Fracture of Composite in Aerospace Application

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ABSTRACT

Primarily there are two branches in the study of fracture, linear and non-linear, presently employed in the aerospace industries. The three modes of K factors in the linear branch, and J and G-Integrals in the non-linear branch are the admissible functions for the assessment of fracture toughness and the energy release rate. The suitability of branch selection depends upon the material characteristics, structural responses and pertinent failure criterion. In this paper, however, additional source of energy in non-continuum mechanics due to material sensitivity to thermodynamics, and interaction of a structure in inhomogeneous medium are to be considered. A new C-Integral complementary to G-Integral represents the cited energy is proposed herein. Limited laboratory results suggest some validity of this investigation.

KEYWORDS

K, stress intensity factor, J and G-Integral, fracture toughness, inhomogeneity, sensitivity.

INTRODUCTION

In the analysis of fracture it can be expected that a structure subject to same environment with the same geometricalconstraint may fail quite differently, depending upon its material properties whether it is of isotropic, homogeneous or anistropic either homogeneous or inhomogeneous. This can partially be explained by their inherent difference in the constitutive relations for composites in addition to their variation in ductility and thermal conductivity. The dissimilar failure of a steel bar and a graphite/aluminum rod is such an example.

TECHNICAL REVIEW

There are chiefly three subjects to be considered in the study of fracture of composite. They are the theory of fracture, appropriate constitutive relations of composite, and interaction of composite with surrounding media. In engineering mechanics, the theory of fracture focuses (Liu, 1983), primarily upon (a) the initiation of crack, (b) growth of a stable crack, and (c) the trajectory of a crack propagation. The fracture criteria in the linear branch consider the stress intensity factors due to geometrical discontinuity in a static case or material inhomogeneity in a dynamic case. There are three modes of failure in this branch, the flatwise tensile debonding between the plies, the interlaminar shear separation and the transverse rupture of composite. In a single ply noted modes are the crack of either fiber or matrix besides the debonding in between.(Fan & Pongracz 1984). While in the non-linear branch J-Integral in strain energy, and G-Integral in kinetic energy are the key functions to govern the design. In the study of incipient crack instabilty fracture toughness and energy release rate are the derivatives from the cited branches (Irwin & Sih, 1986; Kobayashi et al., 1981).

In formulaing the constitutive ralations of a composite, it is necessary to evalaute its strain matrices(Fan, 1988) for the assessment of homogeneity, (Novozhilov, 1953). This deals with either finite or infinitesimal strain with or without the rotation of material points parallel to the presence or absence of vorticities. The homogeneity is defined as the unifrom distribution of material points in size and positioning. For isotropic, homogeneous solid Hooke's law of two elastic coefficients applies in the construction of the constitutive relations, even though the strain matrix may differ because of nonlinear and/or rotational terms to be used in each appropriate class. For isotropic, either homogeneous or inhomogeneous solid (Lekhnitskii,1963), a Generalized Hooke's law, excluding the hygrothermal influence, with 36 elastic coefficients has been accepted for the construction of a constitutive matrix compatible with the prescribed composite. Because of elastic potential, and a plane of symmetry, the number of coefficients can be reduced to 13. In aplane stress case in the absence of any discontinuity, the necessary number becomes 6, and is reduced to 4 in a pure strain condition.They are the moduli of elasticity in the orthogonal directions, Poisson's ratio, and shear modulus. Coupling of shear and axial forces are the other two coefficients when a panel exhibits unsymmetrical stress distribution such as the bearing force in a bolt hole.

The consideration of material sensitivity to thermodynamics, the interactions of inhomogeneity in either composite and/or its surrounding media for the composite are scarcely available at present. Therefore, it seems quite reasonable to examine more recent development in the cited field in order to propose a complementary approach for future design requirement.

ANALYTICAL APPROACH

It is well known that both stress and strain tensors have been used to model the failure criterion for composite. Even though it is more logical to employ the strain criterion for non-ductile materials. Epoxy in a polyner composite or Incoenl in metal composite is such an example. Thus, an energy approach as represented by either J- or G-Integral appears more effective to treat all composite, either ductile orbrittle excepted as noted where possibly additional sources of energy are to be examined.

When an incident wave impinges on a structural component such as pressure pulse or inhomogeneous media lands intermittently on the piping in an engine system, an action in non-continuum mechanics emerges. The induced stress wave has been recognized as a sourceof kinetic energy in G-Integral. However, the potential interaction related to the inhomogeneity of media and the low ductility of material has yet been observed. Therefore, it is important to study their coupling effects in continuum and non-continuum systems. As a Lagrangian is not normally a function in non-conservative system as shown in a Hamilton Integral, thus an extended Hamilton's Princile is employed to show the system coupling.

$$I = \int_{t}^{t_i} \left(L + \sum_{k=1}^{N} G_k \, dq_k\right) dt \qquad (1)$$

With a dimension of J-S (in.-lb-sec), similar to that of a Planck's Constant, the relation of continuum and non-continuum mechanics is thus given. This relation can be illustrated further by noting Schrodinger's wave equation, i.e.

$$i\hat{h}\frac{\partial \psi}{\partial t} = -\frac{\hat{h}^2}{2m}\nabla^2\psi \qquad (2)$$

$$\left(-\frac{\hat{h}^2}{m}\frac{\partial^2}{\partial x^2}\rightarrow\right) \quad i\hat{h}\frac{\partial}{\partial t} \rightarrow E \quad \text{(in.-lb.); (J)}$$

$$-i\hat{h}\,\text{grad} \rightarrow P \quad \text{(lb.-sec.); (N-S)}$$

By (Sih & Tsou,1986;Sih & Chen,1985) it can be seen that whenever there is momentum or impulse the presence of a new energy similar to quanta action is generated.

The extraction of the new source of energy can be accomplished by focusing on the temperature and mass terms in a set of equations of motion with heat conduction and mass transfer (Fan & Pongracz, 1986). The temperature distribution in a wall is given (Carslaw & Jaeger,1959) as below:

$$\Theta = \Theta \left[\left(\frac{b}{x}\right)^{\frac{1}{2}} \operatorname{erfc} \frac{x-b}{2\sqrt{kt}} + \cdots\right]$$

It is clearly seen that for a material with low thermal conductility and relatively poor elongation, it can almost modify the complementary error function to be an impulse function initially. Thus an induced shock due to the material sensitivity to thermodynamics is the response for either a steady-state or transient thermal environment(Fan,1970;Bahr & Weiss,1986; Sih & Bolton,1986).

When there is evidence of mass concentration due to the inhomogeneity in the medium, the mass term then can be perturbed as such that

$$\frac{\partial}{\partial t}\left\{\left[\rho_0 h + \frac{M\delta(x)}{2\pi a h}\right] \frac{\partial w}{\partial t}\right\}$$

It can be seen that vibration, wave propagation etc. in Newtonian mechanics are intact. Yet there is also present an impact or impulse function by the intermittent pulse action, M(x) w/ t, which can contribute to an energy term, eq.(2), in non-continuum mechanics. It is to be noted that both of these cited cases in the material sensitivity and induced impact are of Delta functions.

The responses of this type of equations can be obtained by Green's function which satisfies the differential equations with associated initial and boundary values or by techniques given in (Jahanshashi, 1964;Chen,1963;Lee,1966).

The intensity of these singularities in energy terms under these special conditions justifies their being treated as additional source of energy. LImited tests have tentatively verified that the fracture pattern of superalloy composite or non-ductile high temperature alloy deviates from the criteria governed by J or G-Integral for ductile materials such as Cres, Al, and Ti etc.

Consequently the proposed C-Integral consists of strain, kinetic shock and quantawise energy. They are identified below:

Name	Symbol/ Integral	Function
Strain Energy	E_a/ J, N/cm (lb/in.)	stress function
Kinetic Energy	E_b/ G, cm (in.)	wave function
Quantawise E.	E_c/ C, J-S (in.-lb-sec)	Impulse function
Thermal shock E.	E_d/ C, C-cm (F-in.)	shock function

In energy expression, J (in.-lb)

Strain energy, E_a/J	$\frac{1}{2E} \int_s \varphi_1 \left(\frac{\partial \varphi_1}{\partial \eta}\right) ds$
Kinetic energy, E_b/G	$\frac{W}{A} \int_s \varphi_3 \left(\frac{\partial \varphi_3}{\partial \eta}\right) ds$
Quantawise E., E_c/C	$\frac{1}{wv} \int_s \varphi_2 \left(\frac{\partial \varphi_2}{\partial \eta}\right) ds$
Thermal shock E., E_d/C	$\frac{s^2}{Q_t v} \int_s \varphi_s \left(\frac{\partial \varphi_s}{\partial \eta}\right) ds$

By summation of the total energy represented by C-Integral, or

$$C = G + E_c + E_d$$
$$= J + E_b + E_c + E_d$$

It follows that C is equal to G in the absence of E_c and E_d under special conditions (Sih,1969). Thus, C-Integral is complementary to G-Integral in this case. By making use of Divergence theorem, and the laws ofthermodynamics, the instability of a fissure can be more effectively assessed by energy release rate with the total energy approach under the special condition. It also can be expected that the path of the crack propagation will not be self-similar but path dependent with respect to the unsymmetrical, total energy resistance.

CONCLUSION

Under the special condition as cited previously, the consideration of total energy in the investigation of fracture of composite seems more effective in the assessment of failure criteria. Limited testsresults suggest the influence of (a) material sensitivity to thermodynamics for composite of low thermal conductivity, or the induced thermal shock by the presence of singularities and (b) the induced impulse by the imhomogeneity of medium mass.

This postulation is parallel to the necessary difference in fracture criteria between a brittle and a ductile material (Griffith,1924;Shaw & Avery, 1986). The presence of Delta functions in design certainly can create more problems in the total energy consideration. Thus, it is not surprising to see that the trajectory of crack will not be self-similar because of the potentially unsymmetrical distribution of the energy system.

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NOTATIONS

A	= Area
a,b	= a linear dimension
C	= C-Integral
E	= Modulus of elasticity; energy function
G	= Generalized forces
h	= Wall thickness
$\hat{h}$	= Planck's constant
I	= Extended Hamilton's principle
K	= Thermal conductivity; stress intensity factor
k	= Diffusivity
L	= Lagrangian
l	= Length
M	= Mass
m	= Mass in Schrodunger's
P	= Momentum
q	= Generalized parameter; transverse shear
Q	= Heat
S	= Surface
s	= Entropy
T	= Kinetic energy
t	= Time
u,v,w	= Displacements
V	= Volume
W	= Weight
x,y,z	= Coordinates
α	= Coefficient of thermal expansion
η	= A linear dimension
θ	= Temperature
ν	= Poisson's ratio
ρ	= Density
ϕ	= Function or potential

Subscript

i,j = 1,2,6