes.

The applied bending moment versus φ_{cp} curve was obtained from individual pipe test data by subtracting the pipe elastic rotation due to crack and the pipe rotation in the absence of crack. The area under the moment versus φ_{cp} curve gave the integral in Eq. (12). The second term in this equation was calculated using the point values of the moment and φ_{cp}. The J_p so calculated was then added to the elastic J, giving the total J.

Table 2. Initiation J values for circumferentially flawed pipes in bending

Experiment Number	Pipe Size mm	Θ/π	J at Initiation of Crack Growth, MJ/m^2: Present Work	Zahoor, 1988	Zahoor, et al 1981	NUREG/ CR-4573	Kumar[3] et al 1984
1T	102	0.37	1.38	1.23	1.80	1.17	3.40
2T	102	0.23	1.31	1.07	2.10	---	3.25
6T	51	0.23	1.08	1.04	1.50	---	2.10
8T	406	0.37	6.43	4.50	5.96[1]	4.90[2]	15.79

1. Result from NUREG/CR-4573 where Zahoor et al (1981) method was used.
2. Average result of four independent finite element analyses, values ranged from 4.55 to 5.60 MJ/m^2 (see also Takahashi et al, 1987).
3. EPRI estimation scheme results as reported by Zahoor (1987b).

Table 2 summarizes the J results and includes the pipe test number, pipe size, and crack length. Here, J values calculated from the solution developed in this paper are compared with those calculated from other J-estimation schemes (Zahoor, 1988; Zahoor and Kanninen, 1981; Kumar et al, 1984) and several independent finite element analyses (NUREG/CR-4573, 1986). As shown, the present solution provides an improvement over those from (Zahoor, 1988; Zahoor and Kanninen, 1981), and is in good agreement with finite element results. This comparison provides confidence in the solution developed here. The results from the J estimation scheme of Kumar et al (1984) which is based originally on finite element analyses differ from other finite element studies (NUREG/CR-4573, 1986) as evident from Table 2. This issue is currently receiving close scrutiny in finite element fracture mechanics studies and in leak-before-break evaluations for piping systems.

CONSEQUENCES OF ASSUMPTIONS ON J-ESTIMATION SOLUTION

In this section an attempt is made to assess the influence of certain assumptions on the J estimation solution. The J computations in finite element studies have been performed using the contour integral based definition as well as the pseudo-potential energy interpretation similar to what is used in the present work. It is in the context of this latter method an assessment of the assumptions is pursued.

Separability of Crack Length and φ_{cp}

Equation (10) can be expressed in an alternate form as

$$M = (1/F_b) \cdot g_1(\varphi_{cp}/F_1)$$

where g_1 is some function of the crack length and φ_{cp}. This relationship implies that the crack length and displacement are not totally separable. However, previous J-estimation work on pipe as well as laboratory specimen geometries assumed that these quantities are separable. If such an assumption is made in the above relation, then the function g_1 may be written as $g_1(\varphi_{cp}/F_1) = g_2(\varphi_{cp})/F_1$, where g_2 is another function that depends only on φ_{cp}. In general, the denominator term in the foregoing equation would appear as some function of F_1, but for simplicity such a form is assumed. This gives the φ_{cp} as

$$\varphi_{cp} = g_3(MB_1). \tag{13}$$

Noting that the term in parentheses is proportional to φ_{ce} (see Eq. 4), this result implies that the plastic component is some function of the elastic component of the displacement. A comparison of this result with that given by Eq. (10) indicates that the two results would be identical if F_b is replaced by B_1. With this observation, the J estimation solution is obtained as

$$J_p = (1/2\pi Rt) \cdot (B_1'/B_1) \int_0^{\varphi_{cp}} M \cdot d\varphi_{cp} \tag{14}$$

In contrast to the result in Eq. (12), here only the integral term is retained and $\beta_c = 0$. It is interesting to note that this result is identical to that obtained from elastic considerations. This implies that the J can be calculated using φ_c and there is no need to calculate J_e and J_p separately.

Power Law Hardening Material

The J solutions presented in the foregoing sections are applicable to any elastic-plastic material since they do not make any specific assumption regarding the stress-strain behavior for the material. Below, a J solution is derived for the pure power law material. The impact of separability of crack displacement and crack length terms on the resulting J solution is assessed. The stress-strain relation for a pure power law material behavior may be written as $\varepsilon = \alpha\sigma^n$, where ε and σ are strain and stress, respectively. α is a material parameter and n is the strain hardening index of the material. This stress-strain relationship was considered by Kumar et al (1981, 1984) where J estimation solution is given for a variety of crack geometries and loading. For a pure power law material, the solution for φ_{cp} may be written as

$$\varphi_{cp} = \alpha B_3 M^n \tag{15}$$

where B_3 depends on Θ/π, R/t, and n. This solution indicates that the crack length and bending moment terms are separable. Using the solution in Eq. (15), it can be shown that $\varphi_{cp} \cdot dM = (1/n) \cdot M \cdot d\varphi_{cp}$. Substituting this result in Eq. (12), gives

$$J_p = (1/2\pi Rt) \cdot (\beta + \beta_c/n) \int_0^{\varphi_{cp}} M \cdot d\varphi_{cp} . \tag{16}$$

This solution can now be compared with Eq. (14) to assess the impact of separability assumption. Dividing Eq. (16) by (14) gives the multiplier by which the two J_p solutions differ for the bending moment loading

$$J_p \text{ Multiplier} = (\beta + \beta_c/n)/(B_1'/B_1) \tag{17}$$

It can be shown that the multiplier has a value less than 1. Analyses for the J_p multiplier were carried out for a pipe with R/t = 10, and n = 3 and 10. For n = 3, the multiplier is 0.45 and 0.53 for Θ/π value of 0.2 and 0.5, respectively. For n = 10 and same crack lengths, the multiplier is 0.25 and 0.37. For the range of variables just considered, the multiplier ranges from 0.25 to 0.53.

DISCUSSION

The J-integral estimation solution derived in this paper is applicable to a wide range of crack lengths. The solution includes the effects of pipe R/t on the β factors. The solution is most useful for calculating J directly from the load- displacement curve available from pipe fracture experiment or finite element analysis. A comparison of two special cases of this solution with certain additional assumptions indicated that the solution based on elastic β factor or certain separability assumption may significantly overestimate the plastic component of J.

The solution presented in this paper is suitable for inferring the materials resistance to the initiation of crack growth directly from pipe fracture experiments. J-integral solution suitable for developing J-resistance curve is currently under development. Solutions for axial tension and combined tension and bending loading are needed for application to piping systems. These solutions are currently under development.

It was shown that the solution for φ_{cp} is independent of the plasticity model employed as long as the plastic solution depended on LF where L is the applied load and F is the shape factor appearing in the stress intensity factor solution. The crack displacement solution developed here also can be applied to other crack geometries and loadings (Mode I) when appropriate F and B solutions are used. Work utilizing this approach is being carried out for laboratory fracture specimens (compact tension and three-point bend bar specimens).

The crack displacement solution developed here can also be used to scale load vs. load-point displacement curves from different crack lengths and pipe sizes. This type of scaling has been used successfully for compact tension and bend bar specimens, and is commonly known as the key curve method. For flawed pipes, the scaling relationship is obtained directly from Eq. (10) when all terms in the g function are retained. Eq. (10) in its more general form is

$$\varphi_{cp}/F_1 = g(MF_b/\pi R^2 t).$$

This result indicates that plotting $MF_b/\pi R^2 t$ against φ_{cp}/F_1 would provide suitable scaling of data from different crack lengths and pipes of the same material.

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