Photoelastic Analysis of a Crack at Bimaterial Interface

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ABSTRACT

Photoealsticity technique is used for the investigation of the near crack tip stress field in a bimaterial plate with a crack along the interface. The specimen is made of dissimilar birefringent resins and loaded in a state of remote shear. Both isochromatic and isoclinic fringe patterns are obtained. The LEFM solution to the same problem is evaluated and used to generate the theoretical isochromatics using a computer plotting program. Comparison between the experimental and theoretical solutions shows that there exist regions where the two agree fairly well and regions where they differ substantially. The agreement between the two stress fields seems to be better in the region where the theory predicts crack closure. However,no actual crack closure is observed along the entire crack faces, although the theory implies a contact zone for one half of the crack length.

KEYWORDS

Interfacial cracks; bimaterial; mixed mode fracture; photoelasticity; composites.

INTRODUCTION

The stress field of a crack along the interface of a bimaterial is quite different from that in a homogeneous material. It is characterized by the fact that regardless the mode of the far field load,normal and shear stresses effects coexist and they are coupled in a way that defies separation. As a result, the stress state is not analogeous to that of combined Mode I and Mode II in classical fracture mechanics. However, analytical solutions,within the framemork of LEFM, have been obtained to describe the crack tip displacement and stress fields. Early contributions were given by Williams(1959),Erdogon(1963),England (1965), Sih and Rice(1964) and Rice and Sih(1965). Recent developments along this line include the works by Huchinson et al(1987),Rice(1988) and Shih and Asaro(1988), among others. There have been some intensive discussions on the local oscillatory nature of the solution and the physical meaning of the stress intensity factors. In a recent paper by Rice(1988), the definitions and the interpretations of the characterizing parameters as well as the limitations on the application of the solutions are presented in details. The works by Comninou(1977), Knowles and Sternberg(1983) represent attempts to avoid the local oscillatory nature of the solution by pursuing alternative approaches.

Compared to the advancement in analytical and numerical studies, experimental investigation of the interfacial crack problems seems to be lacking behind. To the best of our knowledge, no previous study has been reported regarding detailed measurement of the crack tip stress, strain or displacement fields, until the recent work of Chiang and Lu(1988) who investigated the displacement field of a crack along the interface of a bimaterial under pure shear, using the technique of laser speckle photography. It was found that while the tangential displacement jump across the crack faces agreed quite well with the theoretical prediction, the opening displacement jump across the crack faces deviated substantially from the analytical values. And there was no observable closure along the entire crack faces. This paper represents another attempt of ours to provide some experimental observation on the interfacial crack behavior. The configuration is again a central interfacial crack under pure shear, because under such a loading the interesting features of the analytical solution manifest themselves most pronouncedly. The technique of photoelasticity was adopted, because it gives stress information directly.

REVIEW ON THEORETICAL SOLUTIONS

Within the framwork of linear elasticity, the derivations based on both Williams' product solution technique(1959) and Rice's analytic function analysis(1988) give the same closed form solutions of stress and displacement fields near a crack tip. However, the solutions predict unbounded oscillatory stresses and mutual penetration of the crack surfaces at the very vicinity of the crack tip. The result has raised the question on the validity of the solutions. England(1965) and Rice(1988) reasoned that so long as the size of this zone is much smaller than the crack length, the solutions neverthless represent the true physical state in the crack tip region outside that small zone. Based on the crack tip displacement solution, Rice(1988) further examined the size of the near tip contact zone under different loading conditions and for different combinations of materials. He showed that the ratio of contact zone size to crack length is very sensitive to the phase angle ψ of the traction acting on the remote surface,but it is insensitive to the variation of the materials, at least within the combinations of the common engineering materials. Thus the applicability of the solutions is largely affected by the nature of the remote loading and to a much less extent to the bimaterial constant. It is reasonable to argue that in order to avoid the oscillatory nature of the solution one should seek fundamentally different routes to solving the problem. The work by Knowles and Sternburg(1983) circumvents the difficulty by assuming material nonlinearity, whereas Comninou(1977,1978) eliminates the obstacle by invoking a frinctionless contact zone between the crack faces near the crack tips. However, neither analysis provides a closed form solution to the crack tip field.

PHOTOELASTIC EXPERIMENT

The specimen geometry and the loading condition are as shown in Fig.1, together with

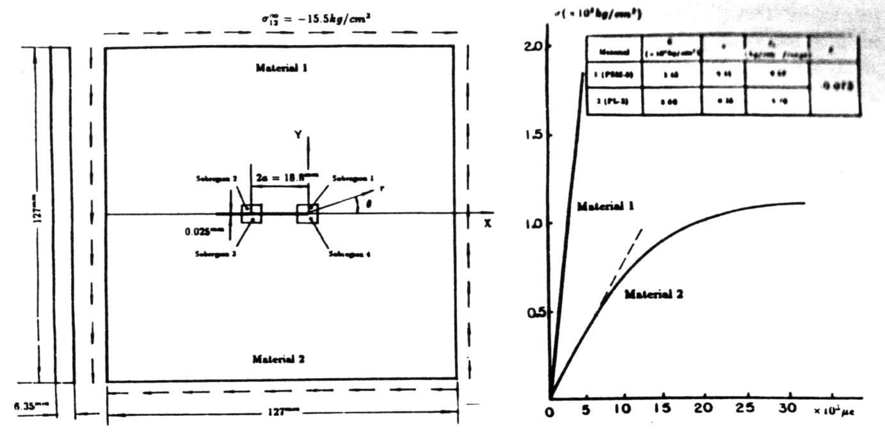
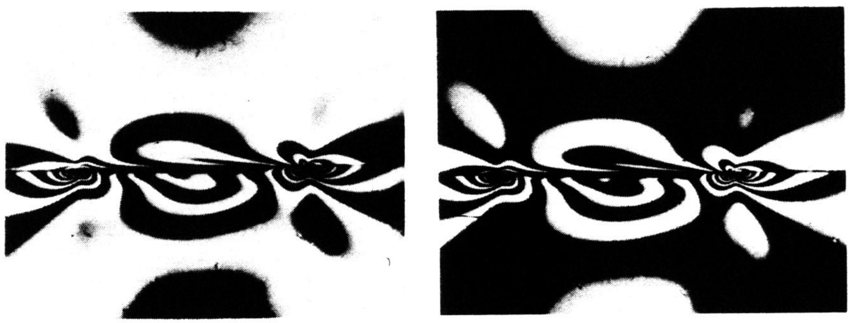


Fig. 1 Specimen Configuration and Material Constants

materials' constants with associated calibration curves. Ideally one would like to have two photoelastic materials having substantial difference in mechanical property but identical in optical property(i.e. with the same birefringent figure of merit). But this condition is difficult to achieve. What we eventually selected were the PSM-5 and PL-2 photoelastic materials (from Measurement Group, Inc.). To avoid the use of a third material to bond the two together, the PL-2 was cast directly onto a PSM-5 sheet of about 0.64 cm thick. Before casting a thin Teflon strip (about 10 μm thick and 18.8 mm long) was placed near the center of the bonding edge of the PSM-5 sheet. After curing the Teflon strip was taken away resulting in a simulated central crack at the interface of the bimaterial. PL-2 resin is a room curing material (curing time was about 24 hrs) with fairly low modulus. The low curing temperature resulted in negligible residual stress at the interface and the crack tip. When the curing process was completed,the specimen was machined into the final dimensions as shown in Fig.1. Calibration of the materials' optical and mechanical properties was performed during the same time when testing of the specimen was being carried out to avoid the possibility of time effect on the photoelastic materials. Remote pure shear was applied via a specially designed loading jig(Chiang and Lu,1988). The state of pure shear was checked by using a single photoelastic plate and observed by the existence of an uniform fringe in an area much larger than that surrounding the central crack.

The main part of the loading device was a modified four-bar linkage which was placed into a Tinius Olsen testing machine and loaded in tension along one of the diagonals. To avoid loading rate effect, the slowest cross-head speed (1 in/min) was used. The whole loading system was placed inside a polariscope and the resulting isoclinic patterns (at every 10°) and isochromatic patterns were recorded. (The isochromatic patterns were obtained at a load of 400 lbs (181.6 kg) whereas the isoclinics were recorded at a load of 100 lbs (45.4 kg) so as to minimize the effect of interfering isochromatics.) Figure 2 shows the resulting isochromatic fringes in both dark and light fields. Since the optical fringe value is different for the two materials, the fringes are not continuous across the interface. Detailed isochromatics at the two crack tips are shown in Fig. 3(a), and the map of isoclinic parameters at the crack tip are shown in Fig.4(a).



dark field light field

Fig. 2 Whole Field Isochromatic Fringe Patterns

COMPUTER GENERATED PHOTOELASTIC FRINGE PATTERNS BASED ON LEFM SOLUTION

For easy comparison with the theoretical solution we generated the theoretical isochromatic and isoclinic fringe patterns based on the LEFM solution given by Williams(1959) and others (*e.g.* Rice and Sih,1965; Rice,1988). We note from Rice's analysis(1988) that this solution may not properly describe the true state of the stress field, especially when the remote traction is pure shear. However, in the absence of a better model we felt that such a comparison would still be useful in the interpretation of experimental result, and could be used as a reference for further theoretical development.

If μ_1, μ_2, ν_1 and ν_2 denote the shear moduli and the Poisson's ratios of the two materials of the upper and lower half panels, the so-called bimaterial constant may be defined as

$$\epsilon = \frac{1}{2\pi} ln\left(\frac{\frac{\kappa_1}{\mu_1} + \frac{1}{\mu_2}}{\frac{1}{\mu_1} + \frac{\kappa_2}{\mu_2}}\right), \tag{1}$$

where $\kappa_1 = \frac{3-\nu_1}{1+\nu_1}$ and $\kappa_2 = \frac{3-\nu_2}{1+\nu_2}$ in the case of plane stress. For an infinite bimaterial plate with a central crack along the interface under remote tension σ_{22}^{∞} and remote shear σ_{12}^{∞}, the stress state at the crack tip, following Shih and Asaro(1988), may be characterized as follows,

$$\sigma_{ij} = \sum_{\alpha=1}^{2} \frac{Q_\alpha}{\sqrt{2\pi r}} g_{ij}^{(\alpha)}[\theta, \epsilon ln(r/2a); \epsilon], \tag{2}$$

where (i,j) denote the polar coordinates (r, θ) centered at the right side crack tip, ϵ is the bimaterial constant and a is the half length of the crack, Q_α are the stress intensity factors and $g_{ij}^{(\alpha)}$ are functions defined in the appendix I of the paper of Sih and Asaro(1988).

We note that there exist different definitions of the complex stress intensity factors introduced to the interface problem (Sih and Rice, 1964; Rice,1988; Shih and Asaro,1988), and seeking a proper definition is still an issue under discussion. However, the solution itself is definite and independent from the form of the stress intensity factor. According to Shih and Asaro(1988), the complex stress intensity factor $\mathbf{Q} = Q_1 + iQ_2$ is related to the far field stresses, bimaterial constant and the crack length parameter by

$$\mathbf{Q} = [(\sigma_{22}^{\infty} - 2\epsilon\sigma_{12}^{\infty}) + i(\sigma_{12}^{\infty} + 2\epsilon\sigma_{22}^{\infty})]\sqrt{\pi a}. \tag{3}$$

Under pure shear $\sigma_{22}^{\infty} = 0$, Q_1 and Q_2 are given by $Q_1 = -2\epsilon\sigma_{12}^{\infty}\sqrt{\pi a}$ and $Q_2 = \sigma_{12}^{\infty}\sqrt{\pi a}$, respectively. In the present investigation $\epsilon = -0.073$ and $\sigma_{12}^{\infty} = -15.5 kg/cm^2$. The stress components σ_{rr},$\sigma_{r\theta}$ and $\sigma_{\theta\theta}$ of the crack tip field are then determined using Eqn.(2), and the maximum shear stress τ_{max} calculated according to:

$$\tau_{max} = \sqrt{(\frac{\sigma_{rr} - \sigma_{\theta\theta}}{2})^2 + \sigma_{r\theta}^2}. \tag{4}$$

Theory of photoelasticity states that the order of the isochromatic fringes N at a point on the model is proportional to the difference of the principal stresses, i.e. the maximum shear stress, at that point. This relation can be expressed as

$$2\tau_{max} = \sigma_1 - \sigma_2 = \frac{N f_\sigma}{h}, \tag{5}$$

where f_σ is the material fringe value and h the thickness of the model. For a given stress field, if f_σ and h are known, the fringe order N at any point is readily determined by

$$N = \frac{2\tau_{max} h}{f_\sigma}. \tag{6}$$

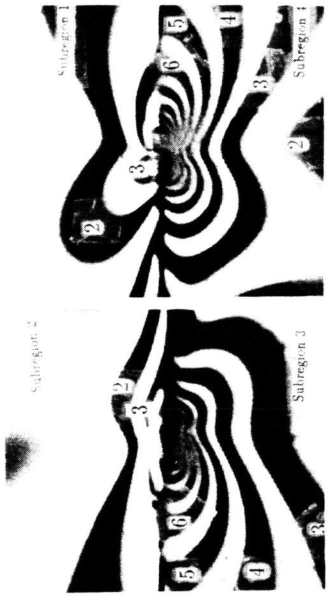
The principal stress directions at any point can be calculated using

$$\tan 2\beta = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}, \tag{7}$$

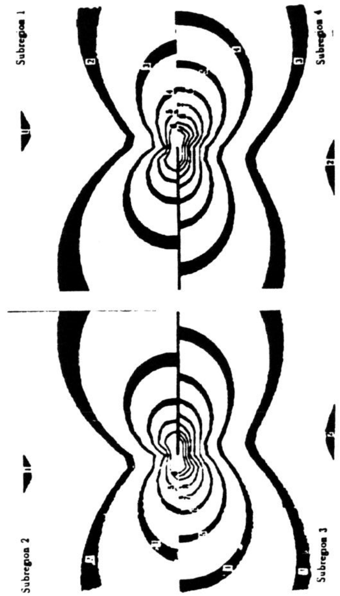
where β is the angle between the principal stress σ_1 and x axis. The polar components of the stress in Eqn. (2) can be converted into cartesian components via stress transformation. Thus, the contours of the calculated isochromatic fringes and the principal direction β near the crack tip can be drawn automatically using a plotting program. The computer generated isochromatic and isoclinic fringe patterns are shown in Fig.3(b) and Fig.4(b),respectively. In Fig.3 and 4 the same magnification is maintained for both the experimental and theoretical fringe patterns.

COMPARISON BETWEEN EXPERIMENTAL RESULT AND THEORETICAL SOLUTION

The results are divided into four different subregions as marked in Fig. 1. A glance of the comparison shown in Fig. 3 and 4 indicates a qualitative similarity between the two solutions. While the isochromatic fringe loops are quite similar in appearance to those of Mode II fringes in a homogeneous material, they do differ in detail. A valley is formed as the fringe loops along the crack tip. The radial line that goes through all the lowest

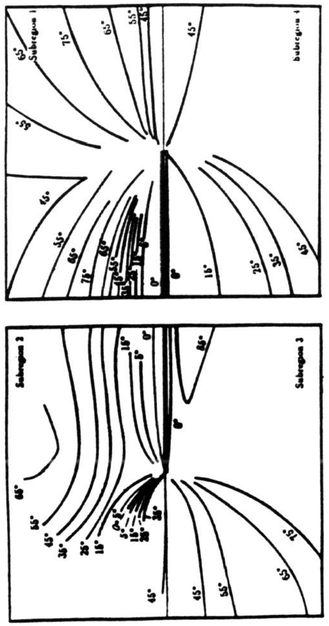


(a) Enlarged Near Crack Tip Isochromatic Patterns

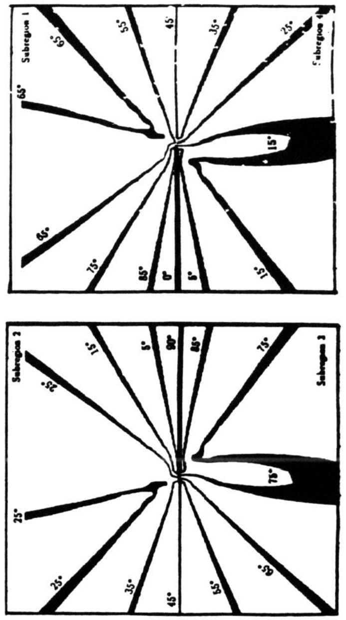


(b) Computer Generated Isochromatic Patterns in The Same Regions

Fig. 3



(a) Maps of Isoclinics in Near Crack Tip Regions



(b) Computer Generated Isoclinics in The Same Regions

Fig. 4

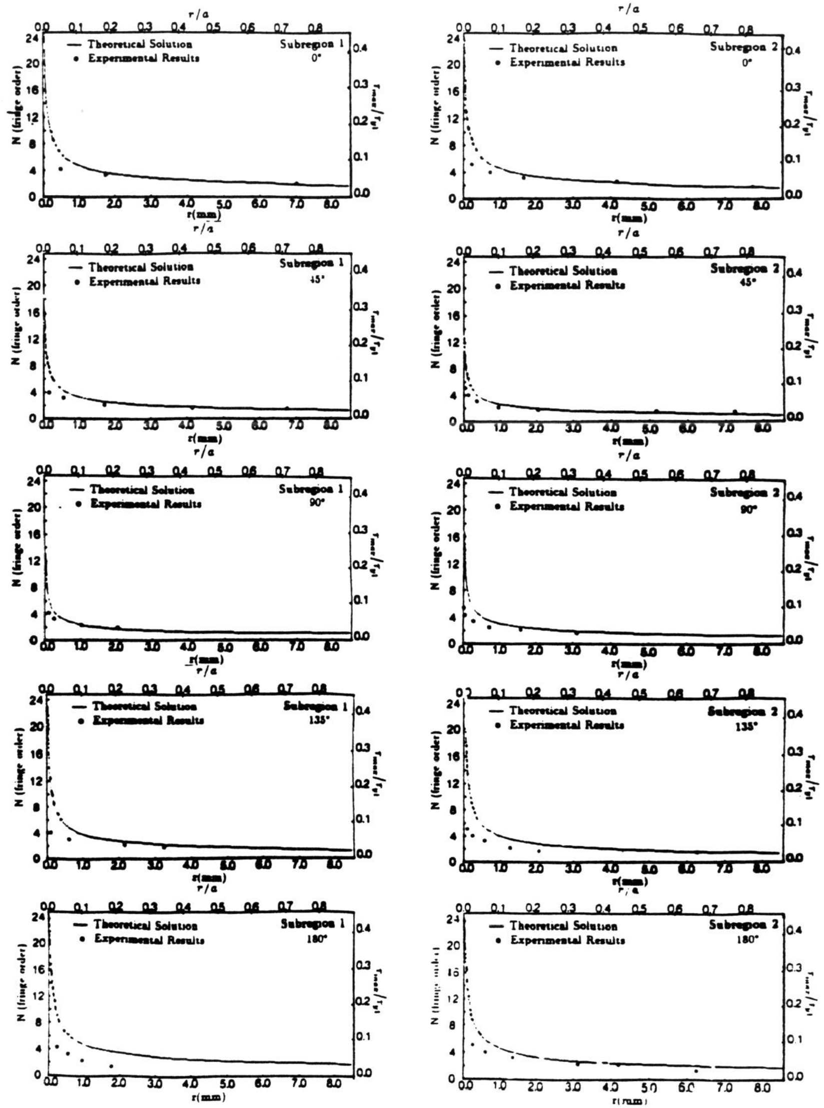


Fig. 5 Comparison of The Experimental And Theoretical Maximum Shear Stress Values along Radii at Different Angles (a) Subregions 1 and 2

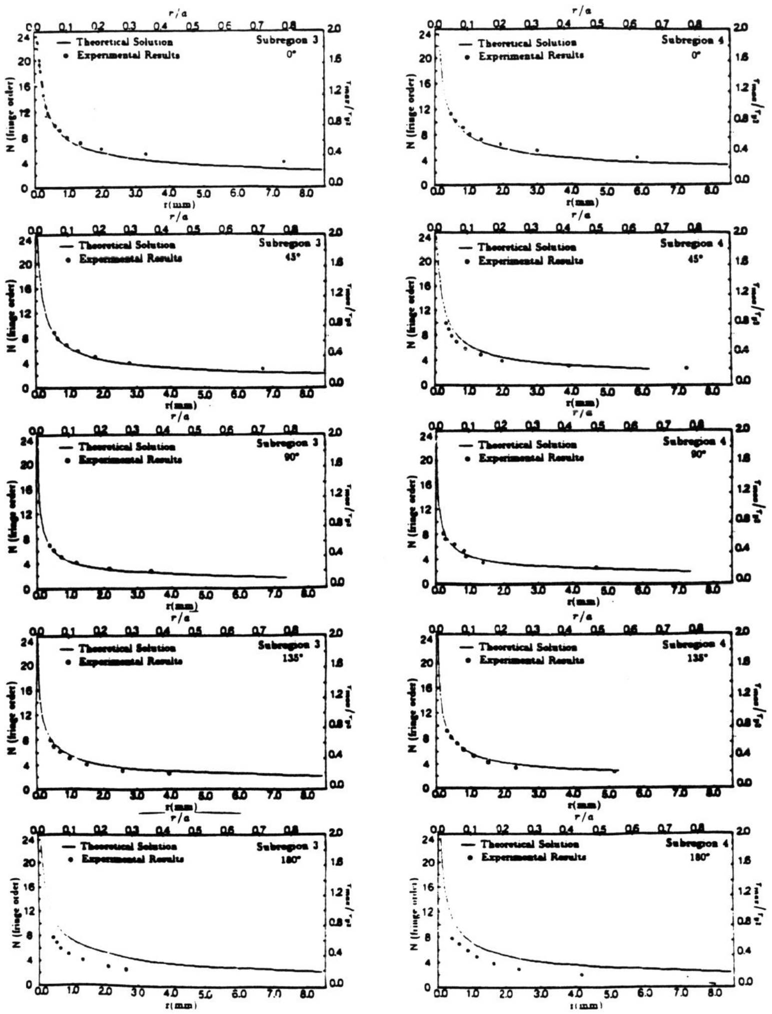


Fig. 5 Comparison of The Experimental And Theoretical Maximum Shear Stress Values along Radii at Different Angles (b) Subregions 3 and 4

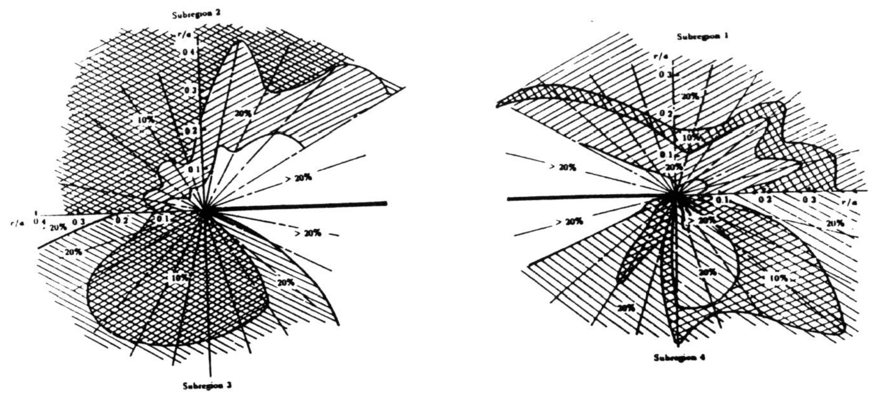


Fig. 6 Regions Where Theoretical Solution and Experimental Results Coincide with Different Tolerence

points of the valley is the direction along which the gradient of maximum shear stress is the largest, and in the present study this direction is near $\pm\pi/2$ from the interface. Indeed it is this deviation from $\pm\pi/2$ that is characteristic of the bimaterial, for in a homogeneous material this angle would have been exactly $\pm\pi/2$.

The comparison of the isochromatics also indicates that there is a better agreement between theory and experiment in the soft (subregions 3 and 4 in the lower half) plate than that in the hard (subregions 1 and 2 in the upper half) plate. Furthermore, at the left side crack tip (subregions 2 and 3) where the theory predicts crack closure, the agreement is better, whereas at the right side crack tip (subregion s 1 and 4)where the theory predicts crack opening, the theoretical and experimental results deviate substantially. As in our previous study (Chiang and Lu,1988), we did not observe crack closure anywhere, although the theory indicate the closure of the entire left half of the crack. If there were contact between the two crack faces, it would have manefested itself in the shape of isochromatic fringes.

Better agreement is observed between the experimental and theoretical isoclinics. This, however, is expected, because isoclinics only indicate the direction of principal stresses and they are not influenced by the magnitude of stresses. A quantitative comparison of the maximum shear stresses along different radial lines from the crack tip is presented in Fig. 5. In order to provide an overall picture of the quantitative comparison we plotted a map for each crack tip indicating in numbers the percentage of difference between the theoretical and experimental results. And they are shown in Fig. 6.

CONCLUSIONS

We have presented a photoelastic study of the stress field of a crack lying along the interface of a bimaterial under remote pure shear. While the fringe loops bear resemblance of the Mode II fringes in a homogeneous material, they show the characteristic asymmetry indicating the existence of mixed mode stress state as predicted by theory. However, detailed comparison between experimental and theoretical results show a substantial difference in certain regions, while good agreement is obtained in others. Better agreement seems to occur in the region where the theory predicts crack closure. There is no observable crack closure anywhere along the crack, while the theory implies that left half of the crack is closed. The isochromatic fringes do not posseses a skew symmetry as dictated by theory.

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