On the Expansion of a Crack Layer's Active Zone

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ABSTRACT

The expansion of the active zone of a crack layer is considered. The corresponding force is taken as a difference between active and resistive parts. The active part is expressed in the form of the M integral of fracture mechanics, and is calculated using a newly developed semi-empirical method for energy calculations. The analysis suggests that Arrhenius type kinetic equation describes well the expansion of the active zone.

KEYWORDS

Crack Layer, Expansional force, Arrhenius equation

I. INTRODUCTION

It is a well documented phenomenon to date, that structural transformations (damage) precede the process of fatigue fracture in a variety of engineering materials (Haglan et al., 1973; Evans and Heuer, 1980; Mills and Walker, 1980; Donald and Kramer, 1981; Takemori and Kambour, 1981; Andrews and Barnes, 1982; Carpinteri, 1982; Botsis, et al., 1983; Chudnovsky and Bessendorf, 1983a); Chudnovsky, et al., 1983c. In these cases, the crack and it associated damage are inreparable.

The conventional approach to fatigue fracture is based on concepts of fracture mechanics, i.e., fatigue fracture can in general be correlated with either the stress intensity factor K_1, or the energy release rate J_1 (Hellan, 1984; Kanninen and Popelar, 1985). The critical values of these parameters are assumed to be a measure of fracture toughness. On the other hand both K_1 and J_1, are macro-parameters. Their use is justified as long as they uniquely characterize the stress and strain fields in the vicinity of a crack tip. However, in many cases K_1 or J_1 fail to correlate with the fracture process. Examples are large scale plasticity, deceleration of a large crack and the behavior of small cracks. Such discrepancies are attributed to the effect of damage (microcracking, slip planes, crazes, voids, etc.) and its interactions with the main crack. Therefore, damage accumulation seems to be an important characteristic of a fracture process which ought to be considered.

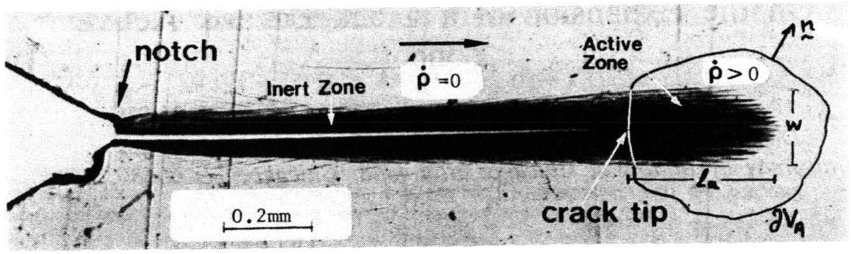

Fig. 1: An optical micrograph of a CL in polystyrene (ρ: damage density)

Within the crack layer (CL) model, a crack interacting with the surrounding damage is considered as a thermodynamic system. Within a CL we distinguish an active and an inert zones (Fig. 1). The model is based on a hypothesis of self-similarity of damage evolution and describes a fracture process in terms of elementary movements of the active zone, namely, translation, rotation, and deformation (Chudnovsky, 1984; 1986). The rates of these movements are treated as thermodynamics fluxes. The corresponding driving forces have been identified (Chudnovsky, 1984; 1986). The model derives the kinetic equation for active zone translation and proposes criteria of instability on the basis of the thermodynamics of irreversible processes. Assuming that translation of an active zone and crack extension coincide, the kinetic equation has been successfully applied to correlate fatigue fracture kinetics in different materials (Bessendorf and Chudnovsky, 1984; Bakar, 1985; Haddaoui et al., 1985; Nguyen and Moet, 1985; Botsis, et al., 1987a).

In this paper we discuss constitutive equations for active zone expansion as a basic life time and reliability predictions.

II. EXPERIMENTAL

Single edge notched tension specimens of 80mm gauge length, 20mm in width and 0.25mm thickness of plane isotropic polystyrene (PS) are used in this investigation. This material besides being transparent, preserves damage patterns induced during fracture for a relatively long period of time, thus facilitating post failure analysis. Details of the specimen geometry and loading conditions can be found elsewhere (Botsis, 1984; Botsis et.al., 1987b). Crack growth and the kinetics of damage are observed by an optical microscope and recorded in real time using a high resolution video system which is attached to the optical microscope.

Figure 2 illustrates the evolution of the active zone as a function of crack length for two loading levels (Botsis, 1988). The contours are traced from optical micrographs taken during the experiments and the vertical markers correspond to the respective crack tip. Note here that the shape of the active zone is loading history dependent. In these experiments craze density $\rho[mm^2/mm^3]$ is characterized by the area of middle planes of crazes per unit volume of the material.

The evolution of R_o and $\dot{e}$ (Botsis 1984, Botsis, 1988) as a function of dimensionless crack length, for the two experiments referred in Fig. 2 are shown in Figs. 3 and 4, respectively.

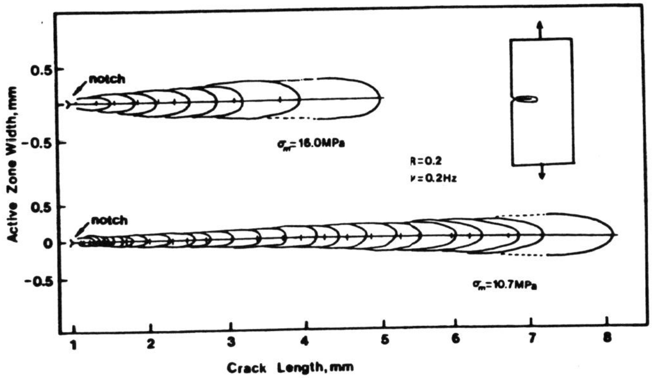

Fig. 2: Evolution of the active zone of a crack layer in polystyrene for two loading conditions.

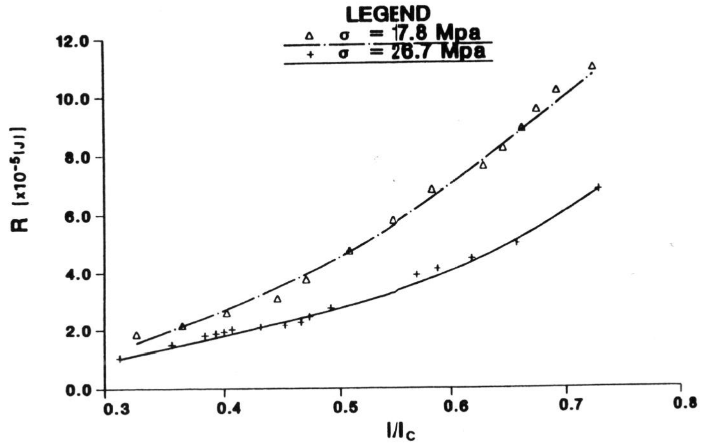

Fig. 3: Resistance moment R_o, plotted against dimensionless crack length for two loading conditions.

III. SEMI-EMPIRICAL METHOD FOR ENERGY CALCULATIONS

The thermodynamic force X^{exp}, for expansion of the active zone is expressed as the difference between active and resistive parts (Chudnovsky 1984; 1986), i.e.,

$$X^{exp} = M - \gamma R_o \tag{1}$$

Here M is the energy release rate due to expansion of the active zone, γ is the specific energy of damage and R_o is the resistance moment associated with the expansion of the active zone.

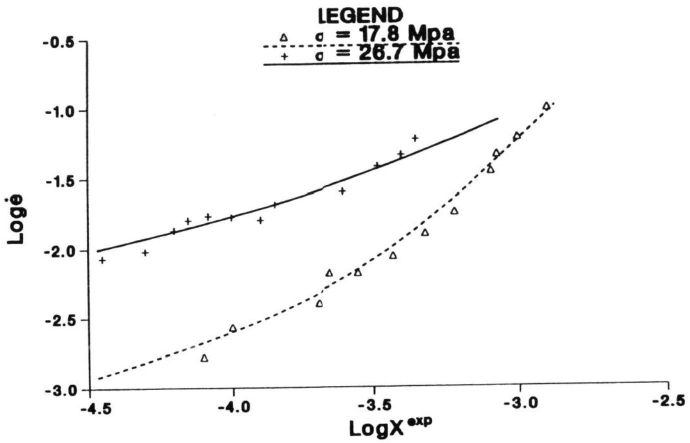

Fig. 4: Rate of expansion of the active zone as a function of dimensionless crack length.

Calculation of the energy release rate M, requires a detailed knowledge of the stress and strain fields resulting from the crack damage interaction. Solutions of the problem have been formulated for a few simple types of crack-microcrack array configurations (Hoagland and Embry, 1980; Chudnovsky and Karchanov, 1983b; Kunin, 1983; Chudnovsky et al., 1984; Shiue and Lee, 1985; Rose, 1986; Rubinstein, 1986; Chudnovsky et.al., 1987).

For a large number of microcracks and a random configuration, solutions are extremely tedious and time consuming numerical procedures.

An alternative approach to energy calculations based on recent developments of stress analysis of a CL (Chudnovsky et al., 1987) and experimental measurements will be employed herein to calculate the energy release rate M.

a. Energy Release Rate Associated with Damage Zone Expansion

The problem of a elastic medium with damage zone (crack layer) around the main crack tip is considered. Using the same method as (Chudnovsky and Shaofu Wu, 1988b), the relation between Energy Release rate for isotropic expansion and Eshelby's energy momentum tensor P_{ij} may be expressed as (while the main crack is stationary).

$$-\frac{\partial \Pi}{\partial e} = \int_{\partial V_A} x_k P_{ki}\, n_i\, d\Gamma \tag{2}$$

where

$$P_{ki} = f\,\delta_{ki} - \sigma_{ij} u_{j,k} \tag{3}$$

Π is the potential energy of the solid, f is strain energy density. x_k is the component of rectangular Cartesian coordinate system, n_i is the components of normal vector on the integral path and ∂V_A is the boundary of damage zone.

b. Semi-Empirical Analysis of Crack-Damage Zone Interaction Problem

In order to calculate the energy release rate associated with damage zone expansion, one needs to know the stress $\underset{\sim}{\sigma}(\underset{\sim}{x})$ and displacement $\underset{\sim}{u}(\underset{\sim}{x})$ fields. Solutions of crack-damage interaction problem are known for just a few relatively simple models of damage. For an intensive damage in a form of microcrack (craze) array similar to that shown in Fig. 1, the solution is not known. Therefore, we employ the semi-empirical analysis (Chudnovsky and Owenzdou, 1988a). Then the stress and displacement can be expressed as

$$\underset{\sim}{u}(\underset{\sim}{x}) = \underset{\sim}{u}^o(\underset{\sim}{x}) + \int_{-L} \underset{\sim}{\delta}(\xi)\, \underset{\sim}{\Phi}(\xi,\underset{\sim}{x})\, d\xi + \int_{V_A} \underset{\sim}{C}(\xi)\, \underset{\sim}{\Phi}(\xi,\underset{\sim}{x})\, d\xi \tag{4}$$

$$= \underset{\sim}{u}^c(\underset{\sim}{x}) + \underset{\sim}{u}^A(\underset{\sim}{x})$$

$$\underset{\sim}{\sigma}(\underset{\sim}{x}) = \underset{\sim}{\sigma}^o(\underset{\sim}{x}) + \int_{-L} \underset{\sim}{\delta}(\xi)\, \underset{\sim}{F}(\xi,\underset{\sim}{x})\, d\xi + \int_{V_A} \underset{\sim}{C}(\xi)\, \underset{\sim}{F}(\xi,\underset{\sim}{x})\, d\xi \tag{5}$$

$$= \underset{\sim}{\sigma}^c(\underset{\sim}{x}) + \underset{\sim}{\sigma}^A(\underset{\sim}{x})$$

where $\underset{\sim}{u}^o$, $\underset{\sim}{\sigma}^o$ is the displacement and stress field due to remote loading, $\underset{\sim}{\delta}(\xi)$ is the crack opening displacement of main crack, $\underset{\sim}{\Phi}$ is the second Green's tensor. $\underset{\sim}{C}(\xi)$ is the craze opening density in the damage zone. In the case under consideration it was observeed that the crazes are parallel to the main crack $(n(0,1))$ and the vector of craze opening $\underset{\sim}{b}$ has also only one (b_2) nonzero component, i.e., $\underset{\sim}{C}(0,C_2)$. As the same as (Chudnovsky and Shaofu Wu, 1988), the average craze opening density $\langle C_2 \rangle$ in the core of damage zone is obtained by the assumption that the total stress intensity factor of main crack $K^{tot} = 0$, the average craze opening density $\langle C_2 \rangle$ at peripheral part of the damage zone is measured experimentally. Using equation (4) and (5), the energy momentum tensor can be rewritten as:

$$P_{ij} = P_{ij}^{A} + P_{ij}^{AC} \tag{6}$$

where:

$$P^{A}_{ij} = f^{A}\delta_{ij} - \sigma^{A}_{jk} u^{A}_{k,i}$$

$$P^{AC}_{ij} = f^{AC}\delta_{ij} - \sigma^{C}_{jk} u^{A}_{k,i} \tag{7}$$

and

$$f^{A} = \frac{1}{2E}\left[\left(\sigma^{A}_{11}\right)^2 + \left(\sigma^{A}_{22}\right)^2 - 2\nu\,\sigma^{A}_{11}\sigma^{A}_{22} + 2(1+\nu)\left(\sigma^{A}_{12}\right)^2\right] \tag{8}$$

$$f^{AC} = \frac{1}{2E}\left[\sigma^{C}_{11}\sigma^{A}_{11} + \sigma^{C}_{22}\sigma^{A}_{22} - 2\nu\sigma^{C}_{11}\sigma^{A}_{22} + 2(1+\nu)\,\sigma^{C}_{12}\sigma^{A}_{12}\right]$$

Then the energy release rate associated with damage zone expansion may be expressed as

$$\frac{\partial \Pi}{\partial e} = -\left[\int_{\partial V_A} x_k P^{A}_{ki} n_i d\Gamma + \int_{\partial V_A} x_k P^{AC}_{ki} n_i d\right]$$

$$= -\left[M^{A} + M^{AC}\right] = -M^{Atot} \tag{9}$$

In the evaluation of M^{Atot}, the parameters of material and geometry of specimen is the same as experiment. The M^{Atot} with respect to dimensionless crack length (which normalized by critical crack length) for two cases of loading condition is shown in Fig. 5. The expansion force X^{exp} with respect to dimensionless crack length for two cases of loading condition is shown in Fig. 6.

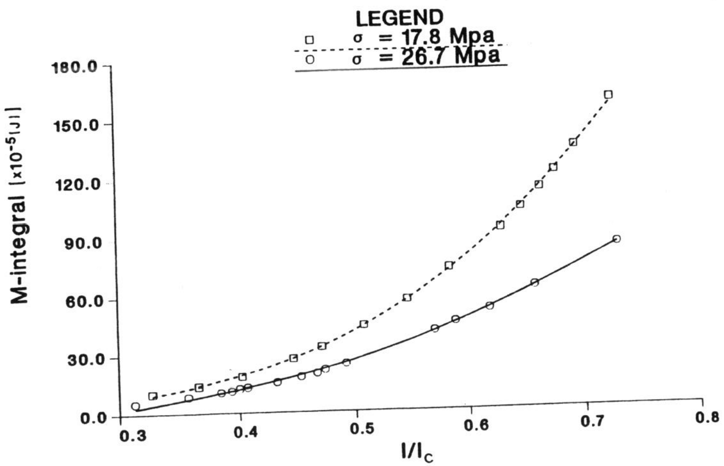

Fig. 5: Energy release rate for expansion of the active zone plotted against dimensionless crack length.

On the basis of experimental observations, various propositions can be made with respect to the constitutive equations for expansion, distortion and rotation of the active zone. Herein we concentrate on the expansion of the active zone only.

In this respect we have examined two candidates for constitutive relations, Onsanger's type linear relation

$$\dot{e} = LX^{exp} \tag{10}$$

and Arrhenius type exponential relation

$$\dot{e} = C_1 \exp\left[-C_2 X^{exp}\right] \tag{11}$$

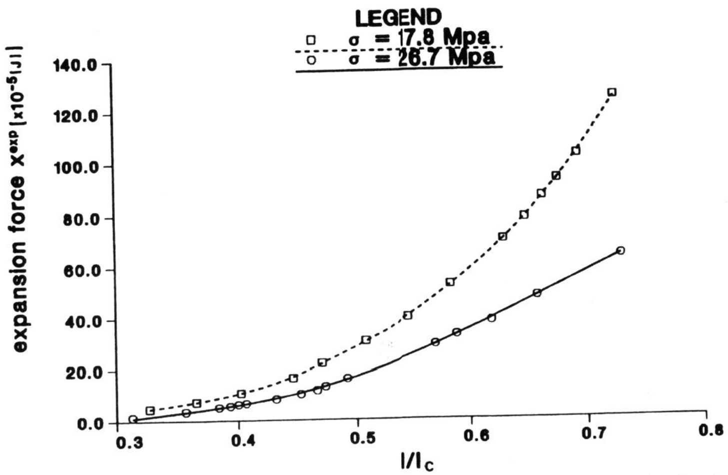

Fig. 6: Variation of the expansion force with dimensionless crack length.

between the rate of expansion $\dot{e}$, and the corresponding force X^{exp}. The coefficients L[sec.Joules^{-1}] (Eq. 10), C_1[sec] and C_2[Joules^{-1}] (Eq. 11) are obtained from linear regression analysis.

Analysis of CL stability has shown that the critical energy release rate A_{1c}, can be expressed as the product of the specific enthalpy of damage γ^*, and the resistance moment at critical propagation, namely, $A_{1c} = \gamma^* R_{1c}$. This process is a manifestation of mainly new damage nucleation. Thus γ^* is directed related to the corresponding specific energy (i.e., energy of nucleation of new damage). On the other hand, γ which appears in Eq. (1) corresponds to the energy of a relative slow process which follows the nucleation. Furthermore, it is well recognized that the difference between the energy of new phase nucleation and the energy of phase growth is essential (Wyatt and Dew-Hughes, 1974). Accordingly, it is expected that γ^* is significantly greater than γ.

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For our model material, γ^* has been found to be of the order of 30 J/m^2 (Botsis et al., 1987b). An experimental method to evaluate γ is being developed. To compare the two candidates (Eqs. 10 and 11) for constitutive equations for active zone expansion, we considered three different values of γ, namely, 10%, 15% and 25% of γ^* respectively.

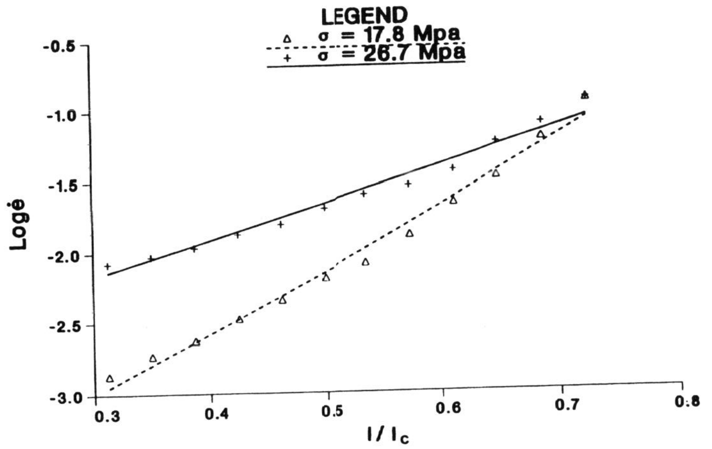

Fig. 7: Log $\dot{e}$ vs. Log X^{exp} for two loading conditions.

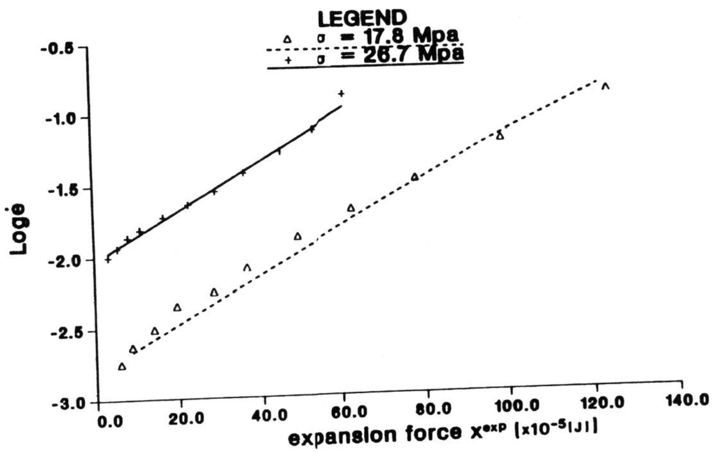

Fig. 8: Log $\dot{e}$ vs. X^{exp} for two loading conditions.

Fracture propagation kinetics are observed under two different fatigue load histories. Whereas the frequency and load ratio were the same, the level of mean stress is 16.0 MPa and 10.7 MPa, respectively.

Figure 7 shows the logarithm of the rate of expansion as a function of the logarithm of X^{exp}. The non-linear trend as well as the values of the slope of the straight lines suggests that simple Onsanger relationship is not adequate.

The solid lines in Fig. 8 represent the right-hand side of 11. The data points are measurements of the rate of expansion of the active zone. The correlation coefficient for both set of data was of the order of 0.9 and the coefficient C_2 was found to be the same for both experiments and each value of γ. If an Arrhenius type constitutive relationship was to describe the expansion rate, C_2 should be the same in this case since both experiments were performed under different load levels and the same temperature. This is very strong evidence that Eq. 11 could be adopted to describe the expansion rate of the active zone. In order, however, to unquestionably assess the applicability of an Arrhenius type kinetic equation for active zone expansion, experiments with different loading rates and temperatures should be available.

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