The Line-spring Model Analysis of Surface Crack at Weld Joint of Two Tubes

LU YINCHU LIU XI

Department of Aerospace Engineering and Mechanics, National University of Defence Technology, Changsha, Hunan, PRC

ABSTRACT

The line-spring model analysis of surface crack at weld joint of tubes or structures is developed. The numerical results show its validity.

KEYWORDS

Surface Crack, Line-spring model, Weld joint, tube and shell structure.

INTRODUCTION

The failure of pressure vessels and pipes frequently originates from the surface crack at weld joints in welded structures. Because of the Complexities in geometry, material behavior and loading Conditions at weld joints of tubes, accurate analysis is Very difficult. Even if the numerical method, such as the finite element method or the boundary integral equation, is used, great difficulty still remains. This paper uses the line-spring model to analyse this problem and proves it validity. The line-spring model is an approximate and Simplified model for analysis of surface crack. In this model, the local effect of surface crack in a structure is replaced by the distributed line-springs based on the solution for edge cracked strip. For a surface crack at a weld joint of tubes, the local material properties caused by welding and the local stress distribution effect caused by the stress concentration at the intersection of two tubes can be simply considered in the solution of line-spring, i.e. edge cracked strip. Therefore, this model can be used intuitively and appropriately to imitate the surface crack at the weld joint of two tubes and offers a simple method for analyzing surface crack at a weld joint of tubes. This method can be also used for the analysis of surface crack at other joints of structures.

PRESENT METHOD

Consider the intersection of the two tubes, shown in Fig.1, where tube 1 is perpendicular to tube 2 and is welded on the surface of tube 2 . The upper end of tube 1 is subjected to tensile and bending loadings. A surface crack at the intersection A is considered. This crack may be located in the weld metal, the parent metal of tube 1 or 2 , or the fusion line. Based on the line-spring model, the problem is reduced to solving a shell structure with a through crack, of which the distributed line-springs are embedded between the two surfaces. The constitutive relations of the line-spring are derived from the solution of the edge cracked strip.

Through Cracked Shell

If the solution of the corresponding problem without the crack is known, we only need to find the disturbance solution caused by the crack. Because the disturbance solution is localised in the small region around the crack, only an analysis in this region is needed. In this small region, tube 1 and tube 2 can be regarded as shallow shells, as shown in Fig.2(b). For the loading of mode I, We only need to consider the cracked shell subjected to membrane force $N-N^{\infty}$ and bending moment $M-M^{\infty}$ on the crack surfaces. N^{∞} and M^{∞} are the crack surfaces value of the solution for the corresponding problem without the crack, and N and M are the reaction from the line-spring. According to the Reissner-type shallow shell theory, the equations of shell 1 are (Tang and Lu, 1987)

$$\nabla^4 F_1 - \frac{E_1 h_1}{R_1}\frac{\partial^2 W_1}{\partial y^2} = 0 \qquad (1)_a$$

$$\nabla^4 W_1 + \frac{1}{R_1 D_1}\left(1-\frac{2\alpha_1^2}{1-\nu_1}\nabla^2\right)\frac{\partial^2 F_1}{\partial y^2} = 0 \qquad (1)_b$$

$$\nabla^2 \psi_1 - \frac{1}{\alpha_1^2}\psi_1 = 0 \qquad (1)_c$$

Where $D_1 = E_1 h_1^3/12(1-\nu_1^2)$, $\alpha_1 = h_1/\sqrt{10}$, and E_1, ν_1 and h_1 is material elastic modulus, poisson's ratio and wall thickness respectively.

Having solved eqs.(1) for F_1, W_1 and ψ_1, the stress resultants, the couples and the deformations of the shell can be given by

$$\beta_{1x} = -\frac{\partial G_1^*}{\partial x} + \frac{\partial \psi_1}{\partial y}, \qquad \beta_{1y} = -\frac{\partial G_1^*}{\partial y} - \frac{\partial \psi_1}{\partial x} \qquad (2)_{a,b}$$

$$G_1^* = W_1 + \frac{2\alpha_1^2}{1-\nu_1}\nabla^2 W_1 - \frac{4\alpha_1^2}{(1-\nu_1)2R_1D_1}\frac{\partial^2 F_1}{\partial y^2} \qquad (3)$$

$$M_{xx}=D_1\left(\frac{\partial\beta_{1x}}{\partial x}+\nu_1\frac{\partial\beta_{1y}}{\partial y}\right), \qquad M_{yy}=D_1\left(\frac{\partial\beta_{1y}}{\partial y}+\nu_1\frac{\partial\beta_{1x}}{\partial x}\right) \qquad (4)_{a,b}$$

$$M_{xy}=M_{yx}=\frac{(1-\nu_1)D_1}{2}\left(\frac{\partial\beta_{1x}}{\partial y}+\frac{\partial\beta_{1y}}{\partial x}\right) \qquad (4)_c$$

$$N_{xx} = \frac{\partial^2 F_1}{\partial y^2}, \qquad N_{yy} = \frac{\partial^2 F_1}{\partial x^2}, \qquad N_{xy} = -\frac{\partial^2 F_1}{\partial x \partial y} \qquad (5)_{a,b,c}$$

Equations of shell 2 are

$$\nabla^4 F_2 - \frac{E_2 h_2}{R_2}\frac{\partial^2 W_2}{\partial z^2} = 0 \qquad (6)_a$$

$$\nabla^4 W_2 + \frac{1}{R_2 D_2}\left(1-\frac{2\alpha_2^2}{1-\nu_2}\nabla^2\right)\frac{\partial^2 F_2}{\partial z^2} = 0 \qquad (6)_b$$

$$\nabla^2 \psi_2 - \frac{1}{\alpha_2^2}\psi_2 = 0 \qquad (6)_c$$

The boundary conditions are stresses vanish as $y=\infty$ or $z=\pm\infty$

and $\lim_{y\to+0} N_{yy} = N-N^{\infty}$, $\lim_{y\to+0} M_{yy} = M-M^{\infty}$ as $|x|<c$ $\qquad (7)_{a,b}$

The conditions of continuity are

$$u_1 = u_2, \quad \beta_{1x} = \beta_{2x}, \quad W_1 = W_2 \quad \text{as } y=0 \qquad (8)_{a,b,c}$$

$$v_1 = v_2, \quad \beta_{1y} = \beta_{2y}, \quad \text{as } y=0 \text{ and } |x|>c \qquad (8)_{d,e}$$

and the corresponding resultants and couples must be equal.

Solving this shell problem, we can obtain

$$\begin{Bmatrix}\sigma_M\\ \sigma_B\end{Bmatrix} = \begin{Bmatrix}\sigma_M^{\infty}\\ \sigma_B^{\infty}\end{Bmatrix} + \int_{-1}^{+1}\left[F(\bar{x},\bar{t})\right]\begin{Bmatrix}\bar{\delta}'(\bar{t})\\ \bar{\theta}'(\bar{t})\end{Bmatrix}d\bar{t} \qquad (9)$$

here $\sigma_M = N/h_1$, $\sigma_B=6M/h_1^2$, $\sigma_M^{\infty}=N^{\infty}/h_1$, $\sigma_B^{\infty}=6M^{\infty}/h_1^2$, $\bar{\delta}'(\bar{t})=\frac{C}{h_1}\frac{d\delta}{dt}$, $\bar{\theta}'(\bar{t}) = \frac{C}{6}\frac{d\theta}{dt}$, $\delta= v_1(t,0)-v_2(t,0)$, $\theta = \beta_{1y}(t,0)-\beta_{2y}(y,0)$, $\bar{t} = t/C$, and $[F(\bar{x},\bar{t})]$ is a matrix of known functions.

Constitutive Relations of Line-Spring

The constitutive relations of the line-spring can be derived from the edge cracked strip shown in Fig.3. If the surface crack is located in the weld metal or in the parent metal of tube 1 or 2, the edge cracked strip shown in Fig.3(a) can be taken. The material constants are E_3 and ν_3 or E_1 and ν_1 or E_2 and ν_2 respectively and the thickness are h_1 or h_1 or h_2 respectively. If the surface crack is located in the fusion line between the weld and the parent metal, the edge cracked strip shown in Fig.3(b) can be used.

Solving the edge cracked strip shown in Fig.3(a) results in the following cauchy-type singular integral equation for the dislocation density function (t)

$$\int_0^{a_1}\left(\frac{1}{t-z_1} + k(z_1, t)\right)\phi(t)\,dt = -\frac{\pi}{m_1}p(z_1) \qquad (10)$$

Where $\phi(z_1) = \frac{\partial}{\partial z_1}v(0,z_1)$, $m_1 = \frac{\mu}{1-\nu}$, μ = shear modulus, ν = Poisson's ratio, and $p(z_1)=\sigma_M+\sigma_B f(z_1)$. If the $p(z_1)$ is linearly distributed in the z_1 direction, then $f(z_1) = 1-\frac{2z_1}{h}$, $\sigma_M = N/h$ and $\sigma_B=6M/h^2$. But $p(z_1)$ is nonlinear because of the stress concentration effect, this leads to $\int_0^h f(z_1)dz_1=0$, $\sigma_M=N/h$ and $\sigma_B=M/\int_0^h f(z_1)(\frac{h}{2}-z_1)dz_1$, and $f(z_1)$ is taken as a quadratic in this paper.

With the help of the transformations $t=\frac{a_1}{2}(\bar{t}+1)$ and $z_1=\frac{a_2}{2}(\bar{z}_1+1)$, eq.(10) becomes

$$\int_{-1}^{+1}\left(\frac{1}{\bar{t}-\bar{z}_1}+h_{11}(\bar{z}_1,\bar{t})\right)\phi_1(\bar{t})d\bar{t} = -\frac{\pi}{m_1}p_1(\bar{z}_1) \tag{11}$$

Where $\phi_1(\bar{t}) = \phi(\frac{a_1}{2}(\bar{t}+1))$ and $p_1(\bar{z}_1) = p(\frac{a_1}{2}(\bar{z}_1+1))$. The solution of eq.(11) is in the form of

$$\phi_1(\bar{t}) = (1-\bar{t}^2)^{-\frac{1}{2}}G(\bar{t}) \tag{12}$$

The stress intensity factor of the edge crack is then

$$K_I = \sqrt{h}\,[\sigma_M g_M(\zeta_1) + \sigma_B g_B(\zeta_1)] \tag{13}$$

here $g_M(\zeta_1) = -G_M(1)\sqrt{\frac{\pi}{2}\zeta_1}$, $g_B(\zeta_1) = -G_B(1)\sqrt{\frac{\pi}{2}\zeta_1}$, and $\zeta_1 = a_1/h$.

The constitutive relation of line-spring can be obtained from the eq.(13) (Lu and Tang, 1985)

$$\begin{Bmatrix} \delta/h \\ \theta/6 \end{Bmatrix} = \frac{2(1-\nu^2)}{E}\begin{bmatrix}\alpha_{MM}, & \alpha_{MB}\\ \alpha_{BM}, & \alpha_{BB}\end{bmatrix}\begin{Bmatrix}\sigma_M\\ \sigma_B\end{Bmatrix} \tag{14}$$

here $\alpha_{\lambda\mu} = \int_0^{\zeta_1} g_\lambda(\zeta_1)g_\mu(\zeta_1)\,d\zeta_1$, λ,μ = M, B.

Expressed in terms of the dimensionless dislocation densities $\bar{\delta}'(\bar{t})$ and $\bar{\theta}'(\bar{t})$, eq.(14) becomes

$$\frac{2(1-\nu^2)}{E}\begin{bmatrix}\alpha_{MM}, \alpha_{MB}\\ \alpha_{BM}, \alpha_{BB}\end{bmatrix}\begin{Bmatrix}\sigma_M\\ \sigma_B\end{Bmatrix} = \int_{-1}^{+1}\begin{Bmatrix}\bar{\delta}'(\bar{t})\\ \bar{\theta}'(\bar{t})\end{Bmatrix}H(\bar{x}-\bar{t})\,d\bar{t} \tag{15}$$

The combination of eq.(9) and eq.(15) can lead to the governing equation for the dislocation densities $\bar{\delta}'(\bar{t})$ and $\bar{\theta}'(\bar{t})$. First the dislocation densities $\bar{\delta}'(\bar{t})$ and $\bar{\theta}'(\bar{t})$ are obtained by solving the governing equation, then the stresses σ_M and σ_B are calculated from eq.(15). Finally the stress intensity factor for the surface crack is given by putting these stresses into eq.(13).

SURFACE CRACK AT WELD JOINT

To simplify the calculation, we further assume that the displacements of tube 1 are fully restrained by tube 2 . The conditions of continuity (9) then become

$$u_1=0, \quad \beta_{1x}=0 , \quad \psi_1=0 \quad \text{as} \quad y=0 \tag{16$_{a,b,c}$}$$

$$v_1=0, \quad \beta_{1y}=0 \quad \text{as} \quad y=0 \ \text{and} \ |x|>c \tag{16$_{d,e}$}$$

The solution of eq.(1) by Fourier transform in the half plane $y\geqslant 0$ is given as

$$W_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\sum_{j=1}^{4}R_j(s)e^{m_j|s|\,y}e^{-isx}\,ds \tag{17$_a$}$$

$$F_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sum_{j=1}^{4}\frac{E_1h_1}{R_1}\frac{m_j^2}{p_j^2 s^2}R_j(s)e^{m_j|s|\,y}e^{-isx}\,ds \tag{17$_b$}$$

$$\psi_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}R_5(s)e^{-\gamma|s|\,y}e^{-isx}\,ds \tag{17$_c$}$$

here $p_j=m_j^2-1$, $\gamma = \sqrt{1+\frac{1}{\alpha_1^2 s^2}}$, m_j(j=1, 2, 3, 4) is the root satisfying $R_e\{m_j\}$ <0 of the following equation

$$(m_j^2-1)^4 + \frac{E_1h_1}{R_1^2D_1s^4}\left[1-\frac{2\alpha_1^2s^2}{1-\nu_1}(m_j^2-1)\right]m_j^4 = 0 \tag{18}$$

and the five unknown functions $R_1(s)$, $R_2(s)$, ..., $R_5(s)$ are determined from the continuity conditions $(16)_{a,b,c}$ and the mixed boundary conditions $(16)_{d,e}$ and $(7)_{a,b}$. It follows from the eq.$(16)_{a,b,c}$ that

$$R_j(s)=(-1)^{j+1}if_1R_1\frac{A_j}{B_o} + (-1)^{j+1}i\frac{\gamma_1f_2}{s^2}\frac{B_j}{B_o}\ (j=1,2,3,4) \tag{19$_a$}$$

$$R_5(s) = \sum_{j=1}^{4}\left[(-1)^j\frac{A_{5,j}}{B_o}L_jf_1R_1 + (-1)^{j+1}\frac{B_j}{B_o}L_j\frac{f_2}{s^2}\right] \tag{19$_b$}$$

Where $f_1 = \frac{1}{\sqrt{2\pi}}\int_{-c}^{c}\delta'(t)e^{ist}\,dt$, $\delta = v_1(x,o)$, $f_2 = \frac{1}{\sqrt{2\pi}}\int_{-c}^{c}\theta'(t)e^{ist}dt$, $\theta = \beta_{1y}(x,o)$, and B_o, B_j, A_j, $A_{5,j}$ and L_j are known functions.

It follows from the mixed boundary conditions $(16)_{d,e}$ and $(7)_{a,b}$ that

$$\begin{Bmatrix}\sigma_M\\ \sigma_B\end{Bmatrix} = \begin{Bmatrix}\sigma_M^\infty\\ \sigma_B^\infty\end{Bmatrix} + [C]\int_{-1}^{+1}\left(\frac{1}{\bar{t}-\bar{x}}\begin{Bmatrix}\bar{\delta}'(\bar{t})\\ \bar{\theta}'(\bar{t})\end{Bmatrix} + [k(\bar{x},\bar{t})]\begin{Bmatrix}\bar{\delta}'(\bar{t})\\ \bar{\theta}'(\bar{t})\end{Bmatrix}\right)d\bar{t} \tag{20}$$

here $[C] = \frac{2E_1h_1}{(1+\nu_1)(3-\nu_1)c}\begin{bmatrix}1, & 0\\ 0, & 3\end{bmatrix}$,

$$[k(\bar{x},\bar{t})] = \frac{E_1h_1}{C}\begin{bmatrix}-k_{11}(\bar{x},\bar{t}) & , & -\frac{6h_1}{R_1}k_{12}(\bar{x},\bar{t})\\ -\frac{1}{2(1-\nu_1^2)}k_{21}(\bar{x},\bar{t}) & , & \frac{6}{(1+\nu_1)(3-\nu_1)}k_{22}(\bar{x},\bar{t})\end{bmatrix}$$

and $k_{ij}(\bar{x},\bar{t})$ are known functions.

Eliminating σ_M and σ_B from eq.(20) and eq.(15), we get

$$\int_{-1}^{+1}\left(\frac{1}{\bar{t}-\bar{x}}\begin{Bmatrix}\bar{\delta}'(\bar{t})\\ \bar{\theta}'(\bar{t})\end{Bmatrix} + [K(\bar{x},\bar{t})]\begin{Bmatrix}\bar{\delta}'(\bar{t})\\ \bar{\theta}'(\bar{t})\end{Bmatrix}\right)d\bar{t} + [C]^{-1}\begin{Bmatrix}\sigma_M^\infty\\ \sigma_B^\infty\end{Bmatrix} = 0 \tag{21}$$

here $[K(\bar{x},\bar{t})] = [C]^{-1}([k(\bar{x},\bar{t})] - \frac{E_1}{2(1-\nu_1^2)}[\alpha_{\lambda\mu}]^{-1}H(\bar{x}-\bar{t}))$. Eq.(21) is solved by the Lobatt-Chebyshev's method (Theocaris and Ioakimidis, 1977) and $\bar{\delta}'(\bar{t})$ and $\bar{\theta}'(\bar{t})$ are obtained. Then σ_M and σ_B are given from eq.(15) and finally the stress intensity factor K_I is calculated from eq.(13).

NUMERICAL RESULTS AND CONCLUSIONS

The results for the surface crack at the weld metal are presented in Fig.4-11. The effects of Young's modulus E_3 and poisson's ratio ν_3 of the weld metal are given in Fig.4-7 and Fig.8-11 respectively, Figs.4,5,8 and 9 are for pure tension and the Figs.6,7,10 and 11 for pure bending. The results for the surface cracks at the parent metal of tube 1 are presented in Figs. 12-15, and the effects of the tube radius R_1 are also given in these figures, of which Figs.12 and 13 are for pure tension, and Figs.14 and 15 for pure bending. $Q=1+1.464(\frac{a_o}{c_o})^{1.65}$. It can be found from the above results that the proposed model is promising.

In fact, the analysis with this model for the surface crack in the complicated structures is a method which divides the problem into three levels on the scale. In the first level, It is the solution of the shell structure without the crack, such as the above resultants N^{∞} and M^{∞}. In the second level, It is the disturbance solution from the crack, Such as the solution of the shell with a through crack shown in Fig.2. In the third level, it is the solution of the edge cracked strip shown in Fig.3, which corresponds to the scale of the stress singular field around the crack tip. The connection between first level and second level is by the superposition principle. The connection between second level and third level is by the asymptotic match. The events in the different level can be only considered in the respective level. For example, we put the effects of weld metal property and local stress concentration on the stress intensity factor into the third level and the effects of shell intersection feature on the disturbance field into the second level. Therefore, this method has many advantages in analyzing a surface crack at weld joint of a complicated structure.

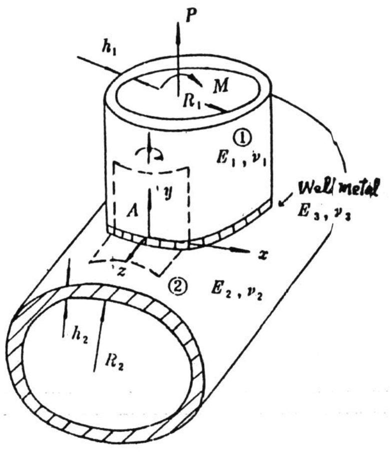

Fig. 1

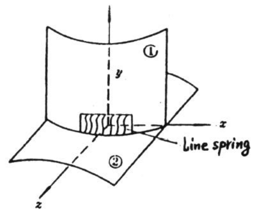

Fig.2 (a)

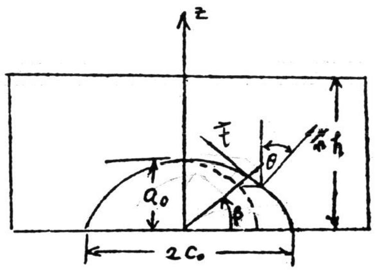

Fig.2 (b)

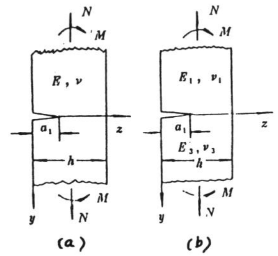

Fig. 3

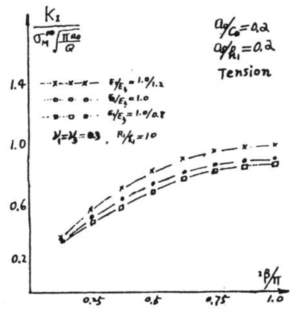

Fig. 4

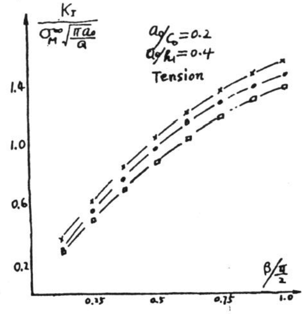

Fig. 5

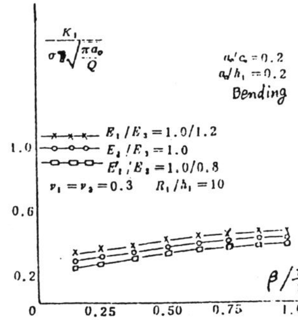

Fig. 6

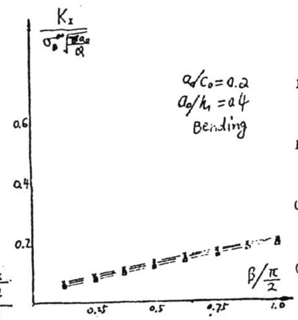

Fig. 7

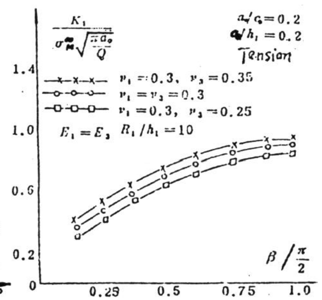

Fig. 8

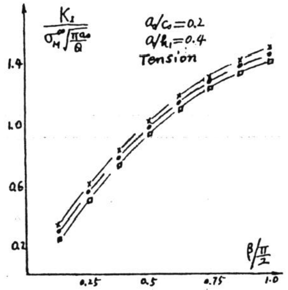

Fig. 9

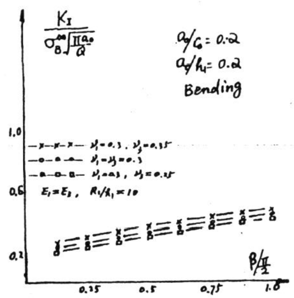

Fig. 10

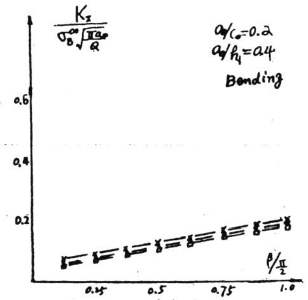

Fig. 11

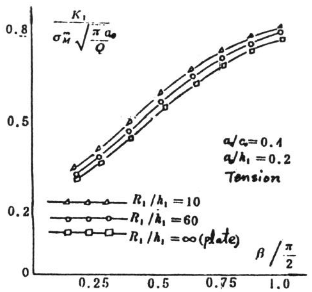

Fig. 12

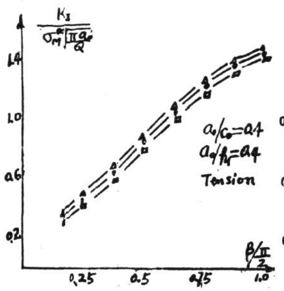

Fig. 13

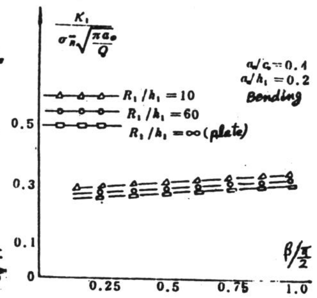

Fig. 14

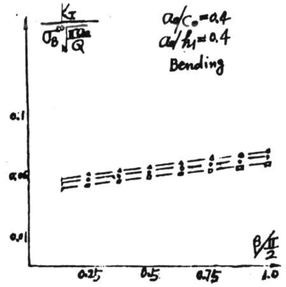

Fig. 15

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