Transient Fracture Resistance in the Weak Toughening Limit

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ABSTRACT

A weak toughening limit is defined for brittle materials possessing increases in fracture resistance comparable to the intrinsic resistance associated with bond rupture. The micromechanics of toughening by ligamentary bridging in this limit is then coupled to crack wall displacements, perturbed from their values in the untoughened state in magnitude alone. A self-consistent scheme is demonstrated for calculating fracture resistance as a function of crack length using this perturbation, allowing strengths of polycrystalline ceramics to be predicted from a knowledge of microstructural variables.

KEYWORDS

Alumina, J-integral, ligamentary bridging, R-curves, strength.

INTRODUCTION

A brittle material is characterized by a tendency to separate into two or more pieces, i.e. to fracture, on the application of a tensile load. At failure, the system (the material plus the loading) is at a point of instability, and if the kinetics of crack propagation are rapid with respect to the rate of increase of the loading, the load at failure is well defined and is regarded as the "strength". Clearly, mechanical energy is being exchanged with the work necessary to create the new surfaces at the instability point, and any process which increases the surface work will stabilize the system, thereby increasing the fracture strength (Cook and Clarke, 1988).

The microstructures of many ceramics (long regarded as the prototypical brittle materials) may be modified so as to produce increases in the surface work on crack propagation, with enhanced strength properties. The most spectacular example of this is probably the transformation toughened zirconia materials (Marshall, 1986), in which a dilatant phase transformation is activated in the highly stressed crack-tip region of the material, shielding the crack and absorbing energy which would otherwise have gone into bond rupture. A somewhat more prevalent toughening mechanism, occurring in polycrystalline materials, especially alumina (Knehans and Steinbrech,

1982; Swanson *et al.*, 1987), is that arising from the ligamentary bridging of the crack behind the tip, by interlocking or unruptured grains. In this latter case, which we will focus on here, energy is absorbed in the rupturing of the bridges, increasing the mechanical energy needed to propagate the crack above that required for bond rupture.

In order to understand the relationship between strength and microstructure the explicit form of the increase in the surface work with crack length is needed, the connection being made through the micromechanics of the bridging process. Here, we consider this problem in the limit of weak bridging - identifying a "weak toughening limit" in a general sense. The parameters required to define a strength are considered first, followed by an examination of the mechanics of toughening by ligamentary bridges. The J-integral is introduced and used in its usual sense in the calculation of the steady-state increment of the surface work. An explicit statement of the weak toughening limit is then made and a self-consistent scheme demonstrated for the calculation from the J-integral of transient fracture properties in terms of microstructural variables.

ENERGETICS OF THE STRENGTH TEST

Figure 1 plots the total potential energy of a body containing a crack of area A loaded by a uniform applied stress σ_a. The surface potential energy is assumed linear in the crack area:

$$U_S = 2\Gamma_0 A \tag{1}$$

where the proportionality constant $2\Gamma_0$ is the surface work or (linear) fracture resistance. The mechanical potential energy is given by (Sneddon, 1946)

$$U_M = -\psi^2 \sigma_a^2 A^{3/2}/E \tag{2}$$

where E is the Young's modulus and ψ is a geometrical constant of order unity. An equilibrium point for this system is defined by a zero in the first derivative of the total energy, or

$$\mathrm{d}U_S/\mathrm{d}A = -\mathrm{d}U_M/\mathrm{d}A \qquad (\textit{Equilibrium}) \tag{3a}$$

and the equilibrium is unstable if the second derivative is negative at this point, or

$$\mathrm{d}^2U_S/\mathrm{d}A^2 < -\mathrm{d}^2U_M/\mathrm{d}A^2 \qquad (\textit{Instability}) \tag{3b}$$

The applied stress for the system in Fig. 1 has been chosen so as to create a a point of unstable equilibrium for the crack area A_0. Any increase in σ_a beyond σ_f makes it energetically favorable for the crack to increase in area, and if the kinetics of this non-equilibrium propagation are rapid σ_f is perceived as a strength.

Obviously the mechanics of the system become more complicated if the surface potential energy is a general, non-linear function of crack area. Thus it is usually more convenient to operate with the first derivatives of the energy functions: the mechanical energy release rate,

$$G = -\mathrm{d}U_M/\mathrm{d}A \tag{4}$$

and the fracture resistance,

$$R = \mathrm{d}U_S/\mathrm{d}A \tag{5}$$

The point of equilibrium instability defined by (3) is now given by

$$G = R \textbf{ and } \mathrm{d}G/\mathrm{d}A > \mathrm{d}R/\mathrm{d}A \qquad (\textit{Unstable Equilibrium}) \tag{6}$$

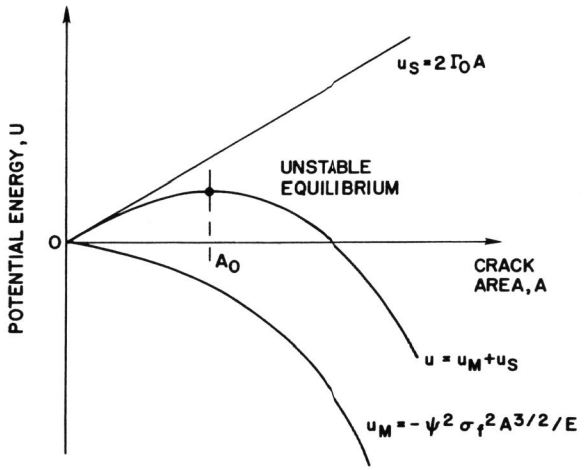

Fig. 1. Plot of potential energy as a function of crack area for a body with a linear surface work under uniform applied stress.

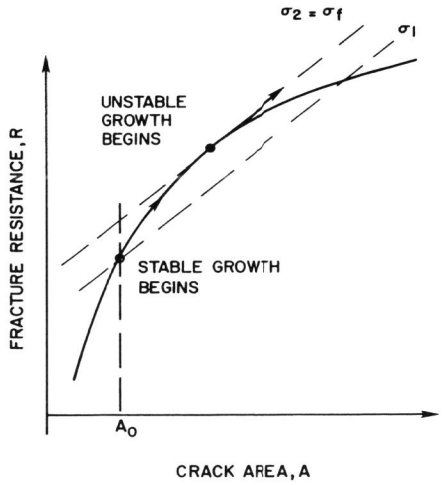

Fig. 2. Plot of fracture resistance as a function of crack area for a body with non-linear, increasing surface work (logarithmic coordinates). The dashed lines indicate mechanical energy release rates at the beginning of stable and unstable crack growth.

As we are usually interested in the changes in the independent variable σ_a giving rise to the fulfillment of (6), and as σ_a appears only as an amplitude term in (2) it is convenient to work in logarithmic co-ordinates, so as amplitude changes appear as linear shifts. A strength prediction from an R-curve plot is shown in Fig. 2. For stresses below σ_1 the system is unchanged, as none of the requirements of (6) are met. As the stress is increased above σ_1 stable growth of the crack begins, as the first but not the second condition of (6) is filled, until at $\sigma_2 = \sigma_f$ the conditions of (6) are both met and the system fails. The shape of the R-curve has dictated the extent of stable growth beyond A_0, and the increase in strength from σ_1 to σ_f The next section examines the reasons underlying R-curve behavior leading to this enhanced strength for materials toughened by ligamentary bridges.

WEAK TOUGHENING BY LIGAMENTARY BRIDGES

The increasing R-curves of a steel, a zirconia, and a polycrystalline alumina are shown in Fig. 3 (Gudas and Alexander, 1981; Swain, 1986; Readey *et al.*, 1987). (From this point forward we will discuss crack propagation in terms of the linear variable c, the crack length, under the assumption that the crack area may be expressed in terms of this single parameter, and that the mechanical energy release rate is similarly expressible, e.g. as for linear through-cracks or circular cracks.) All three materials show a tendency to an upper plateau of R at large crack extensions. In the case of steel, the sample was tested under plane stress conditions and the increasing value of R reflects the growing size of the plastic deformation zone around the crack tip during propagation. The plateau occurs when the zone size becomes fully constrained at the onset of plane strain conditions or by the sample size. For zirconia, the increase in R reflects the gradual formation of a transformation zone about the crack, followed by the dissipation of energy as the transforming particles are irreversibly unloaded in the crack wake. Steady-state is reached when the initially transformed particles are completely unloaded. The mechanism of increasing R for the alumina is as follows. Up to a crack extension of d the resistance is that of the weak interfaces in the material (typically the grain boundaries), $R = 2\Gamma_0$. For $c \geq d$ bridges form behind the crack tip by either interlocking or unruptured grains, probably aided by any residual stresses fixed in the structure during processing, at average separation d. As the crack propagates more and more bridges are formed and the increasing fracture resistance reflects the work done deforming the bridges, or overcoming the friction during bridge pullout, $R > 2\Gamma_0$. Eventually, at a crack length c^*, the first bridge formed ruptures as a consequence of the increased crack opening and a bridging zone of constant size $(c^* - d)$ propagates with the crack giving rise to a plateau resistance $R = R_\infty$.

The inherent brittleness of the alumina is reflected in the low absolute values of R at similar crack extensions for those in the steel and zirconia. However, more important to us here is the relatively low increase of R to the steady-state value: about a factor of three for the alumina, compared with an order of magnitude for the zirconia, and over two orders of magnitude for the steel. The small increase is an indication of the weak energy absorption by the bridging ligaments in comparison to the interatomic bonds. Hence we may refer to this material, and others which behave similarly, as "weakly toughened", as the fracture resistance increases are comparable to the intrinsic fracture resistance associated with bond rupture. In the next section we present the mathematical framework to describe the increases in fracture resistance by bridging, and relate the steady-state fracture resistance to the work necessary to rupture a ligament, thereby permitting an exact definition of a "weak toughening limit". Explicit advantage will then be taken of the weak toughening in these materials to derive transient fracture resistance curves.

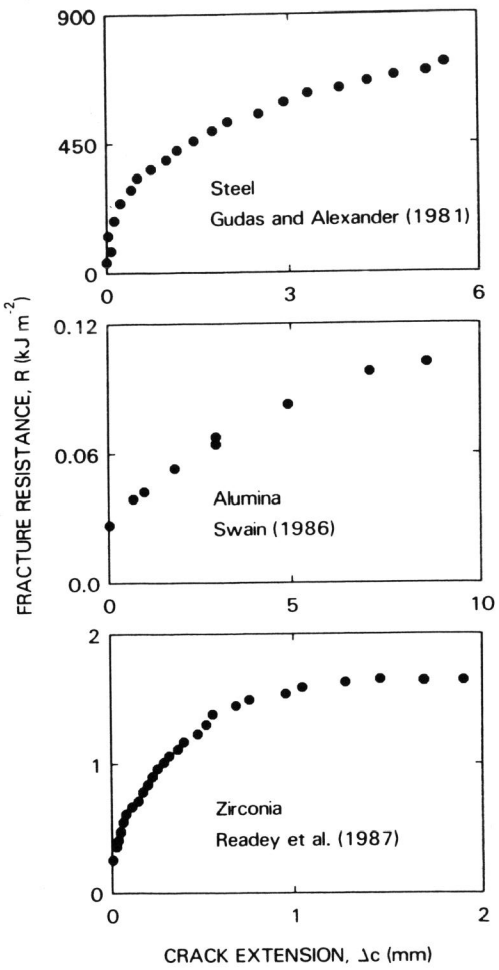

Fig. 3. Plots of fracture resistance as a function of crack extension for steel, alumina, and zirconia.

THE J-INTEGRAL AND STEADY-STATE FRACTURE RESISTANCE

A powerful method for calculating the fluxes of energy arising during crack propagation is the use of the J-integral which involves the integration of the strain energy density and the work done by tractions in a body, along paths beginning and ending on opposite crack faces. As the integral is path-independent, equilibrium conditions may be generated by appropriate summation of different integrated paths. The integral is defined over a path S by (Kanninen and Popelar, 1985)

$$J = \int_S [\mathscr{U} \mathrm{d}z - \mathbf{T} \cdot (\partial \mathbf{u}/\partial r) \mathrm{d}s] \tag{7}$$

where $\mathscr{U}$ is the strain energy density and $\mathbf{T}$ is the traction vector (defined by $T_i = \sigma_{ij} n_j$, where σ_{ij} is the stress and n_j is the normal to the integration path element ds). $\mathbf{u}$ is the displacement, over which the tractions do work, and dr and dz are coordinate system elements. (We concentrate on a system with circular symmetry, where r is a radial co-ordinate in the crack plane.) Making a circuit beginning at the crack mouth, around the exterior of the specimen, followed by return to the starting point traversing the crack surface leads to

$$J_a - J_\mu - J_0 = 0 \tag{8}$$

where $J_a = G_a'$ is the experimentally determined contribution to the path from the applied loadings, $J_0 = 2\Gamma_0$ the contribution from the interatomic bond rupture processes , and J_μ the contribution from the ligamented zone. (Here we have taken the usual convention that anticlockwise paths are positive. The sum over the paths is zero as there are no sinks of energy within the complete circuit.) For paths along the crack face there is no contribution from the strain energy ($dz = 0$) and the J-integral is simply expressed in terms of the tractions exerted:

$$J_\mu = \int_{c-b}^{c} - \mathbf{T} \cdot (\partial \mathbf{u}/\partial r) \mathrm{d}s \; - \int_{c}^{c-b} - \mathbf{T} \cdot (\partial \mathbf{u}/\partial r) \mathrm{d}s \tag{9}$$

where c is the crack length, b is the length of the bridged zone, and $\mathbf{T}$ represents the tractions exerted by the ligaments in the zone. An individual ligament exerts a force resisting extension approximated by $\sigma(u)d^2$, where d is an average ligament separation and hence $\sigma(u)$ an effective ligament stress. If we assume that the ligament forces act only normal to the crack face, such that the only non-zero component of $\mathbf{T}$ is $T_u = \sigma(u)$, where we now take u to be the local displacement of a crack wall, and $ds = dr$, (9) may be written as

$$J_\mu = - 2 \int_0^{\alpha} \sigma(u) \mathrm{d}u \tag{10}$$

where α is the semi crack opening at the end of the bridged zone (ie at $r = c - b$). (10) relates the J_μ term to the area under the $\sigma(u)$ curve. The ligament at $r = d$ undergoes the cycle from $u = 0$ at $c \leq d$ to $u = u^*$ at $c = c^*$ during crack extension.

In steady-state crack propagation the crack opening at the end of the bridging zone is equal to that necessary to cause ligament rupture. If we write this critical crack opening as $2\alpha = 2u^*$, the integral in (10) may then be evaluated explicitly:

$$J_\mu^{\max} = - 2 \int_0^{u^*} \sigma(u) \mathrm{d}u = 2\Gamma_\ell \tag{11}$$

The maximum increment in the work of fracture is equal to the work per unit area necessary to rupture a ligament, $2\Gamma_\ell$. The steady-state work of fracture is given by $R_\infty = J_0 + J_\mu^{\max} = 2\Gamma_0 + 2\Gamma_\ell$. A weakly toughened material is then defined as one in which Γ_0 and Γ_ℓ are comparable.

THE WTL AND TRANSIENT FRACTURE RESISTANCE

The material property which controls toughening by ligamentary bridging at the most fundamental level is the stress-extension function for the bridges, $\sigma(u)$. We choose here for illustration ligaments dominated by a decreasing component, in which case the constitutive equation of the ligament may be written (Cook *et al.*, 1987)

$$\sigma(u) = - \sigma^*(1 - u/u^*)^m \tag{12}$$

where σ^* is the magnitude of the peak restraining stress, and m is an exponent characterizing the nature of the restraint. Inserting (12) into (10) and integrating yields

$$J_\mu = 2\Gamma_\ell[1 - (1 - \alpha/u^*)^{m+1}] \tag{13}$$

However, the equilibrium profile of the crack depends on the total fracture resistance, R, which includes the contribution from J_μ.That is, $\alpha = \alpha(J_\mu)$, and we are left with an implicit equation for J_μ. Basically we require $\alpha(c)$, remembering that α is the crack opening at $r = d$, to obtain $J_\mu(c)$. To this point our analysis has been exact and it is the specification of $\alpha(c)$ in the weak toughening limit that our approximation is introduced.

We may write an equilibrium relation for $\alpha(c)$ for fracture resistance forces infinitesimally localized at the crack tip (Sneddon, 1946):

$$\alpha = \psi(G_a'/Ec)^{1/2}(c^2 - d^2)^{1/2} \tag{14}$$

(14) has two exact limits for a bridged crack. The first is that there is no bridging at all, $\Gamma_\ell = 0$, in which case $G_a' = 2\Gamma_0$ and

$$\alpha = \alpha_0 = \psi(2\Gamma_0/Ec)^{1/2}(c^2 - d^2)^{1/2} \tag{15}$$

The second limit is that the bridging is totally constrained to a region about the crack tip, $c >> c^*$, in which case $G_a' = 2\Gamma_0 + 2\Gamma_\ell$ and $\alpha = \alpha^* > \alpha_0$. (15) is really a first-order perturbation on the untoughened profile (and somewhat equivalent to the assumption made in the earlier work of Cook *et al.*, (1987)) but is not self-consistent in that increasing the toughening does not alter the crack opening. A second-order, and self-consistent, perturbation maintains the shape of the profile, but iteratively solves for the opening displacements, relaxing the constant bridging constraint and the localization constraint.

We begin our derivation of a self-consistent weak toughening limit (WTL) by writing the equilibrium relation (8) generally, as

$$\begin{aligned} G_a' &= 2\Gamma_0 + J_\mu \\ &= 2\Gamma_0[1 + (\Gamma_\ell/\Gamma_0)(J_\mu/2\Gamma_\ell)] \end{aligned} \tag{16}$$

and hence we may write α using (14) and (15), as

$$\alpha = \alpha_0[1 + (\Gamma_\ell/\Gamma_0)(J_\mu/2\Gamma_\ell)]^{1/2} \tag{17}$$

where we relax the constraint that J_μ maintain constancy at either of its extreme values, but still imagine that the microstructural influence is localized to a crack tip region. (17) inverts to yield

$$J_\mu = 2\Gamma_\ell[(\alpha/u^*)^2/(\alpha_0/u^*)^2 - 1]/[\Gamma_\ell/\Gamma_0] \tag{18}$$

Simultaneous solution of (13) and (18) self-consistently yields the extent of fracture resistance increase, $J_\mu/2\Gamma_\ell$, as a function of the degree of ligament opening, α/u^*. Crack length is introduced by the parameter α_0/u^* via (15). (Coupling the degree of fracture resistance increase to the crack opening at $r = d$ essentially breaks the localization condition.) There is an upper bound to this latter parameter of

$$(\alpha_0/u^*)^{\max} = (1 + \Gamma_\ell/\Gamma_0)^{-1/2} < 1 \tag{19}$$

Combining (15) and (19) then specifies the crack length at which the fracture resistance attains its steady-state value

$$c^* \sim u^{*2}E/\psi^2 R_\infty \tag{20}$$

Figure 4 plots (13) and (18) for different crack lengths indicating the points of simultaneous solution. Figure 5 plots the R-curve derived from these points and also plots the R-curve derived from the first-order perturbation which does not maintain self-consistency. As can be seen the self-consistent WTL reaches steady-state fracture resistance at shorter crack lengths than the first-order WTL, a consequence of the greater crack openings required to maintain equilibrium in the former. Also plotted in Fig. 5 are the mechanical energy release rates at the points of unstable equilibrium for the two R-curves. The self-consistent WTL predicts a greater strength, and greater crack length at failure than the first-order WTL, for cracks in a material of initial length $\sim d$ (as might be expected in a typical microstructure), highlighting the sensitivity of strength prediction to the exact shape of the R-curve.

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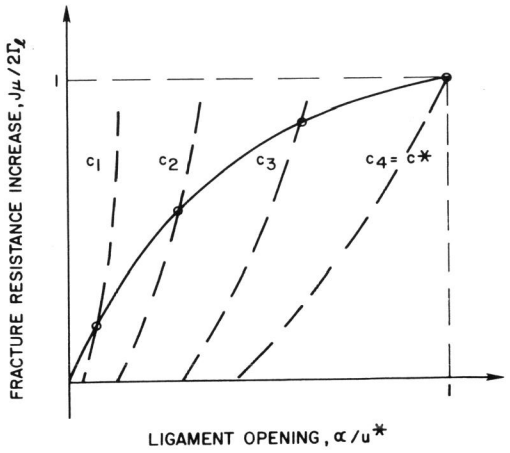

Fig. 4. Plot of the extent of fracture resistance increase as a function of the extent of ligament opening. The solid line is an exact prediction from the J-integral, the dashed lines indicate points of solution as a function of crack length in the self-consistent weak toughening limit.

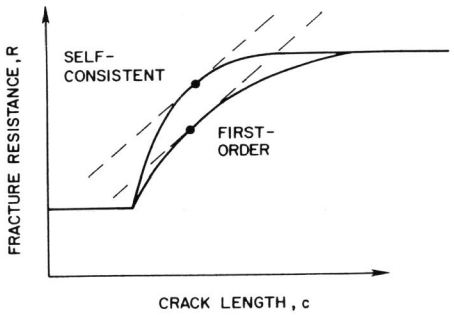

Fig. 5. R-curve plots for the self-consistent and first-order weak toughening limits. The dashed lines indicate the points of instability for uniform applied stresses, and show the greater strengths predicted by the self-consistent solution.