Statistical Failure Approach to a Heterogeneous Material

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ABSTRACT

A brittle failure model of a syntactic foam is proposed. The studied material is composed of hollow glass microspheres in an epoxy resin matrix. In tension, the biggest microspheres break, leading to local damage in the structure. The proposed model allows us to take into account the large scatter of the failure strength related to the microstructure. A Weibull statistical approach is used and correlated with the microspheres size distribution. The fracture toughness and its scatter can then be determined in relation with the microstructure.

KEYWORDS

Syntactic foam; failure; model; statistics; toughness.

Introduction

The material studied is a syntactic foam made of hollow glass microspheres embedded in an epoxy resin. A large scatter of the tensile strength is observed experimentally. This is to be related to the heterogeneity of the properties of the individual components and to the size distribution of the microspheres. Their diameter ranges between 20 and 200 μm for a wall thickness of 1 to 2μm. We studied the brittle failure of this material using Weibull statistics (1) , which allow to predict size effects and which can be used to evaluate the fracture toughness and its scatter, in relation with the microstructure of the material.

Characteristics of the material

The mean characteristics of the material are given in table I

Table I

volume fraction Fv:	0,63
microsphere diameter Ø	120 µm <Ø <20µ m
microsphere wall thickness e:	1 µ m<e<2 µ m
density ρ :	0,63 Kg/dm^3
Young's modulus E:	2700 MPa
tensile strength Σ_R	28 MPa
compressive strength Σ_{rc}	85 MPa
Specific compressive strength: Σ_{rc}/ρ (steel: 30, concrete 12)	135
toughness K_{IC}	0,78 MPa$\sqrt{m}$

The material is macroscopicaly isotropic (2) . Figure 2 shows the size distribution of the glass microspheres as measured either by seaving or by quantitative micrography. A volume element Vo of side length equal to 3 times the biggest diameter or 15 times the mean diameter, is considered as being homogeneous (Vo = 0,216 mm^3).

STATISTICAL APPROACH TO A SYNTACTIC FOAM FAILURE.

The failure probability P_{fv} of a volume V according to Weibull is given as:

$$P_{fv} = 1 - \exp\left[- \int \left(\frac{(\sigma - \sigma_s)}{\sigma_u} \right)^m \frac{dv}{v_0} \right]$$

Where σ is the maximum principal stress, σ_s, σ_u and m are material parameters. This expression is valid if $\sigma \geq \sigma_s$.

As initially, owing to the manufacturing process, the material contains large microcracks, cavities and broken microspheres, we consider the threshold stress σ_s as being equal to zero.

For tensile specimens of volume V the failure probability is given as

$$P_{fv} = 1 - \exp\left[- (\sigma/\sigma_u)^m \frac{dV}{Vo} \right]$$

allowing the determination of σ_u and m. Twenty two tensile specimens were used, yielding m = 13,9 and σ_u = 60 MPa, as shown on figure 1. (Vo = 0,216 mm^3)

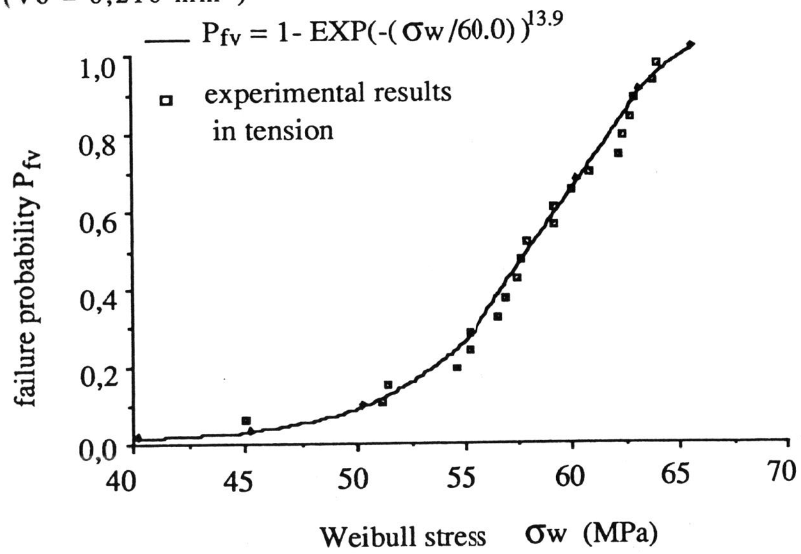

Fig. 1. Failure probability of foam FM 280.

This failure probability must be related to the defect size distribution. Let n(σ) be the number of defects per unit volume corresponding to a fracture stress less than σ. It follows that the failure probability is given as

$$P_{fv} = 1 - \exp(-nV)$$

So that $n(\sigma)$ should be equal to $(\sigma/\sigma_u)^m/V_o$.

On the other hand $n(\sigma)$ is a function of the size distribution of the defects[3]

$$n(\sigma) = \int_{\varnothing_c(\sigma)}^{\infty} f(\varnothing)\, d\varnothing$$

where $\varnothing_c$ is the critical defect size for a stress σ.
If these defects are considered as penny shaped crack in a material of fracture toughness K_{IC}.

$$\varnothing_c = \frac{\Pi}{2\alpha^2} \left(\frac{K_{IC}}{\sigma}\right)^2$$

where α is a parameter taking into account the interaction between defects.
The derivative of $n(\sigma)$ with respect to σ, yields.

$$dn/d\sigma = (\pi/\alpha^2)\ (K_{IC}/\sigma)^2\ f(\varnothing c)/\sigma$$

So that from the above relation between $n(\sigma)$ and the Weibull parameters the following relation should be observed.

$$f(\varnothing) = (\Pi/2\alpha^2)^{\frac{m}{2}+1}\ (m/\Pi V_o)\ (K_{IC}/\sigma_u)^m\ /\varnothing^{\frac{m}{2}+1}$$

This distribution function of the defects can be compared with the histogram of the size distribution of microspheres. (fig2).
A fair fit is obtained for the larger microspheres by adjusting the α interaction parameter at a value of 1.4 (fig2). Indeed, it is experimentally observed that the larger microspheres only are the ones which break.

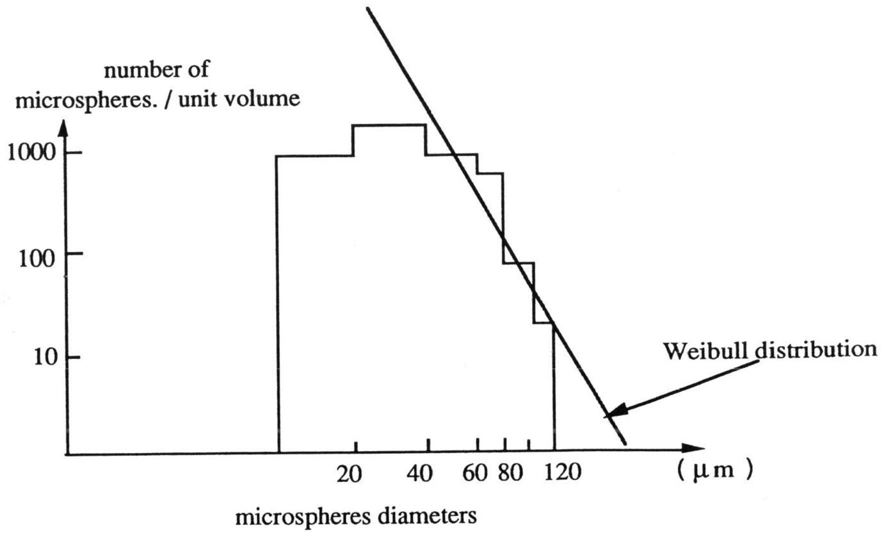

Fig. 2. Comparison between the Weibull and the microspheres diameters distributions.

FRACTURE TOUGHNESS.

The statistical approach just described can be used to deduce the fracture toughness of the material, as well as its scatter in relation with microstructural parameters.
The maximum principal stress at a small distance r from the crack tip is given by:

$$\Sigma_1 = K_1 \left(1+\sin\theta/_2\right)\left(\cos\theta/_2\right) \sqrt{2\pi r}$$

where K_I is the stress intensity factor and θ the polar angle from the tip.
The Weibull stress as previously defined is given in that case by

$$\sigma_w^m = \int_v \Sigma_1^m\, dv/V_0 = \frac{B}{V_0} \int_{-\pi}^{+\pi} \int_0^{\infty} \Sigma_1^m\, r\,dr\,d\theta$$

The damage in the material at the crack tip limits the value of Σ_1 for r smaller than a certain distance r_c, so that this integral is bounded.
In a uniform uniaxial stress field, the stress concentration factor K_T in the microsphere walls was computed by a finite elements method [4]. It was found that

$$K_T = \sigma/_{\Sigma} = \beta\,(\mathrm{Eeff})\sqrt{\phi/_e}$$

where β (Eeff) is a decreasing function of the effective modulus of the syntactic foam, ϕ the diameter of the microspheres and e their wall thickness. Hence largest microspheres are the first ones to break, and, as the applied stress increases, smaller and smaller microspheres are destroyed. This reduces the stiffness of the syntactic foam, thus increasing K_T. A catastrophic increase of the damage follows for a stress of about 26 MPa. The final fracture occurs when the resistance of the resin is reached, leaving the microspheres with a diameter smaller than about 25 µm, unbroken.
We now assume that, near the tip, the stress Σ_1 keeps a constant value Σ_R corresponding to the fracture stress σ_R of the microsphere glass walls equal to about 700 MPa. With β = 2,3, the value achieved for the undamaged modulus E_o, ϕ = 25µm and e = 1,5 µm, Σ_R equal to 72 MPa. Here the undamaged modulus is used because the material is only damaged in a very small area near the tip.
The distance r_c at which Σ_1 reaches Σ_R is given as in IRWIN's model by:

$$r_c = \frac{1}{\Pi}\left(\frac{K_I}{\Sigma_R}\right)^2 f^2(\theta)$$

where $f(\theta) = (1+\sin\theta/2)\cos\theta/2$
Hence

$$\sigma_w^m = \frac{B}{V_0}\int_{-\pi}^{+\pi}\left\{\int_0^{r_c}\Sigma_R^m\, r\, dr\, d\theta + \int_{r_c}^{\infty}\left[\frac{K_{IC} f(\theta)}{\sqrt{2\pi(r - r_c/2)}}\right]^m r\, dr\, d\theta\right\}$$

yielding (m>4):

$$K_{IC} = \left[\frac{4\pi}{1+\frac{35(m-3)}{8(m-2)(m-4)}}\,\frac{V_0}{B}\right]^{1/4}\left(\frac{\sigma_u}{\Sigma_R}\right)^{m/4}\Sigma_R\left[\mathrm{Ln}\left(\frac{1}{1-P_{fv}}\right)\right]^{1/4}$$

taking m = 13.9, Vo = 0.296 mm^3, B = 10 mm, σ_u = 60 MPa, Σ_R = 72 MPa.
For the mean value of the toughness, in front of the crack where $f(\theta) = 1$, the distance r_{co} is equal to 38 µm, corresponding to a volume fraction f_{25} = 0.2 of microspheres 25 µm in diameter, if

$$r_{co} = L = \left(\frac{\pi}{3\sqrt{2}\, f_{\phi}}\right)^{1/3}\phi$$

L being the mean distance between the microspheres in a FCC arrangement.

By replacing Σ_R by $K_{IC}/\sqrt{\pi r_{co}}$, the formula for K_{IC} can also be written in the following way:

$$K_{IC} = \left[\frac{4/\pi}{1+\frac{35}{8}\frac{m-3}{(m-2)(m-4)}}\,\frac{V_0}{B\, r_{co}^2}\right]^{1/m}\sigma_u\sqrt{\pi r_{co}}\left(\mathrm{Ln}\left(\frac{1}{1-P_{fv}}\right)\right)^{1/m}$$

This relating K_{IC} to the size ϕ and the volume fraction fv of the largest unbroken microspheres, which can be measured on a fractograph.
The bounds given in table 2 are found for a failure probability equal to 0.1 and 0.9 respectively.

P_{fv}	0.1	0.5				0.9
K_{IC} $MPa\sqrt{m}$ theory	0.69	0.79				0.85
K_{IC} experimental		0.70	0.78	0.79	0.83	

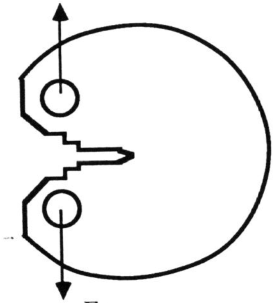

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Table 2. Fracture toughness values and failure probabilities.

The measured fracture toughness values all fall within these bounds and are close to the predicted value for a failure probability of 0.5.

CONCLUSION

This brittle failure model, based on the Weibull statistic, gives a good estimation of the failure probability of a structure made with a syntactic foam. The fracture toughness and its scatter is determined from the Weibull parameters identified with a large number of tensile tests on simple specimens, the thickness of the structure, and the characteristic microstructure size, which is the smallest microspheres diameter remaining unbroken, surrounded by the corresponding volume fraction of resin.

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