The Stress Intensity Factors at the Tip of a Kinked and Curved Crack

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ABSTRACT

This paper studies the expansion of the stress intensity factors (SIF's) at the tip of a kinked and curved crack in powers of the crack extension length, and more specifically the influence of the curvature parameters of the extension on these SIF's. Using "universality " properties established elsewhere (Leblond, 1988), one shows that general results can be obtained by considering the special case of a straight initial crack extended in an arbitrary direction by a slightly curved secondary branch, in an infinite body loaded by uniform forces at infinity. A perturbation technique analogous to that of Cotterell and Rice (1980) is used to reduce the original problem to one involving a simpler geometry, namely a crack composed of two straight branches. This problem is then solved by means of Muskhelishvili's (1953) method and conformal mapping. Formulas for the curvature parameters of the crack extension usable for numerical predictions of crack paths are finally given.

KEYWORDS

Stress intensity factors; kinked crack; curved crack; perturbation technique; conformal mapping.

INTRODUCTION

Consider (Fig.1) a two-dimensional elastic body under plane strain conditions, containing a curvilinear crack with a small kinked and curved extension of length s. Let πm ($-1<m<+1$) denote the kink angle, and let the shape of the extension be described by

$$h = a^* x^{3/2} + \frac{C^*}{2} x^2 + O(x^{5/2}) \tag{1}$$

where h denotes the distance from the point considered to its projection on the tangent to the extension at the angular point O, x the distance from the point O along the extension, and a^* and C^* parameters (Fig.1). It has been shown in Leblond (1988) by general arguments that the stress intensity factors (SIF's) $k_p(s)$ (p=1,2) at the tip of the extended crack admit an expansion of the form

$$k_p(s) = k_p^* + k_p^{(1/2)} \sqrt{s} + k_p^{(1)} s + O(s^{3/2}) \tag{2}$$

where k_p^*, $k_p^{(1/2)}$ and $k_p^{(1)}$ are given by

$$k_p^* = F_{pq}(m)k_q \quad ; \quad k_p^{(1/2)} = \left[k_p^{(1/2)}\right]_{a^*=0}^{\pi m} + a^* H_{pq}(m)k_q \quad ;$$

$$k_p^{(1)} = \left[k_p^{(1)}\right]_{C^*=0}^{\pi m, a^*} + C^* M_{pq}(m)k_q \ . \tag{3}$$

In these equations the k_q's (q=1,2) are the original SIF's at the point O, the F_{pq}'s, H_{pq}'s, M_{pq}'s functions of m, $\left[k_p^{(1/2)}\right]_{a^*=0}^{\pi m}$ the value of $k_p^{(1/2)}$ for a zero a^*, and $\left[k_p^{(1)}\right]_{C^*=0}^{\pi m, a^*}$ that of $k_p^{(1)}$ for a zero C^*. The functions F_{pq}'s describe the asymptotic form of the SIF's when s tends to zero; the functions H_{pq}'s and M_{pq}'s describe the influence of the curvature parameters a^*, C^* of the extension on the next terms of the expansion of the SIF's. The validity of formulas (2,3) is established in Leblond (1988) in the most general situation: arbitrary geometry of the body and the crack, arbitrary loading; for this reason the functions F_{pq}'s, H_{pq}'s, M_{pq}'s of the kink angle are termed "universal" in Leblond (1988).

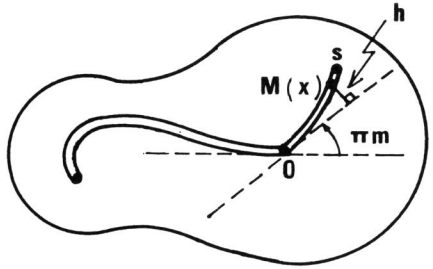

Fig. 1 : General problem considered.

Previous works concerning the analytic determination of the functions F_{pq}'s, H_{pq}'s, M_{pq}'s can be summarized as follows. The F_{pq}'s have been calculated by various authors (Bilby *et al.*, 1977; Wu, 1978 a,b; Amestoy *et al.*, 1979) by considering the special case of a crack composed of two straight branches in an infinite body loaded by uniform forces at infinity; the simplest solution is that of Amestoy *et al.* (1979), which uses Muskhelishvili's (1953) method and conformal mapping. On the other hand, the H_{pq} and M_{pq} functions have been determined only to the 0^{th} order in m, by Sumi *et al.* (1983), and to the 1^{st} order, by Karihaloo *et al.* (1981); these authors considered a nearly straight crack (small parameters πm, a^*, C^*) and used Cotterell and Rice's (1980) perturbation method to reduce the problem to one involving a rectilinear crack.

This paper presents a method of calculation of the H_{pq}'s and M_{pq}'s for all values of the kink angle πm. Formulas expressing the curvature parameters of the crack extension in terms of these functions are given, using the "principle of local symmetry" (Goldstein and Salganik, 1974) as a propagation criterion. These formulas can be used for numerical predictions of crack paths.

PRINCIPLE OF THE METHOD

Because of their "universality" property, the functions H_{pq}'s and M_{pq}'s can be determined by studying the particular case of a straight main crack extended in an arbitrary direction by a curved extension (of shape described by eqn.(1)) in an infinite body loaded by uniform forces at infinity (Fig.2). Since the expressions (3_2,3_3) of $k_p^{(1/2)}$ and $k_p^{(1)}$ are linear in a^* and C^* respectively, *exact results* can be obtained by using a 1^{st} order perturbation technique with respect to

these parameters. Such a technique will be used to reduce the problem to one concerned with a crack with two straight branches, namely the main crack plus a secondary branch identical to the secant AB or BC (Fig.2). The latter problem will be solved with the aid of Muskhelishvili's formalism and conformal mapping. This method combines thus the main ideas of the works of Sumi *et al.* (1983) and Karihaloo *et al.* (1981) on the one hand, and Amestoy *et al.* (1979) on the other hand. The kink angle will be allowed to take arbitrary values instead of being restricted to small ones like in Sumi *et al.* (1983) and Karihaloo *et al.* (1981), since the problem of a crack with two straight branches can be solved exactly for all values of this angle (Amestoy *et al.*, 1979).

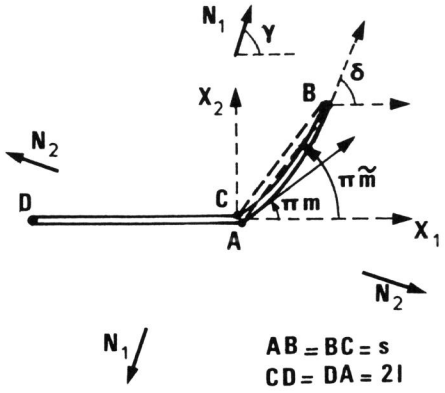

Fig. 2 : Particular case studied.

EQUATIONS OF THE PROBLEM IN THE PHYSICAL PLANE

The problem in the physical Z-plane consists in finding Muskhelishvili's potentials Φ and Ψ, analytic everywhere except on the (real) crack, and subject to the boundary condition and to the conditions at infinity

$$\Phi(Z) + Z\overline{\Phi'(Z)} + \overline{\Psi(Z)} = Cst \qquad \text{on the crack;} \tag{4}$$

$$\Phi(Z) = \Gamma Z + O(1) \ ; \ \Psi(Z) = \Gamma' Z + O(1). \tag{5}$$

In the latter equation Γ and Γ' are given in terms of the stress tensor at infinity by

$$\Gamma = (N_1 + N_2)/4 \ ; \ \Gamma' = (N_2 - N_1)\, e^{-2i\gamma}/2, \tag{6}$$

N_1 and N_2 being the principal stresses at infinity and γ the angle between the OX_1 axis and the first principal direction (Fig.2).

Following Cotterell and Rice (1981), we associate to Φ and Ψ some functions Φ^a and Ψ^a which are identical to Φ and Ψ except that their cut is along the secant AB or BC instead of the real crack, and we expand Φ^a and Ψ^a to the 1^{st} order in a^* and C^* :

$$\Phi^a = \Phi_0 + \Phi_1 + \dots \ ; \ \Psi^a = \Psi_0 + \Psi_1 + \dots \ . \tag{7}$$

Let η denote the gap between the secant AB or BC and the real crack. If Z is on the secant, $Z + \eta(Z)$ is on the real crack, and $\Phi[Z + \eta(Z)] = \Phi^a(Z) + \Phi^{a\prime}(Z)\eta(Z) = \Phi_0(Z) + \Phi_0'(Z)\eta(Z) + \Phi_1(Z)$ to the 1^{st} order in η; similar equations hold for Φ' and Ψ. Thus the boundary condition on the deviated branch can be written

$$\Phi_0(Z) + \Phi_0'(Z)\eta(Z) + \Phi_1(Z) + [Z + \eta(Z)][\overline{\Phi_0'(Z)} + \overline{\Phi_0''(Z)}\eta(Z) + \overline{\Phi_1'(Z)}]$$
$$+ \overline{\Psi_0(Z)} + \overline{\Psi_0'(Z)}\eta(Z) + \overline{\Psi_1(Z)} = \text{Cst} \qquad \text{for } Z \in AB \text{ or } BC.$$

This equation holds also for $Z \in CD$ or DA, taking $\eta(Z) = 0$ in that case. Identifying terms of order 0 and 1, we get the boundary conditions for Φ_0, Ψ_0, Φ_1, Ψ_1 :

$$\left.\begin{aligned} &\Phi_0 + Z\overline{\Phi_0'} + \overline{\Psi_0} = \text{Cst} \\ &\Phi_1 + Z\overline{\Phi_1'} + \overline{\Psi_1} + \Phi_0'\eta + \overline{\Phi_0'}\eta + Z\overline{\Phi_0''}\eta + \overline{\Psi_0'}\eta = \text{Cst} \end{aligned}\right\} \quad \text{for } Z \in ABCDA. \quad (8)$$

Expansion of eqns.(5) to the 1st order in η yields similarly the conditions at infinity for Φ_0, Ψ_0, Φ_1, Ψ_1 :

$$\Phi_0(Z) = \Gamma Z + O(1) \; ; \; \Phi_1(Z) = O(1) \; ; \; \Psi_0(Z) = \Gamma' Z + O(1) \; ; \; \Psi_1(Z) = O(1). \quad (9)$$

The principle of the treatment will be to solve the problem for the 0^{th} order functions Φ_0, Ψ_0, then that for the 1st order functions Φ_1, Ψ_1, which is quite analogous except that eqn.(8_2) involves a "second member" expressed in terms of the 0^{th} order solution.

CONFORMAL MAPPING ONTO THE z-PLANE; REDUCTION OF THE PROBLEM TO INTEGRAL EQUATIONS

Conformal mapping-Problem in the z-plane. The exterior of the crack ABCDA can be mapped onto the exterior of the unit circle $\mathcal{U}$ in a new z-plane (Fig.3), by defining (see e.g. Dudukalenko and Romalis, 1973) :

$$Z = \omega(z) = Re^{i\tilde{m}\alpha} (z-e^{i\alpha})^{1-\tilde{m}} (z-e^{-i\alpha})^{1+\tilde{m}} / z \; . \quad (10)$$

In this equation $\pi\tilde{m}$ is the angle between the OX_1 axis and the secant AB or BC (see Fig. 2), and R and α are related to $\tilde{m}$, s and the length 2ℓ of the principal crack by

$$\ell = 2R \cos^{1-\tilde{m}}\left[\frac{\alpha+\beta}{2}\right] \cos^{1+\tilde{m}}\left[\frac{\alpha-\beta}{2}\right] \; ; \; s = 4R \sin^{1+\tilde{m}}\left[\frac{\alpha+\beta}{2}\right] \sin^{1-\tilde{m}}\left[\frac{\alpha-\beta}{2}\right] ;$$

$$\sin\beta = \tilde{m} \sin\alpha \quad (11).$$

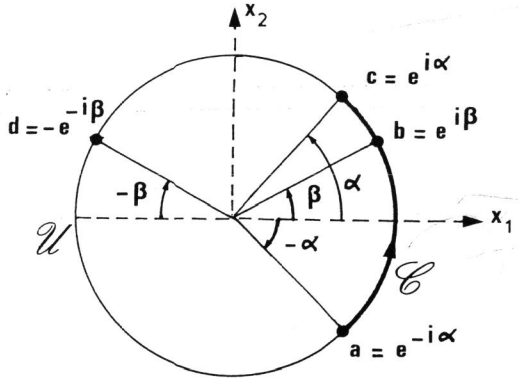

Fig. 3 : The z-plane.

Defining $\phi_i(z) = \Phi_i(Z)$ and $\psi_i(z) = \Psi_i(Z)$ (i = 0,1), eqns.(8,9) become

$$\left.\begin{aligned} &\phi_0 + \frac{\omega}{\overline{\omega'}}\overline{\phi_0'} + \overline{\psi_0} = \text{Cst} \\ &\phi_1 + \frac{\omega}{\overline{\omega'}}\overline{\phi_1'} + \overline{\psi_1} + \left[\left(\frac{\phi_0'}{\omega'} + \frac{\overline{\phi_0'}}{\overline{\omega'}}\right)\eta + \left(\omega\frac{\overline{\phi_0''}}{\overline{\omega'^2}} - \omega\frac{\overline{\phi_0'}\,\overline{\omega''}}{\overline{\omega'^3}} + \frac{\overline{\psi_0'}}{\overline{\omega'}}\right)\overline{\eta} = \text{Cst}\right. \end{aligned}\right\} \quad \text{for } z \in \mathcal{U}; \quad (12)$$

$$\phi_0(z) = \Gamma Re^{i\tilde{m}\alpha} z + O(1) \; ; \; \phi_1(z) = O(1) \; ; \; \psi_0(z) = \Gamma' Re^{i\tilde{m}\alpha} z + O(1) \; ; \; \psi_1(z) = O(1). \quad (13)$$

Integral equation for the 0^{th} order problem. It has been shown in Amestoy (1987), Hussain *et al.* (1974), Chatterjee (1975), using Plemelj's formula, that eqns.(12_1,13_1,13_3) for the 0^{th} order functions ϕ_0, ψ_0 lead to an integral equation for ϕ_0' :

$$\phi_0'(z) = \phi_0'^0(z) + \mathcal{L}\phi_0'(z) \quad (14)$$

where the integral operator $\mathcal{L}$ and the function $\phi_0'^0$ are given by

$$\mathcal{L}\chi(z) = \frac{1-e^{2i\pi\tilde{m}}}{2i\pi} \int_{\mathcal{C}} \frac{(t-e^{i\alpha})\,(t-e^{-i\alpha})\,\overline{\chi(t)}\,dt}{t\,(t-e^{i\beta^-})\,(t+e^{-i\beta})\,(t-z)^2} \; ; \quad (15)$$

$$\phi_0'^0(z) = \Gamma Re^{i\tilde{m}\alpha} + (\Gamma + \overline{\Gamma'})\, Re^{-i\tilde{m}\alpha}/z^2 \; . \quad (16)$$

The integral in eqn.(15) is taken over the arc $\mathcal{C}$ = abc (Fig.3); the pole $e^{i\beta}$ of the integrand is slightly displaced towards the exterior of $\mathcal{U}$, i.e. it must be understood as $e^{i\beta^-} = e^{i(\beta-i\epsilon)}$, $\epsilon>0$, $\epsilon\to0$. It is also shown in Amestoy (1987) that ψ_0 can be expressed in terms of ϕ_0' as

$$\psi_0(z) = (\Gamma + \Gamma')\, Re^{i\tilde{m}\alpha}\, z - \frac{\Gamma Re^{-i\tilde{m}\alpha}}{z} - \frac{z\,(z-e^{i\alpha})\,(z-e^{-i\alpha})}{(z-e^{i\beta})\,(z+e^{-i\beta})}\,\phi_0'(z)$$

$$+ \frac{1-e^{-2i\pi\tilde{m}}}{2i\pi} \int_{\mathcal{C}} \frac{z\,(t-e^{i\alpha})\,(t-e^{-i\alpha})\,\phi_0'(t)\,dt}{(t-e^{i\beta^+})\,(t+e^{-i\beta})\,(z-t)} + \text{Cst} \quad (\text{where } \beta^+ = \beta+i\epsilon) \; . \quad (17)$$

Integral equation for the 1st order problem. The treatment of eqns.(12_2,13_2,13_4) for the 1st order functions ϕ_1, ψ_1 is quite analogous to that for the 0^{th} order functions ϕ_0, ψ_0, and leads to an integral equation for ϕ_1' analogous to eqn.(14) for ϕ_0' :

$$\phi_1'(z) = \phi_1'^0(z) + \mathcal{L}\phi_1'(z), \quad (18)$$

where the "second member " $\phi_1'^0$ is expressed in terms of the 0^{th} order functions ϕ_0', ψ_0' :

$$\phi_1'^0(z) = \frac{1}{2i\pi} \int_{\mathcal{C}} \left[\left(\frac{\phi_0'}{\omega'} + \frac{\overline{\phi_0'}}{\overline{\omega'}}\right)\eta + \left(\frac{\omega\overline{\phi_0''}}{\overline{\omega'^2}} - \frac{\omega\overline{\omega''}\,\overline{\phi_0'}}{\overline{\omega'^3}} + \frac{\overline{\psi_0'}}{\overline{\omega'}}\right)\overline{\eta}\right] dt \,/\, (t-z)^2 \; . \quad (19)$$

Like in eqn.(15), the integrand has a pole at $e^{i\beta}$, which must be understood as $e^{i\beta^-}$.

Stress intensity factors. Once the functions ϕ_0', ϕ_1' are known, the SIF's at the tip of the extended crack can be obtained through Andersson's (1969) formula :

$$k_1(s) - ik_2(s) = 2\sqrt{\pi}\,[\phi_0'(e^{i\beta}) + \phi_1'(e^{i\beta})]\, e^{-i\delta/2}\, [\omega''(e^{i\beta})]^{-1/2} \; , \quad (20)$$

where δ is the angle between the OX_1 axis and the tangent to the crack at its tip (see Fig.2).

Expansion in powers of s. Formulas (14-20) allow for the calculation of the SIF's for arbitrary values of the crack extension length s. To obtain the expansion of the SIF's in powers of s, and hence the H_{pq} and M_{pq} functions, one must perform an expansion of these formulas in powers of s. This is equivalent to an expansion in powers of α since eqns.(11) imply that $\alpha = O(s^{1/2})$. This is achieved by a "double scale technique", using the following change of variable and functions :

$$z = e^{i\alpha\zeta} \; ; \; \phi_i'(z) = \ell e^{-i\alpha\zeta} \, [U_i(\zeta) + \alpha V_i(\zeta) + \alpha^2 W_i(\zeta) + O(\alpha^3)] \quad ;$$

$$\psi_i'(z) = \ell e^{-i\alpha\zeta} \, [X_i(\zeta) + \alpha Y_i(\zeta) + O(\alpha^2)] \qquad (i = 0,1). \tag{21}$$

This is the same change of variable and functions as that used previously by Amestoy *et al.* (1979) for the study of the F_{pq} functions.

The calculation is straightforward but heavy, and therefore will not be presented. It leads to integral equations for the unknown functions which involve the same integral operator as that encountered in the study of the F_{pq} functions (see Amestoy *et al.*, 1979); only the "second member" is different. The solutions can be obtained in the form of series, using the fact that the operator is contractant for a certain norm (see Amestoy *et al.*, 1979).

FIRST ORDER EXPRESSIONS AND NUMERICAL CALCULATION OF THE H_{pq}'s AND M_{pq}'s

Exact expansions of the H_{pq}'s and M_{pq}'s to a given order in m can in theory be obtained by truncating the series giving the solutions of the integral equations in a suitable way, and performing the calculations analytically; however, because of the length of the procedure, this is feasible in practice only for low orders in m. One gets thus the following 1^{st} order expressions of the H_{p1}'s and M_{p1}'s, and 0^{th} order expressions of the H_{p2}'s and M_{p2}'s :

$$H_{11} = -\frac{9\pi m}{8} \; ; \; H_{21} = \frac{3}{4} \; ; \; M_{11} = -\frac{3\pi m}{4} \; ; \; M_{21} = \frac{1}{2} \; ; \tag{22}$$

$$H_{12} = -\frac{9}{4} \; ; \; H_{22} = 0 \; ; \; M_{12} = -\frac{3}{2} \; ; \; M_{22} = 0 \; . \tag{23}$$

These equations agree with the results of Sumi *et al.* (1983) and Karihaloo *et al.* (1981), once some calculation errors have been corrected in the latter work.

Numerical values of the H_{pq} and M_{pq} functions for arbitrary values of the kink angle can also be obtained by computing the series numerically. Results are presented in Tables 1 and 2. The (absolute) accuracy is of the order of 10^{-3}.

CONCLUSION : APPLICATION TO THE NUMERICAL PREDICTION OF CRACK PATHS.

Using the (now widely accepted) "principle of local symmetry" of Goldstein and Salganik (1974) as a propagation criterion, one must equate to zero the successive terms k_2^*, $k_2^{(1/2)}$, $k_2^{(1)}$... of the expansion of $k_2(s)$. This yields the value of the geometric parameters πm, a^*, C^* ... of the crack extension:

$$F_{21}(m)k_1 + F_{22}(m)k_2 = 0 \qquad \text{(hence m);} \tag{24}$$

$$\left[k_2^{(1/2)}\right]_{a^*=0}^{\pi m} + a^*[H_{21}(m)k_1 + H_{22}(m)k_2] = 0 \qquad \Longrightarrow$$

Table 1 : Values of the H_{pq} functions.

Angle(°)	H_{11}	H_{12}	H_{21}	H_{22}	Angle(°)	H_{11}	H_{12}	H_{21}	H_{22}
0	0	-2.250	0.750	0	40	-0.669	-1.460	0.474	-1.250
2.5	-0.049	-2.247	0.749	-0.095	42.5	-0.696	-1.369	0.442	-1.294
5	-0.098	-2.236	0.746	-0.189	45	-0.721	-1.276	0.410	-1.334
7.5	-0.146	-2.219	0.740	-0.282	47.5	-0.743	-1.180	0.377	-1.368
10	-0.194	-2.196	0.731	-0.375	50	-0.763	-1.082	0.344	-1.396
12.5	-0.242	-2.166	0.720	-0.465	52.5	-0.781	-0.982	0.310	-1.419
15	-0.288	-2.129	0.707	-0.553	55	-0.796	-0.881	0.276	-1.436
17.5	-0.333	-2.086	0.693	-0.639	57.5	-0.809	-0.779	0.241	-1.448
20	-0.377	-2.037	0.675	-0.723	60	-0.819	-0.677	0.207	-1.454
22.5	-0.420	-1.982	0.656	-0.803	62.5	-0.827	-0.574	0.173	-1.455
25	-0.461	-1.922	0.635	-0.879	65	-0.833	-0.472	0.139	-1.450
27.5	-0.500	-1.856	0.612	-0.952	67.5	-0.836	-0.371	0.105	-1.440
30	-0.538	-1.786	0.587	-1.021	70	-0.837	-0.270	0.072	-1.424
32.5	-0.574	-1.710	0.561	-1.085	72.5	-0.835	-0.171	0.040	-1.404
35	-0.608	-1.631	0.533	-1.145	75	-0.832	-0.073	0.009	-1.378
37.5	-0.639	-1.547	0.504	-1.200	77.5	-0.826	0.022	-0.022	-1.348

Table 2 : Values of the M_{pq} functions.

Angle(°)	M_{11}	M_{12}	M_{21}	M_{22}	Angle(°)	M_{11}	M_{12}	M_{21}	M_{22}
0	0	-1.500	0.500	0	40	-0.446	-0.975	0.328	-0.791
2.5	-0.033	-1.498	0.500	-0.060	42.5	-0.465	-0.915	0.309	-0.819
5	-0.065	-1.491	0.497	-0.119	45	-0.481	-0.853	0.289	-0.845
7.5	-0.098	-1.480	0.494	-0.178	47.5	-0.496	-0.789	0.268	-0.866
10	-0.130	-1.464	0.488	-0.236	50	-0.510	-0.724	0.247	-0.885
12.5	-0.161	-1.444	0.482	-0.294	52.5	-0.522	-0.657	0.226	-0.900
15	-0.192	-1.420	0.474	-0.349	55	-0.532	-0.590	0.205	-0.911
17.5	-0.222	-1.391	0.465	-0.404	57.5	-0.541	-0.522	0.183	-0.919
20	-0.252	-1.358	0.454	-0.457	60	-0.548	-0.454	0.162	-0.924
22.5	-0.280	-1.322	0.442	-0.507	62.5	-0.553	-0.386	0.140	-0.925
25	-0.307	-1.282	0.429	-0.556	65	-0.557	-0.318	0.119	-0.922
27.5	-0.334	-1.238	0.415	-0.602	67.5	-0.559	-0.251	0.978	-0.916
30	-0.359	-1.191	0.399	-0.645	70	-0.560	-0.184	0.077	-0.907
32.5	-0.383	-1.141	0.383	-0.686	72.5	-0.559	-0.118	0.057	-0.895
35	-0.406	-1.089	0.366	-0.724	75	-0.557	-0.053	0.037	-0.879
37.5	-0.427	-1.033	0.347	-0.759	77.5	-0.553	0.011	0.018	-0.861

$$a^* = - \frac{\left[k_2^{(1/2)}\right]^{\pi m}_{a^*=0}}{H_{21}(m)k_1 + H_{22}(m)k_2} ; \qquad (25)$$

$$\left[k_2^{(1)}\right]^{\pi m, a^*}_{C^*=0} + C^*[M_{21}(m)k_1 + M_{22}(m)k_2] = 0 \qquad \Longrightarrow$$

$$C^* = - \frac{\left[k_2^{(1)}\right]^{\pi m, a^*}_{C^*=0}}{M_{21}(m)k_1 + M_{22}(m)k_2} . \qquad (26)$$

These formulas can be used for numerical predictions of crack paths by a step-by-step method, in the following way. At a given step (i.e. for a given crack geometry), k_1 and k_2 can be computed numerically; eqn.(24) gives then the value of πm. Next $\left[k_2^{(1/2)}\right]^{\pi m}_{a^*=0}$ can be computed by comparing the original SIF's with those at the tip of a small straight extension in the direction πm, and the value of a^* is then deduced from eqn.(25). $\left[k_2^{(1)}\right]^{\pi m, a^*}_{C^*=0}$ can also be determined in a similar way, using a small extension having an a^* equal to the value determined but a zero C^*, and the value of C^* is then deduced from eqn.(26). The crack can then be "numerically extended", using a remeshing procedure, according to the values of πm, a^*, C^* determined; the extension is to be stopped at a small, arbitrary distance from the original crack tip. The next step can then be carried out.

In essence, this was the procedure used by Sumi (1986 a,b) for the study of crack paths in some configurations of practical interest; however it was impaired by the incomplete knowledge of the functions H_{pq}'s and M_{pq}'s. The present work fills this gap.

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