The Line Spring Model for Surface Cracked Shell Subjected to Antisymmetric Loadings

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ABSTRACT

The line-spring model for surface cracked shell with arbitrary principal curvatures and under antisymmetric loadings in respect to crack plane are formulated for the unimproved and improved models. The numerical results show that the improved model is more suitable than the unimproved one.

KEYWORDS

Surface crack, Line-spring model, shell, Antisymmetric loadings.

INTRODUCTION

The line-spring model for surface cracked shell with arbitrary principal curvatures and under antisymmetric loadings in respect to crack plane are considered in this paper. The combination of this model and the symmetric one(Tang and Lu, 1987) will furnish the solution of line-spring model for surface cracked shell under the general loadings, i.e. the mixture of mode I, II and III. The numerical results are compared with the existing solutions of the alternative method in the case of plate(Smith and Sorensen, 1974). It is shown that in the range of $0 < a_0/c_0 < 1.0$, the solutions of improved model are better consistent with those of the alternative method(Smith and Sorensen, 1974) than those of unimproved model. The numerical results for the general shallow shell with a surface crack are also given.

THE PRESENT MODEL

As shown in Fig.1, the surface cracked shell can be simulated by the Reissner shallow shell with a through crack, in which the distributed line springs are embedded. The constitutive relations of springs are taken as the relations between the generalized forces and displacements of the plane strain edge cracked strips, Fig.1(c). The stress intensity factors of surface crack are then obtained from the edge cracked strips for the unimproved model. However, the curvature effect of crack front which gives rise to the interaction between springs has been omitted in this model. From the solution of embedded elliptical crack in infinite body(Kassir and Sih, 1975), the improved solutions of edge cracked strip considering the interaction effect can be derived and the improved constitutive relation of the line spring are formulated from energy principle.

THE BEHAVIOR EQUATIONS OF SHALLOW SHELL UNDER ANTISYMMETRIC LOADINGS

The shallow sheel with arbitrary constant principal curvatures and with a through crack and the loadings which are antisymmetric in respect to crack plane are shown in Fig.1(b). The solution based on the Reissner-type shallow shell theory has been obtained by us from Fourier transform. We call it as the behavior equations

$$\int_{-1}^{+1}\left\{\left(\frac{1}{\bar{t}-\bar{x}}+h_{11}(\bar{t},\bar{x})\right)G_1(\bar{t})+h_{12}(\bar{t},\bar{x})G_2(\bar{t})+\left[h_{13}(\bar{t},\bar{x})-\left(\frac{\lambda_x}{\lambda}\right)^2\bar{t}\left(\frac{1}{\bar{t}-\bar{x}}+h_{11}(\bar{t},\bar{x})\right)\right]G_3(\bar{t})\right\}d\bar{t}$$

$$=\frac{2\pi}{E}\left(-\tau_M^{\infty}+\tau_M\right) \tag{1$_a$}$$

$$\int_{-1}^{+1}\left\{\left(\frac{1-\nu}{\lambda^4}\frac{1}{\bar{t}-\bar{x}}+h_{22}(\bar{t},\bar{x})\right)G_2(\bar{t})+h_{21}(\bar{t},\bar{x})G_1(\bar{t})+\left[h_{23}(\bar{t},\bar{x})-\left(\frac{\lambda_x}{\lambda}\right)^2+h_{21}(\bar{t},\bar{x})\right]G_3(\bar{t})\right\}d\bar{t}$$

$$=\frac{\pi}{3E}\left(-\tau_B^{\infty}+\tau_B\right)(-1)^n \tag{1$_b$}$$

$$\int_{-1}^{+1}\left\{\left[\frac{1}{\bar{t}-\bar{x}}+h_{33}(\bar{t},\bar{x})-\left(\frac{\lambda_x}{\lambda}\right)^2\bar{t}\,h_{31}(\bar{t},\bar{x})\right]G_3(\bar{t})+h_{31}(\bar{t},\bar{x})G_1(\bar{t})+h_{32}(\bar{t},\bar{x})G_2(\bar{t})\right\}d\bar{t}$$

$$=\frac{2\pi}{E}\left(-\tau_Q^{\infty}+\tau_Q\right)2(1+\nu) \tag{1$_c$}$$

where $\tau_B=6H/h^2$, $\tau_B^{\infty}=6H^{\infty}/h^2$, $\tau_M=T/h$, $\tau_M^{\infty}=T^{\infty}/h$, $\tau_Q=6Q/5h$, $\tau_Q^{\infty}=6Q^{\infty}/5h$, $\bar{x}=x/c_0$, $\left(\frac{\lambda_x}{\lambda}\right)^2=\frac{C_0}{R_x}$, $\lambda^4=12(1-\nu^2)C_0^2/h^2$, $G_j(\bar{t})$ (j=1, 2, 3) are dimensionless dislocation densities, and n=0 and 1 are corresponding to outer and inner surface crack respectively.

CONSTITUTIVE RELATIONS OF THE LINE SPRINGS

Solving the edge cracked strip as shown in Fig.1(c), the stress intensity factors of the edge crack are found as follow

$$K_{IIz}=\sqrt{h}\,\tau_Q g_Q^{II}(\zeta_1) \tag{2$_a$}$$

$$K_{IIIz}=\sqrt{h}\left[\tau_M g_M^{III}(\zeta_1)+\tau_B g_B^{III}(\zeta_1)\right] \tag{2$_b$}$$

where $\zeta_1=a_1/h$.

For the strain energy of the edge cracked strip U, we have

$$U=U_0+\int_0^{a_1}G_z\,da_1=U_0+\int_0^{a_1}\left(\frac{1-\nu^2}{E}K_{IIz}^2+\frac{1+\nu}{E}K_{IIIz}^2\right)da_1 \tag{3}$$

where U_0 is the strain energy of the strip without the crack. Therefore the constitutive relations of the line spring can be found as follow

$$\delta_x=\frac{1}{h}\frac{\partial U}{\partial \tau_M}=\frac{2h(1+\nu)}{E}\left(\tau_M\alpha_{MM}^{III}+\tau_B\alpha_{MB}^{III}\right) \tag{4$_a$}$$

$$\theta_x=\frac{6}{h}\frac{\partial U}{\partial \tau_B}=\frac{12(1+\nu)}{E}\left(\tau_M\alpha_{BM}^{III}+\tau_B\alpha_{BB}^{III}\right) \tag{4$_b$}$$

$$\delta_z=\frac{1}{h}\frac{\partial U}{\partial \tau_Q}=\frac{2h(1-\nu^2)}{E}\tau_Q\alpha_{QQ}^{II} \tag{4$_c$}$$

where $\alpha_{QQ}^{II}=\int_0^1 g_Q^{II^2}(\zeta_1 t)\zeta_1\,dt$ and $\alpha_{\lambda\mu}^{III}=\int_0^1 g_\lambda^{III}(\zeta_1 t)g_\mu^{III}(\zeta_1 t)\zeta_1\,dt$ $(\lambda,\mu=M,B)$

Introducing the dislocation densities $G_j(\bar{t})$ (j=1,2,3), the constitutive relations become

$$\begin{Bmatrix}\tau_M\\ \tau_B\\ \tau_Q\end{Bmatrix}=\frac{E}{1+\nu}\begin{bmatrix}\gamma_{MM}^{III}, & \gamma_{MB}^{III}, & 0\\ \gamma_{BM}^{III}, & \gamma_{BB}^{III}, & 0\\ 0, & 0, & \frac{1}{1-\nu}\gamma_{QQ}^{II}\end{bmatrix}\int_{-1}^{+1}\begin{Bmatrix}\frac{C_0}{h}G_1(\bar{t})\\ \frac{1}{6}G_2(\bar{t})\\ \frac{C_0}{h}G_3(\bar{t})\end{Bmatrix}H(\bar{x}-\bar{t})\,d\bar{t} \tag{5}$$

where H(t) is the Heaviside unit function and γ_{QQ}^{II} and $\gamma_{\lambda\mu}^{III}$ (λ,μ=M, B) are known functions of $\bar{x}$. In these relations, the curvature effect of crack front which gives rise to the interaction between springs has been omitted, For this reason, there are not transformation of stress intensity factors between mode II and III along the surface crack front in this unimproved model. The constitutive relations considering this effect can be derived from the solution of embedded elliptical crack in infinite body.

For an embedded elliptical crack subjected to shear stresses τ_Q and $\tau_M+\tau_B\left(1-\frac{2z}{h}\right)$ on the crack face, the stress intensity factors are respectively

$$K_{II}(\tau_Q)=\tau_Q f_{2Q}\sin\beta\sqrt{\pi a_0}\left(\sin^2\beta+\frac{a_0^2}{C_0^2}\cos^2\beta\right)^{-\frac{1}{4}} \tag{6$_a$}$$

$$K_{III}(\tau_Q)=\tau_Q f_{3Q}\cos\beta\sqrt{\pi a_0}\left(\sin^2\beta+\frac{a_0^2}{C_0^2}\cos^2\beta\right)^{-\frac{1}{4}} \tag{6$_b$}$$

and

$$K_{II}(\tau_M,\tau_B)=\left[\tau_M f_{2M}\cos\beta+\tau_B\left(f_{2M}\cos\beta+\frac{2a_0}{h}f_{2B}\sin\beta\cos\beta\right)\right]\sqrt{\pi a_0}\left(\sin^2\beta+\frac{a_0^2}{C_0^2}\cos^2\beta\right)^{-\frac{1}{4}} \tag{7$_a$}$$

$$K_{III}(\tau_M,\tau_B)=\left[\tau_M f_{3M}\sin\beta+\tau_B\left(f_{3M}\sin\beta+\frac{2a_0}{h}f_{3B}(\sin^2\beta+f_J\cos^2\beta)\right)\right]\sqrt{\pi a_0}\left(\sin^2\beta+\frac{a_0^2}{C_0^2}\cos^2\beta\right)^{-\frac{1}{4}} \tag{7$_b$}$$

where β is the parametric angle of ellipse.

With the help of directional derivative formulas of strain energy U, we have

$$\frac{dU}{dz}=\frac{dU}{dn}\cos\theta+\frac{dU}{d\tau}\sin\theta \tag{8}$$

where n and τ are normal and tangential variable of elliptical front respectively. Because $\frac{dU}{d\tau} = 0$, eq. (8) becomes

$$\frac{dU}{dz} = \frac{dU}{dn}\cos\theta \qquad (9)$$

where $\cos\theta = \sin\beta / (\sin^2\beta + \frac{a_o^2}{C_o^2}\cos^2\beta)^{\frac{1}{2}}$

From the relations $\frac{dU}{dz} = G_z$ and $\frac{dU}{dn} = G$, we derive

$$K_{IIz}^2 = \left[K_{II}^2(\tau_Q) + \frac{1}{1-\nu}K_{III}^2(\tau_Q)\right]\cos\theta \qquad (10)_a$$

$$K_{IIIz}^2 = \left[(1-\nu)K_{II}^2(\tau_M,\tau_B) + K_{II}^{1}(\tau_M,\tau_B)\right]\cos\theta \qquad (10)_b$$

By introducing the boundary correction, the improved stress intensity factors of edge cracked strip, K_{IIZ} and K_{IIIZ}, is then obtained. Similarly the strain energy of the edge cracked strip takes the form of eq.(3) and therefore the improved constitutive relations of the line spring are

$$\begin{Bmatrix}\tau_M\\ \tau_B\\ \tau_Q\end{Bmatrix} = \frac{E}{1+\nu}\begin{bmatrix}\tilde{\gamma}_{MM}^{III}, & \tilde{\gamma}_{MB}^{III}, & 0\\ \tilde{\gamma}_{BM}^{III}, & \tilde{\gamma}_{BB}^{III}, & 0\\ 0, & 0, & \frac{1}{1-\nu}\tilde{\gamma}_{QQ}^{II}\end{bmatrix} \qquad (11)$$

where $\left[\tilde{\gamma}_{\lambda\mu}^{III}\right] = \left[\tilde{\alpha}_{\lambda\mu}^{III}\right]^{-1}$, $\tilde{\gamma}_{QQ}^{II} = \tilde{\alpha}_{QQ}^{II\,-1}$, $\tilde{\alpha}_{\lambda\mu}^{III} = \int_0^1 A_{\lambda\mu}\, g_\lambda^{III}(\zeta_1 t) g_\mu^{III}(\zeta_1 t)\zeta_1\, dt \ (\lambda,\mu = M, B)$

$$A_{MM} = \left[(1-\nu)f_{2M}^2\cos^2\beta + f_{3M}^2\sin^2\beta\right] / (\sin^2\beta + \frac{a_o^2}{C_o^2}\cos^2\beta)$$

$$A_{MB} = A_{BM} = \Big\{\left[(1-\nu)f_{2M}^2\cos^2\beta + f_{3M}^2\sin^2\beta\right] + 2\zeta_1 t\left[(1-\nu)f_{2M}f_{2B}\cos^2\beta + f_{3M}f_{3B}(\sin^2\beta + f_J\cos^2\beta)\right]$$
$$\Big\}/(1-\zeta_1 t)(\sin^2\beta + \frac{a_o^2}{C_o^2}\cos^2\beta)$$

$$A_{BB} = \left\{(1-\nu)(f_{2M} + 2\zeta_1 t f_{2B})^2\cos^2\beta + \left[f_{3M} + 2\zeta_1 t f_{3B}(1 + f_J\cos^2\beta/\sin^2\beta)\right]^2\sin^2\beta\right\} /$$
$$(1-\zeta_1 t)^2(\sin^2\beta + \frac{a_o^2}{C_o^2}\cos^2\beta)$$

$$\tilde{\alpha}_{QQ}^{II} = \int_0^1 A_{QQ}\, g_Q^{II\,2}(\zeta_1 t)\zeta_1\, dt$$

$$A_{QQ} = (f_{2Q}^2\sin^2\beta + \frac{1}{1-\nu}f_{3Q}^2\cos^2\beta) / (\sin^2\beta + \frac{a_o^2}{C_o^2}\cos^2\beta) .$$

RESULTS AND CONCLUSIONS

The systems of basic integral equations of the unimproved model and the improved one can be obtained by substituting eq.(5) and eq.(11) into eq.(1). For the unimproved model, we have

$$\int_{-1}^{+1}\Big\{\Big[\frac{1}{\bar{t}-\bar{x}} + h_{11}(\bar{t},\bar{x}) - \frac{2\pi}{1+\nu}\frac{C_o}{h}\gamma_{MM}^{III}H(\bar{x}-\bar{t})\Big]G_1(\bar{t}) + \Big[h_{12}(\bar{t},\bar{x}) - \frac{2\pi}{6(1+\nu)}\gamma_{MB}^{III}H(\bar{x}-\bar{t})\Big]G_2(\bar{t}) +$$
$$+\Big[h_{13}(\bar{t},\bar{x}) - \Big(\frac{\lambda_x}{\lambda}\Big)^2\bar{t}\Big(\frac{1}{\bar{t}-\bar{x}} + h_{11}(\bar{t},\bar{x})\Big)\Big]G_3(\bar{t})\Big\}d\bar{t} = -2\pi\frac{\tau_M^\infty}{E} \qquad (12)_a$$

$$\int_{-1}^{+1}\Big\{\Big[h_{21}(\bar{t},\bar{x}) - \frac{(-1)^n 2\pi}{6(1+\nu)}\gamma_{BM}^{III}H(\bar{x}-\bar{t})\Big]G_1(\bar{t}) + \Big[\frac{1-\nu^2}{\lambda^4}\frac{1}{\bar{t}-\bar{x}} + h_{22}(\bar{t},\bar{x}) - \frac{(-1)^n 2\pi}{36(1+\nu)}\frac{h}{C_o}\gamma_{BB}^{III}H(\bar{x}-\bar{t})$$
$$\Big]G_2(\bar{t}) + \Big[h_{23}(\bar{t},\bar{x}) - \Big(\frac{\lambda_x}{\lambda}\Big)^2\bar{t}\, h_{21}(\bar{t},\bar{x})\Big]G_3(\bar{t})\Big\}d\bar{t} = -2\pi\frac{h}{C_o}\frac{\tau_B^\infty}{6E}(-1)^n \qquad (12)_b$$

$$\int_{-1}^{+1}\Big\{h_{31}(\bar{t},\bar{x})G_1(\bar{t}) + h_{32}(\bar{t},\bar{x})G_2(\bar{t}) + \Big[\frac{2}{\bar{t}-\bar{x}} + h_{33}(\bar{t},\bar{x}) - \Big(\frac{\lambda_x}{\lambda}\Big)^2\bar{t}\, h_{31}(\bar{t},\bar{x}) -$$
$$-\frac{4\pi}{1-\nu}\frac{C_o}{h}\gamma_{QQ}^{II}H(\bar{x}-\bar{t})\Big]G_3(\bar{t})\Big\}d\bar{t} = -2\pi\frac{\tau_Q^\infty}{E}2(1+\nu). \qquad (12)_c$$

This is a system of cauchy-type singular integral equations and it can be solved numerically by Gauss-chebyshev quadrature(Erdogan and Gupta, 1972). Using these solutions, the line-spring forces, τ_Q, τ_M and τ_B, can be obtained from eq.(5) and the stress intensity factors K_{IIZ} and K_{IIIZ} can be caluclated from eq.(2). For the improved model, the only need is the substitution of $\tilde{\gamma}_{QQ}^{II}$ and $\tilde{\gamma}_{\lambda\mu}^{III}$ for γ_{QQ}^{II} and $\gamma_{\lambda\mu}^{III}$.

The numerical results of surface cracked plate or shallow shell with h/R_x = 0.0001 are illustrated in Fig.2(a), (b) and (c). It can be seen that the results of improved model are fairly consistant with those of alternating method in the range $0 < a_o/c_o < 1.0$, but the results of unimporved model are rather unsatisfactory, particularly for mode II. In these figures, $M_2 = K_{II}/\tau_M^\infty\sqrt{\frac{\pi a_o}{Q}}$, $M_3 = K_{III}/\tau_M^\infty\sqrt{\frac{\pi a_o}{Q}}$, $Q = 1+1.464(a_o/c_o)^{1.65}$ and $\theta = \frac{\pi}{2} - \beta$. The stress intensity factors for the outer and inner surface crack in general shallow shell are given in table 1 to table 4. In these tables, $e = R_x/R_y$ and R_x and R_y are two principal curvature radiuses of the shell.

The combination of this model under antisymmetrical loading with one under symmetrical loading can carry out the analysis of surface crack in varions shell structures under complex loadings, i.e. the combination of mode I, II and III loadings.

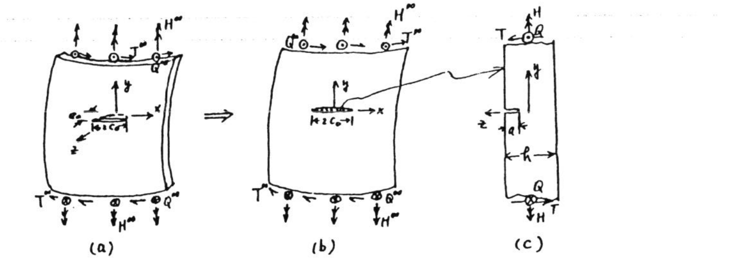
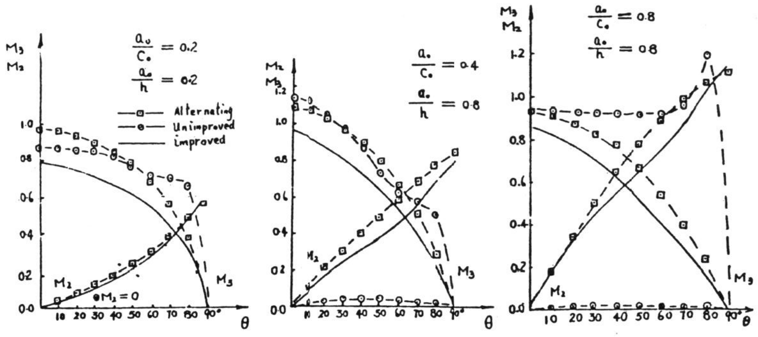


Fig. 1



(a) (b) (c)

Fig.2

Table 1. $M_{o2}=K_{oII}/\sqrt{\pi a_o/Q}$, $M_{o3}=K_{oIII}/\sqrt{\pi a_o/Q}$, $a_o/c_o=0.2$, $a_o=0.2h$, $\tau_M^\infty=1.0$, $\tau_B^\infty=0.0$, $\tau_Q^\infty=0.0$

e	h/R_x	θ	0°	20°	40°	60°	80°	90°
0.0	0.05	M_{o2}	0.0000	0.0734	0.1527	0.2631	0.5060	0.6095
		M_{o3}	0.7190	0.7008	0.6437	0.5363	0.3112	0.0000
	0.1	M_{o2}	0.0000	0.0733	0.1526	0.2630	0.5033	0.6040
		M_{o3}	0.7187	0.7008	0.6435	0.5359	0.3100	0.0000
	0.2	M_{o2}	0.0000	0.0732	0.1526	0.2628	0.5011	0.6026
		M_{o3}	0.7185	0.7003	0.6431	0.5355	0.3082	0.0000
0.5	0.05	M_{o2}	0.0000	0.0735	0.1530	0.2644	0.5296	0.6426
		M_{o3}	0.7194	0.7015	0.6448	0.5389	0.3257	0.0000
	0.1	M_{o2}	0.0000	0.0734	0.1528	0.2640	0.5266	0.6332
		M_{o3}	0.7189	0.7013	0.6437	0.5371	0.3211	0.0000
	0.2	M_{o2}	0.0000	0.0732	0.1526	0.2637	0.5233	0.6206
		M_{o3}	0.7188	0.7005	0.6436	0.5364	0.3124	0.0000

Table 2. $M_{i2}=K_{iII}/\sqrt{\pi a_o/Q}$, $M_{i3}=K_{iIII}/\sqrt{\pi a_o/Q}$, $a_o/c_o=0.2$, $a_o/h=0.2$, $\tau_M^\infty=1.0$, $\tau_B^\infty=0.0$, $\tau_Q^\infty=0.0$

e	h/R_x	θ	0°	20°	40°	60°	80°	90°
0.0	0.05	M_{i2}	0.0000	0.0742	0.1544	0.2644	0.5049	0.6172
		M_{i3}	0.7204	0.7018	0.6444	0.5362	0.3009	0.0000
	0.10	M_{i2}	0.0000	0.0749	0.1556	0.2654	0.5094	0.6226
		M_{i3}	0.7206	0.7021	0.6446	0.5364	0.3075	0.0000
	0.20	M_{i2}	0.0000	0.0770	0.1597	0.2679	0.5128	0.6237
		M_{i3}	0.7209	0.7030	0.6447	0.5360	0.3104	0.0000
0.5	0.05	M_{i2}	0.0000	0.0742	0.1543	0.2645	0.5050	0.6073
		M_{i3}	0.7203	0.7021	0.6446	0.5364	0.3100	0.0000
	0.10	M_{i2}	0.0000	0.0749	0.1559	0.2671	0.5322	0.6451
		M_{i3}	0.7213	0.7032	0.6461	0.5393	0.3263	0.0000
	0.20	M_{i2}	0.0000	0.0771	0.1600	0.2705	0.5207	0.6271
		M_{i3}	0.7222	0.7038	0.6457	0.5373	0.3174	0.0000

Table 3. $M_{o2}=K_{oII}/\sqrt{\pi a_o/Q}$, $M_{o3}=K_{oIII}/\sqrt{\pi a_o/Q}$, $h/R_x=0.2$, $a_o/c_o=0.2$, $a_o/h=0.2$, $\tau_M^\infty=1.0$, $\tau_B^\infty=0.0$, $\tau_Q^\infty=0.0$

θ	e=0.0 M_{o2}	M_{o3}	e=0.2 M_{o2}	M_{o3}	e=0.4 M_{o2}	M_{o3}	e=0.6 M_{o2}	M_{o3}
0°	0.0000	0.7185	0.0000	0.7187	0.0000	0.7186	0.0000	0.7191
20°	0.0732	0.7003	0.0733	0.7005	0.0733	0.7005	0.0734	0.7012
40°	0.1526	0.6431	0.1527	0.6436	0.1527	0.6432	0.1532	0.6450
60°	0.2628	0.5355	0.2633	0.5364	0.2632	0.5363	0.2648	0.5393
80°	0.5011	0.3082	0.5088	0.3129	0.5087	0.3128	0.5364	0.3298
90°	0.6026	0.0000	0.6134	0.0000	0.6133	0.0000	0.6521	0.0000

Table 4. $M_{i2}=K_{iII}/\sqrt{\pi a_o/Q}$, $M_{i3}=K_{iIII}/\sqrt{\pi a_o/Q}$, $h/R_x=0.25$, $a_o/c_o=0.2$, $a_o/h=0.2$, $\tau_M^\infty=1.0$, $\tau_B^\infty=0.0$, $\tau_Q^\infty=0.0$

θ	e = 0.0 M_{i2}	M_{i3}	e = 0.2 M_{i2}	M_{i3}	e = 0.4 M_{i2}	M_{i3}	e = 0.6 M_{i2}	M_{i3}
0°	0.0000	0.7215	0.0000	0.7219	0.0000	0.7222	0.0000	0.7230
20°	0.0770	0.7030	0.0770	0.7035	0.0769	0.7040	0.0773	0.7046
40°	0.1597	0.6447	0.1598	0.6455	0.1599	0.6461	0.1604	0.6463
60°	0.2697	0.5358	0.2705	0.5376	0.2710	0.5387	0.2710	0.5392
80°	0.5094	0.3010	0.5218	0.3182	0.5334	0.3255	0.5337	0.3269
90°	0.6111	0.0000	0.6286	0.0000	0.6453	0.0000	0.6456	0.0000

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