Cracks in Bimaterial Interface – An Overview

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ABSTRACT

Problems of cracks on interfaces are reviewed. Topics discussed are contact zone models, models of interfaces including continuous variations in elastic moduli, and numerical results of elastic-plastic analysis.

KEYWORDS

Contact zones, interface models, bimaterial strip, elastic-plastic analysis.

OVERVIEW

Consider the situation shown in Figure 1 where the figure either represents a plane sheet or a plane cross-section for plane strain deformation. The unbroken welded interface (infinitesimally thin) is modelled by the continuity of tractions and displacements of the two media across it, whereas the cracked region is loaded by point loads as shown. It has been known since the work of (England 1965) and (Malyshev and Salganik 1965) that for such a problem, where media 1 and 2 are dissimilar and linear elastic, the analysis predicts the peculiar phenomenon of interpenetration of the crack faces. (England actually considered a finite length crack loaded internally and worked out details for a uniform pressure loading). This is, of course, unsatisfactory from the physical point of view but is usually defended on the grounds that only a very small region near the crack tip is affected

The solution of the problem shown in Figure 1 (where the crack is semi-infinite) can be written near the crack tip $r = 0$, as

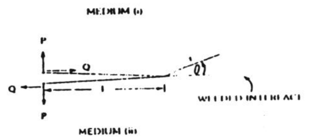

Fig.1 Semi-infinite crack (no contact zone) oscillatory singularity at $r = 0$.

$$(\sigma_{r\theta} + i\sigma_{\theta\theta})_{\theta=0} \sim \frac{(\ell r)^{-1/2}(Q+iP)(A_1+B_1)}{2\pi(A_1B_1)^{1/2}} \left(\cos\left\{\frac{1}{2\pi}\log\left[\frac{B_1}{A_1}\right]\log\left[\frac{r}{\ell}\right]\right\} - i\sin\left\{\frac{1}{2\pi}\log\left[\frac{B_1}{A_1}\right]\log\left[\frac{r}{\ell}\right]\right\}\right) \qquad (1)$$

$$\left[\frac{\partial u_r}{\partial r} + i\frac{\partial u_\theta}{\partial r}\right]_2^1 \sim \frac{-(\ell r)^{-1/2}(Q+iP)(A_1+B_1)}{4\pi} \left(\cos\left\{\frac{1}{2\pi}\log\left[\frac{B_1}{A_1}\right]\log\left[\frac{r}{\ell}\right]\right\} - i\sin\left\{\frac{1}{2\pi}\log\left[\frac{B_1}{A_1}\right]\log\left[\frac{r}{\ell}\right]\right\}\right) .$$

Equation (1) represents the jump in these displacement gradients across $\theta = \pm\pi$. A_1 and B_1 are given as

$$A_1 = \frac{1}{\mu_1} + \frac{\kappa_1}{\mu_2}, \qquad B_1 = \frac{1}{\mu_2} + \frac{\kappa_1}{\mu_1} \qquad (2)$$

where μ_1, μ_2 are the shear moduli of medium 1 and 2 respectively and $\kappa = 3-4\nu$ (plane strain) ($= \frac{3-\nu}{1+\nu}$ plane stress).

Expressions (1) show that near the crack tip the jump in normal displacement may change sign infinitely often, leading to interpenetration of the crack faces. However, evaluating the energy release rate (G) by a local work argument at the crack tip gives the result

$$G = \frac{(A_1+B_1)}{8\pi\ell}(Q^2+P^2) \qquad (3)$$

Thus even though the stress and displacement fields have the oscillatory behaviour at the crack tip the energy release rate is well behaved. Note further that setting Q equal to zero in equations (1) still produces mode 1 and 2 stress so that the failure modes are interlinked. We have presented results for a specific example as an illustration but the above features are generic.

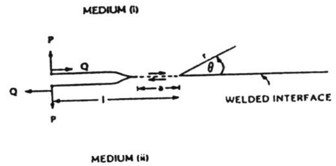

Fig.2 Comninou's contact zone model. Smooth crack closure at $r = a$, $\theta = \pm\pi$.

A large number of boundary value problems all with the oscillatory stress, interpenetration characteristic have been solved (e.g.(Williams 1959), (Erdogan 1963,1965), (Sih 1965), (Rice & Sih 1965), (Willis 1972) and many others). It is worth noting that although these solutions have this invalid feature and cannot thus hold at the crack tip itself the solutions may still be useful as outer solutions, in the sense of matched asymptotic expansions, valid away from the crack tip. This feature is noted in (Atkinson 1977) and used in (Atkinson 1982b) to obtain an analytic solution to the (Comninou 1977) model of the interface crack with a contact zone. Comninou attempted to satisfy the condition that the crack faces should not interpenetrate by postulating a region behind the crack tip on which the faces would be in contact but could slide. The length of this contact region is to be determined by the conditions that the open part of the crack closes smoothly and that the stress on the closed part is compressive, the resulting picture looks something like Figure 2. The ratio of length of contact zone to crack length is very small when the contact zone is due to the "interpenetration" effect (e.g. for the situation of Figure 1 with $Q = 0$, $a/\ell < 10^{-3}$). The smallness of the contact zone has lead some recent authors to pay little attention to this model while being careful to admit the possibility of contact. This is unfortunate since the model does take care of the interpenetration anomaly and for some loadings (e.g. the finite length crack in an applied shear stress field) the asymmetry of the problem causes a much longer contact region at one end of the crack. As shown in Figure 2 the crack tip which is at the origin now behaves like a shear crack with an energy release rate given by equation (3). This is shown in (Atkinson 1982) where other "possible" models are discussed. (Comninou and co-workers 1977, 1978, 1979) have applied her model to a variety of situations including mixed mode loading and three dimensional problems. In (Atkinson 1982b) we show how matched asymptotic expansions can be used to derive the contact zone solution for a given problem using the semi-infinite "inner problem" of (Atkinson 1982a) together with the appropriate "incorrect interpenetration" solution. Thus the large number of boundary value problem solutions mentioned above can be turned into contact zone solutions without too much effort. More recently (Gautesen & Dundurs 1987) have given a solution to the interface crack in a tension field in the form of a convergent series of elliptic functions. They emphasise the necessity of determining, for any crack problem, whether or not the crack faces make contact or not. Questions of uniqueness and existence are also important, of course, although often taken for granted. Thus the model of Comninou is unique provided the two inequalities, crack open or closed and stress compressive on the closed region, are imposed (Shield 1982). The standard solution with interpenetration is also unique with an integrability condition on the energy density so uniqueness does not necessarily mean there will not be objections to a solution. The alternative models discussed in (Atkinson 1982a) fall into this category, a referee claimed they were as objectionable as the original 'interpenetration' one. However, such models can be constructed which remove interpenetration and redistribute the available energy release rate into either pure mode 1 crack tip deformation or a mixture of modes, see the cited paper for details. Although the contact zone model may be the most desirable model from a physical point of view it would be nice to have experimental support. Note that the contact model, implies that the available energy release goes into a local mode 2 crack tip for remote tensile loading even if the bimaterial is only slightly dissimilar.

The discussion above has centred on interface cracks where the interface is modelled as infinitesimally thin, it might be appropriate however to explore other models of the interface region. Some simple situations have been discussed in (Atkinson 1977). These include models where the crack lies inside an interface region which may have constant or spatially varying moduli or the interface region itself is diffuse having moduli which vary continuously from one medium to the other. These two situations are shown in Figure 3 for the situation of a displacement loaded strip. The situation shown in figure 3a could be a description of two unlike media 1 and 2 intentionally glued together with a layer of adhesive 3. The situation of Figure 3b could be a description of a simple or glued interface between two materials which had suffered a gradual transition of the physical properties (interdiffusion of metals, reaction of the adhesive with the adherends, etc.). Either situation (3a or 3b) might also hold on a microscopic scale at an interface which was nominally a simple unglued junction between two materials. Note that a very similar justification of precisely these models has been given recently by Delale & Erdogan (1988).

In order to illustrate these models consider the situation where the crack lies in a bimaterial strip of thickness 2b. Figures 3a and 3b, the crack is assumed to be semi-infinite, the components of the bimaterial and the interface are isotropic elastic and plane strain conditions are assumed to exist so that the displacements do not varying in the x_3 direction. To simplify the problem still further at this stage we assume two special kinds of loading.

Fixed displacements on the strip side, with boundary conditions

$$u_1 = 0 \quad \text{on} \quad x_2 = \pm b \;;$$

$$u_2 = u_{20}, \quad \text{on} \quad x_2 = +b, \quad u_2 = u_{21} \quad \text{on} \quad x_2 = -b \quad \text{for all } x_1 \tag{4}$$

(u_1, u_2) are the displacement components in the (x_1, x_2) co-ordinate system, plane strain conditions are assumed and u_{20} and u_{21} are constants. For

this situation one can determine the energy flow into the crack tip for either model of the interface using results given in (Atkinson 1975, 1977).

The results are:
For model (i) (Figure 3a)

$$2G = (u_{20}-u_{21})^2 \left[\left[\frac{1}{(\lambda+2\mu)_2} + \frac{1}{(\lambda+2\mu)_1} \right] (b-h_1) + \frac{2h_1}{(\lambda+2\mu)_3} \right]^{-1} \tag{5}$$

and, for model (ii) (figure 3b)

$$2G = (u_{20}-u_{21})^2 \left[\frac{(b-h_1)}{(\lambda+2\mu)_2} + \frac{b}{(\lambda+2\mu)_1} + \int_{-h_1}^{0} \frac{dx_2}{(\lambda+2\mu)_3} \right]^{-1} . \tag{6}$$

The corresponding results for the "ideal interface" where the line $x_2 = 0$ is the boundary between media 1 and 2 is

$$2G = (u_{20}-u_{21})^2 \left[\frac{b}{(\lambda+2\mu)_2} + \frac{b}{(\lambda+2\mu)_1} \right]^{-1} \tag{7}$$

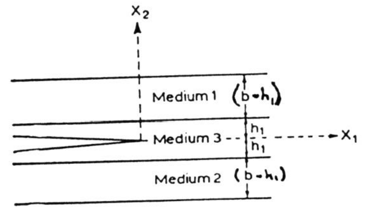

Fig. 3a

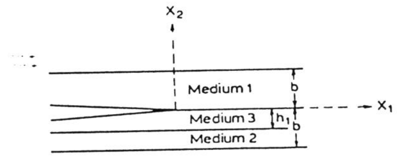

Fig. 3b

Comparing the results (7) with either (5) or (6) we see that for $h_1 \ll b$ the result (7) agrees with (5) or (6) with an error term $O(h_1/b)$. So for a very thin interface in this example at least, the 'ideal interface' will give a suitable first approximation for the energy release even though the stress and displacement at the crack tip will be quite different from those of either of the models we suggest. In particular, of course, the ideal interface model would still show the interpenetration effect whereas the other models would not.

Time harmonic applied displacements and transient loading problems

For time harmonic applied displacements we might have boundary conditions like

$$u_1 = 0 \text{ on } x_2 = \pm b; \quad u_2 = u_{20}e^{iwt} \text{ on } x_2 = +b \quad \text{for all } x_1$$

and $\quad u_2 = u_{21}e^{iwt}$ on $x_2 = -b$ for all x_1.

A comparison with results given in (Atkinson 1977) for a crack in a bimaterial sandwich suggests that the expression for the energy release rate would now involve terms like $\cos(wh/c_L)$ and $\sin(wh/c_L)$ where h is the thickness of the interface layer and c_L is the longitudinal wave speed in it for interface model (Figure 3a). Similar but more complicated results would follow for model (Figure 3b). The important, though perhaps obvious, point is that a limit $h \to 0$ to compare with the ideal interface solution is no longer a uniform limit and depends on the frequency w, the ideal interface solution being a better approximation for the energy release rate if the frequency w is small. Similar general considerations should apply in the case of transient loading.

Simple minded models such as shown in Figure 3a, where medium 3 culd be homogeneous with behaviour all its own, could be useful in the necessarily complicated analysis of elastic-plastic bimaterials where it is not known what the form of the crack tip solution is at the *ideal* bimaterial interface, we return to this point later.

In a recent paper (Rice 1988) has returned to the 'classical' interface crack solution (the one that interpenetrates) and has argued that the complex stress intensity factor K associated with it can be used as a crack tip characterising parameter. We have already noted above that it can be used as an outer solution (i.e. valid sufficiently far from the real crack tip) to deduce the contact zone solution. Referring back to our equations (1) and (2) his notation following (Hutchinson *et al.* 1987) is to write

$$\varepsilon = \frac{1}{2\pi}\ln(B_1/A_1) \tag{8}$$

and to define a complex stress intensity factor K so that the stress along the interface ahead of the crack tip is

$$(\sigma_{yy}+i\sigma_{xy})_{\theta=0} = Kr^{i\varepsilon}/\sqrt{2\pi r} \tag{9}$$

Thus for the situation shown in Figure 1 (equation 1)

$$K = (2\pi\ell)^{-1/2}(P+iQ)\ell^{i\varepsilon}2\cosh\pi\varepsilon \tag{10}$$

and for an interface crack of length L subject to remotely uniform stresses σ^∞_{yy} and σ^∞_{xy}

$$K = (\sigma^\infty_{yy}+i\sigma^\infty_{xy})(1+2i\varepsilon)L^{-i\varepsilon}\sqrt{\pi L/2} \tag{11}$$

at the right-hand crack tip. For this latter case (the remotely loaded crack) a phase angle ψ is defined so that

$$\tan\psi = \frac{\sigma^\infty_{xy}}{\sigma^\infty_{yy}},$$

thus $\psi = 0$ denotes tension and $\psi = \pm\pi/2$ shear in $\pm x$ direction. The conclusions are

(a) similar values of K (the complex stress intensity factor defined above) for two cracked bodies then imply similar states at the crack tip, so that conditions for crack growth can be phrased in terms of K reaching a critical failure locus in a complex plane, and

(b) the maintenance of a similar state at a crack tip under change of crack length is shown to require alteration of both the magnitude and phase angle of a combined tension and shear loading.

Conclusion (b) is, of course, simply a consequence of expressions like (10) or (11).

The problem of elastic-plastic analysis of cracks on bimaterial interfaces has been considered by (Shih & Asaro 1988). They give many numerical results for small scale yielding, the small strain approximation and a medium described by J_2 deformation theory for a Ramberg-Osgood stress-strain behaviour i.e. in uniaxial tension the material deforms according to $\epsilon/\epsilon_0 = \sigma/\sigma_0 + \alpha(\sigma/\sigma_0)^N$ where σ_0 and ϵ_0 are the reference stress and strain, α is a material constant and N the strain hardening exponent. Under small scale yielding they specify remote elastic fields by a complex stress concentration vector $Q = |Q|e^{i\phi}$ with ϕ being the phase angle between the two in-plane stress modes (Q is related to the complex K given above by $Q = (L)^{i\varepsilon}K$, ε being defined as in (8) above, L being the total crack length). They find that the elastic-plastic fields are members of a family parameterised by a new phase angle $\xi \equiv \phi + \varepsilon\ln(Q\bar{Q}/\sigma_0^2L)$ and the fields *nearly* scale with the well defined energy release rate evaluated by the J integral. The authors note that although the near tip fields do not appear to have a separable singular form (e.g. of the HRR type fields as in homogeneous media) they do however, bear interesting similarities to certain mixed mode HRR fields. Some concern is also given in their numerical work to the possibility of contact between the crack faces. However, the question of what is the precise form of the near crack tip field for elastic-plastic bimaterials still remains. Some progress on this has been made recently by (Champion & Atkinson 1988). Perhaps some light will be cast on this question in this session?

The problem of a single slip band emanating from the tip of an interface crack has been considered recently by (Bastero & Atkinson 1988). They considered the special case of an incompressible matrix joined to a rigid material and a flaw or debond at the boundary. On the application of load the crack tip singularity was relaxed by dislocation motion along a single slip band intersecting the crack tip. For given mixed mode loading a unique angle was found for which slip could just relax the crack. The corresponding problem when the matrix is compressible is more complicated due to the oscillatory nature of the (classical) elastic stress-singularity and has just been completed.

The above mentioned papers were all concerned with small deformation, however, (Knowles & Sternberg 1983) made an asymptotic investigation, within the nonlinear theory of plane stress, of a traction free interface crack between two dissimilar semi-infinite neo-Hookean sheets. The results they obtained were free of the oscillatory singularities of the kind predicted by the linear theory. They made the following statement.

> *'The precise approximative status of solutions to (linearised) problems involving interface-cracks remains an intriguing issue'.*

The brackets in this statement have been inserted by us, if it is read ignoring the adjective in brackets it probably still applies to the current position.

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