Unified Damage Approach to Crack Initiation

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SUMMARY

1 - When a structure is subjected to complex loadings, many events contributing to the rupture may occur simultaneously or successively : fatigue, ductile or creep damages in one or three dimensions involving or not mixed modes, non proportional loading effects and non-linear accumulation or interaction. Many models have been proposed for each particular phenomenon but a unified approach is still needed for predictions when the sequence of the events is ignored before any calculation.

2 - The State and Kinetic Couplings theory is used in the framework of the continuum mechanics to derive the complete set of constitutive equations modelling elasticity plasticity and damage coupled together.

- Elasticity is coupled to damage through a State Coupling written in the thermodynamical potential.

$$\varepsilon^{e}_{ij} = \mathbb{E}^{-1}_{ijkl} \frac{\sigma_{kl}}{1-D}$$

Where ε^{e} is the elastic strain tensor, σ the stress tensor, $\mathbb{E}$ the Hooke elasticity operator - D is the isotropic damage scalar variable related to the surface density of micro cracks or micro voids in any plane ($0 \le D \le 1$).

- Plasticity is coupled to damage through a Kinetic Coupling written in the potential of dissipation from which the kinetic laws of evolution derive.

$$F = f + F_D$$

where $f = 0$ is the Von Mises plasticity function.

$$\dot{\varepsilon}^{p}_{ij} = \frac{\partial F}{\partial \sigma_{ij}} \dot{\lambda} \quad \text{if } f = 0 \text{ and } \dot{f} = 0$$

where $\dot{\varepsilon}^{p}$ is the plastic strain rate tensor, $\dot{\lambda}$ is the plasticity multiplier calculated from the consistency condition $\dot{f} = 0$.

- The kinetic law of damage evolution is derived from the potential of dissipation

$$\dot{D} = -\frac{\partial F}{\partial Y}\dot{\lambda} = -\frac{\partial F_D}{\partial Y}\dot{\lambda}$$

where Y is the strain energy density release rate, the variable associated to the damage variable D.

$$\bar{Y} = -Y = \frac{we}{1-D} = \frac{1}{2}\frac{we}{\partial D}$$

where we is the elastic strain energy density. Observations of damage mechanisms and many experimental results allow to choose for F_D a function such that the damage rate is a linear function of $\bar{Y}$ and the accumulated plastic strain rate $\dot{p}$

$$\dot{p} = (\frac{3}{2}\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij})^{1/2}$$

$$\dot{D} = \frac{\bar{Y}}{S}\dot{p} \quad \text{if } p > p_D$$

Only two material constants are introduced viz. a scale factor S and a strain threshold p_D.

3 - This simple kinetic damage law which unifies many particular models contains the following poperties.

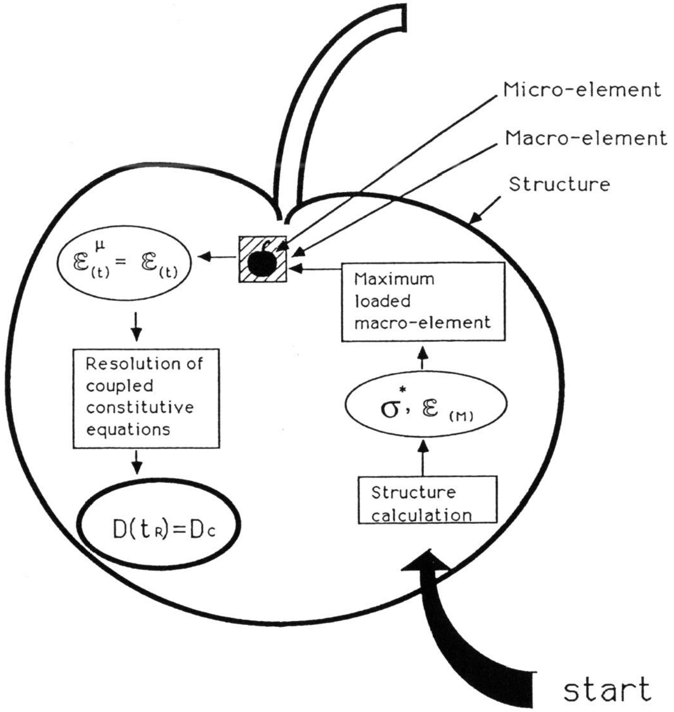
- No damage up to a certain value of accumulated plastic strain (period of nucleation).

- The effect of the triaxiality ratio is contained in the $\bar{Y}$ variable as we is the total elastic strain energy density.
- The crack initiation condition is written through an instability criterion giving rise to a critical value of damage Dc depending upon the type of loading.
- The brittle crack initiation condition is obtained when this instablility occurs for a very small value of the plastic strain
- By integration as a function of the plastic strain one may obtain a model for ductile damage some what similar to the Mac Clintock or Rice and Tracey models.
- By integration, first over one cycle of strain or stress and then, as a function of the number of cycles, models for low cycle fatigue or high cycle fatigue are obtained. They have the properties of the Manson Coffin model or the Woehler-Miner model with the Goodman rule. The non-linear accumulation property is also modelled together with the threedimensional effects such as the influence of non proportional loadings.
- By integration with respect to time, a creep damage model some what similar to the earlier Kachanov-Rabotnov model is derived.

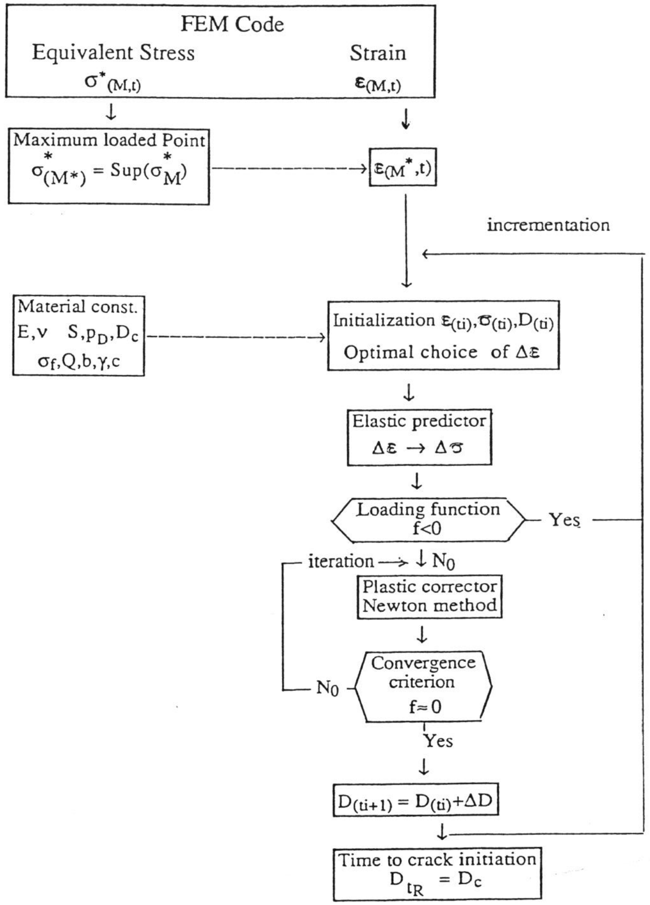
4 - Often, the damage is very localized in such a way that the damaged volume is small in comparison to the usual meso-volume element. This is one of the main reasons of the serious difficulties of numerical strain damage coupled analyses by the finite element method. A micro-mechanical model avoids some difficulties.

This two-scale model is a micro-element subjected to elasticity, plasticity and damage embedded in a classical meso-element subjected to only elasticity and plasticity. The material properties are the same in the two elements except the yield stress which is lower in the micro-element. That allows it to be damaged and cracked before the meso-element. According to the Lin-Taylor's hypothesis the two elements are subjected to the same state of strain.

To predict a crack initiation in a structure the following procedure is followed :
- Due to the localization of damage in a small volume, the damage does not affect the state of stress and strain at macro-scale. Their history may be calculated by a classical calculation in elasticity or elasto-plasticity ignoring the damage.
- The history of strain, at the maximum loaded meso-element, is considered as an imput in the micro-element . At that scale only constitutive differential equations have to be solved that is equations for elasticity, plasticity and damage coupled together.



- A post-processor to any Finite element code has been worked out to integrate step by step in time the set of constitutive equations. It is described below



- Many numerical simulations point out the properties listed in section 3 and particular effects due to internal micro stresses in the micro-element induced by the compatibility of strain between the elastoplastic damageable micro-element and the elastic (plastic) meso-element.

BIBLIOGRAPHY

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