Kinetic Strength of Solids

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ABSTRACT

The kinetic strength of a solid has been analyzed empirically and theoretically. It is found that the statistical reaction rate theory can explain satisfactorily the nonlinear behavior between fracture strength and time while the linear empirical relationship is limited in its scope.

KEYWORDS

Strength of solid bodies, Kinetic strength, Fracture strength, Time dependent strength, Reaction rate theory

The time dependent kinetic strength of solids has been studied for over half of a century. In general, two levels of approach have been employed. One is submicroscopic atomic consideration and the other may be referred to as supermacroscopic continuum investigations. The latter is mostly phenomenological which results in numerous empirical relationships. One of the most extensive investigations is that done by Zhurkov (1965). Under a state of constant stress creep condition more than 50 different kinds of solids including metallic and nonmetallic, amorphous and crystalline, oriented and unoriented systems were recorded the stress dependent of the time-to-break data. Even data on the temperature variations were tested and analyzed. It was found that the logarithm of time-to-break and the applied uniaxial tension were linearly related as

$$t_b = t_o \exp[(U - \gamma\sigma)/kT] \tag{1}$$

where t_b is time-to-break,
t_o is a constant,
U is a constant which may be related to the activation energy of the solid,
γ is a positive definite constant,
σ is the applied constant stress,
k is Boltzmann constant and
T is the absolute temperature.

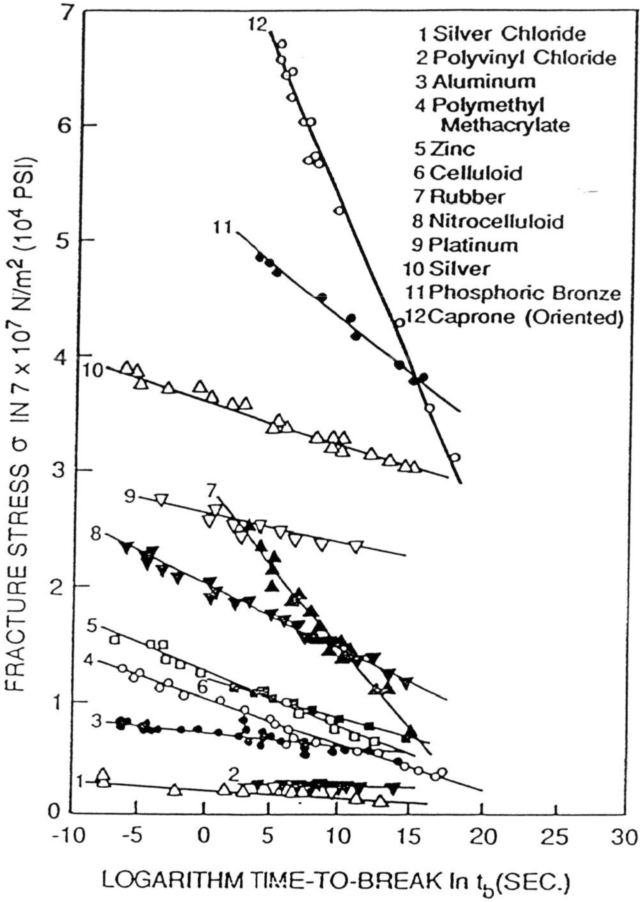

Fig. 1. Time-dependent fracture strength for solids, (after Zhurkov).

This result has been very useful as it was found to be reasonably consistent within a meso-stress range. Figure 1 shows the results of a variety of solids. It is seen that the time dependent fracture strength is indeed linearly related between the logarithm of time-to-break and the stress at fracture.

However, in reality, there is deviation from this empirical linearity when either super high stresses or relatively low stresses beyond the meso-stress range is encountered.

This short report is to address this point using an equation derived from the Tobolsky-Eyring-Hsiao expression (Krausz and Eyring, 1975). Essentially, the theory is based upon considerations at a submicroscopic atomic level. At this level, the statistical nature of any system behavior can be calculated according to the rates of forming and breaking of bonds. To make it easily accessible to engineering applications, the mathematical model used is a matrix of oriented submicroscopic bonds randomly distributed in an arbitrary domain. The fraction of intact bonds "f" measures the degree of integrity of the system. The integrity or strength of a solid body is identified by calculating $\dot{f}$, the rate of change of f as follows:

$$\dot{f} == K_r(1-f) - K_b f \tag{2}$$

where K_r is the rate of reformation of broken bonds,
K_b is the rate of breaking of intact bonds.

These rates can further be expressed in terms of the following submicroscopic quantities:

$$K_r = \omega_r \exp(-U/RT - \rho\psi) \, , \tag{3}$$

$$K_b = \omega_b \exp(-U/RT + \beta\psi) \, . \tag{4}$$

where ω_r is the frequency of motion of the broken bonds,
U is the activation energy,
R is the universal gas constant,
T is again the absolute temperature,
ρ is a positive definite stress modifier,
ψ is the stress in the bonding direction,
ω_b is the frequency of motion of the intact bonds and
β is a positive definite stress modifier.

For an oriented system under an applied stress $\sigma(t)$, the stress may be written as

$$\psi(t) = \sigma(t)/f(t) \, . \tag{5}$$

When expressions (2) and (5) are solved for a limiting value of $\psi(t)$, say at t_b, and let

$$\psi(t_b) = \psi_b \, , \tag{6}$$

and for unit step applied stress $\sigma(t) = \sigma u(t)$

$$\psi(t_b) = \sigma/f(t_b) \, . \tag{7}$$

The following expression for time-to-break can be obtained for the fracture stress σ.

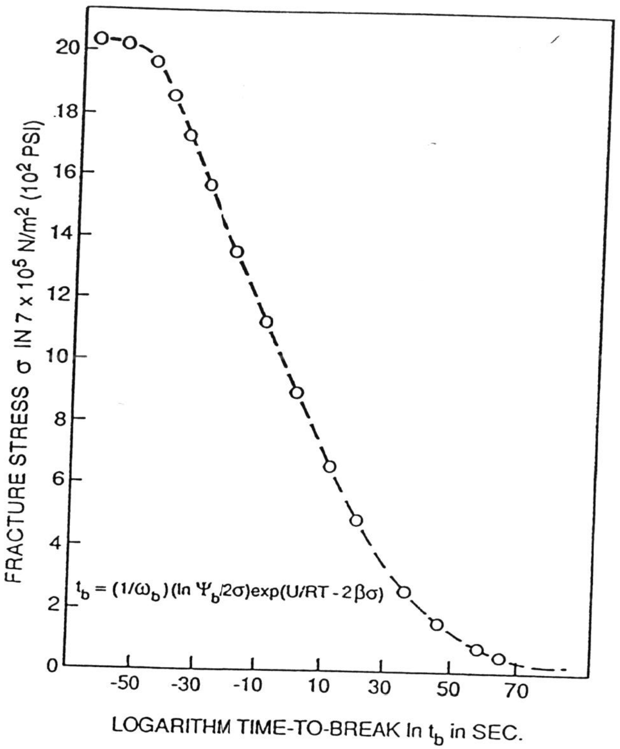

Fig. 2. Fracture stress vs. time-to-break.

$$t_b = (1/\omega_b)(\ln\psi_b/2\sigma)\exp(U/RT - 2\beta\sigma)\ . \qquad (8)$$

Using proper values (Ettouney and Hsiao, 1988) for the various quantities, (8) is plotted in Fig. 2. As can be seen, not only the mesostress range is satisfied, as the central section shows the linear relations between the logarithm of time-to-break and the fracture stress but also the nonlinear portions for both the high and low stresses beyond the linear region. From the expression (8), it is seen that K_r has been assumed zero; otherwise, the low stress region would move up as shown in Fig. 3 with $K_r \neq 0$.

All these tell us that using equation (2), the nonlinear relationships between high and low levels of the fracture stress and the logarithm of time-to-break can be matched. Indeed, this can be illustrated in Fig. 4 in which the data points for solids tested at elevated temperatures when low stress values become dominating were obtained earlier by Zhurkov and the curves were computed to show the possible representation at all stress levels.

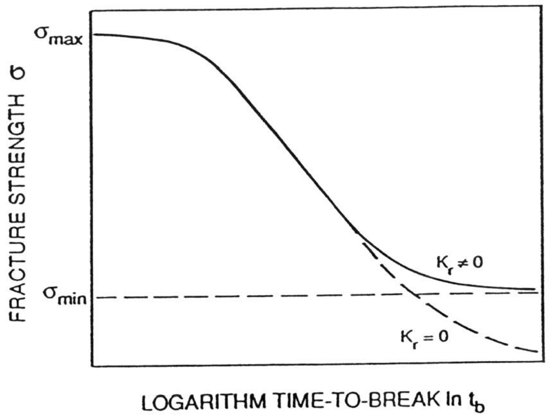

Fig. 3. Time-dependent strength of solids under simple tension.

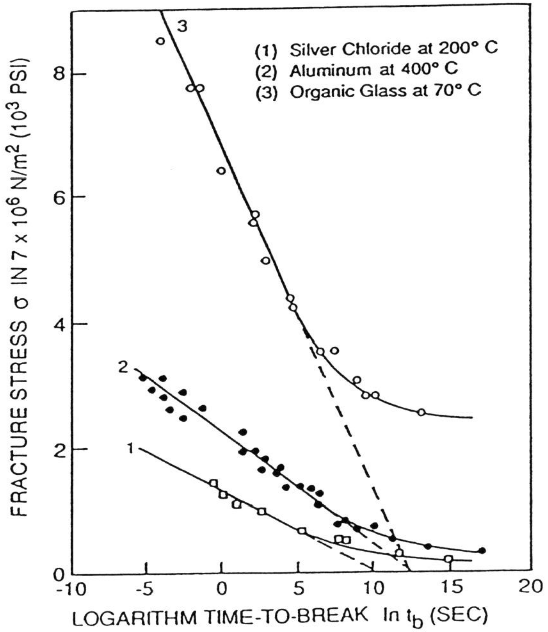

Fig. 4. Fracture stress vs. logarithm time

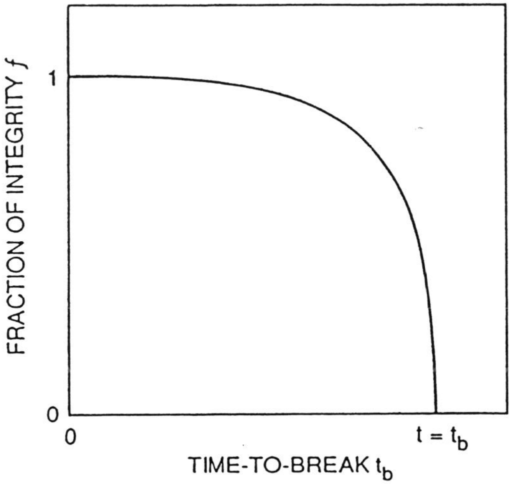

Fig. 5. Qualitative representation of fraction of integrity f versus time t.

It would be interesting to compare the high kinetic strengths with experimental values. Up to now little or no data have been found yet. However, it is felt that expression (2) should predict the kinetic strength behavior adequately. If this is accepted then the time-to-break t_b vs. f, the fraction of integrity, for a solid will be related as shown in Fig. 5.

It should be pointed out that this report seems to give the appearance of the extension of the classical one-dimensional Zhurkov model, however, it is fundamentally different from it. As stated at the beginning of this write-up that Zhurkov's model was and is an empirical relationship whereas the present model is based upon the submicroscopic atomic as well as molecular considerations. It is also quite apart from Hoff's (1953) or Kachanov's (1958) models. Using atomistic approach the current model should not be looked upon as a one-dimensional model as it is easily extended to a three-dimensional situation by introducing molecular orientation mechanism as a result of deformation (Mun and Hsiao, 1986). This mesomechanics approach is considered to be very sound as it makes the connection between microstructure, micromechanics, and macromechanics. Therefore the kinetic strength is given in terms of the basic atomic and molecular quantities, thereby the mechanical properties can be deduced for solids exhibiting creep, diffusion, or dislocation glide and so on as the time, temperature, molecular motion, and elementary bonding stresses, etc. have been incorporated into the model in the first place (Hsiao and Moghe, 1971; Hsiao, 1971).

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