The Damage Criterion for Ductile Fracture and its Applications

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ABSTRACT

In this paper, the damage mechanics criterion for ductile fracture is further investigated and is used to predict the rupture of notched tensile specimens under both proportional and nonproportional loading condition. It is also applied to model macrocrack initiation and the relationships between V_{Dc} and J_{Ic} (or δ_{tc}) are obtained. Also, two equations for K_{Ic} and δ_{tc} are derived from the damage criterion, which are in good agreement with related experimental results.

KEYWORDS

Damage criterion; damage parameter; ductile fracture; microvoids; fracture toughness; crack initiation.

INTRODUCTION

For ductile materials, the rupture processes are strain controlled. So many local criterion with respect to the effective plastic strain $\bar{\varepsilon}_p$ have been proposed. Criteria like "Effective plastic Strain"(Cockroft, 1968a), "Plastic Work"(Cockroft, 1968b), "Maximum Tensile Work"(Cockroft and Latham, 1968), "Plastic-strain, Mean-stress"(Norris et al., 1978), etc, have been advanced to explain the fracture behavior of various materials and have found wide application to ductile fracture. However, most of them do not take into consideration the gradual increase of internal damage during the deformation of ductile materials.

Considering the fact that ductile fracture involves three successive damage processes, i.e. the nucleation of voids from inclusion, void growth and void coalescence, McClintock(1968), firstly, has given an approximate equation for hole growth and proposed a fracture criterion in ductile fracture, whilst Rice and Tracey (1969), later, have given a rather simple expression for the growth of a spherical void in an infinite body, which leads to the critical void growth criterion for ductile rupture (Beremin, 1985, Zheng and Radon, 1985).

On the other hand, the continuum damage theory developed within the framework of thermodynamics and considering systematically the effects of microvoids on the subsequent development of these voids themselves and on the states of stress and strain in materials, has been applied to the problems of ductile plastic damage. Many damage models and damage criteria have been proposed and applied to describe the ductile fracture process of metals(Lemaitre, 1985, Tai and Yang, 1987).

In this paper, the damage mechanics criterion for ductile fracture proposed by Tai and Yang(1987) is further investigated experimentally, and the new damage parameter V_{Dc} with an obvious microscopic physical meaning is emphasized. The damage criterion is used to predict the rupture of the notched tensile specimens under both proportional and nonproportional loading conditions, and a comparison of theoretical results and the experimental data is made. Also, the damage criterion is applied to model macrocrack initiation and the relationships between V_{Dc} and J_{Ic} are derived. Two equations for fracture toughness K_{Ic} and COD at ductile fracture initiation are obtained, which are coincident with experimental results.

DAMAGE CRITERION FOR DUCTILE FRACTURE

Damage Criterion

In ductile metals, inclusions and second-phase particles are generally believed to be principally responsible for the initiation of microvoids, and mechanical damage is mainly caused by the growth and coalescence of voids. For this reason, a new damage parameter V_D indicating the microscopic geometrical deterioration of ductile materials was proposed by Tai and Yang (1987). It is assumed that ductile failure would occur if the damage parameter V_D reaches its critical value V_{Dc} in a ductile material, then the damage criterion is expressed as follows:

$$V_D = V_{Dc} \quad (1)$$

or

$$V_D = \int_{\bar{\varepsilon}_0}^{\bar{\varepsilon}_p} f(\sigma_m/\bar{\sigma}) \cdot d\bar{\varepsilon}_p = V_{Dc} \quad (2)$$

or

$$V_D = C \cdot \ln(D/D_0) = V_{Dc} \quad (3)$$

where

$$f(\sigma_m/\bar{\sigma}) = \frac{2}{3}(1+\nu) + 3(1-2\nu)(\sigma_m/\bar{\sigma})^2 \quad (4)$$

$\sigma_m/\bar{\sigma}$ is the stress triaxiality, $\bar{\varepsilon}_p$ the effective plastic strain, D the damage variable under Kachanov-Lemaitre(Lemaitre, 1985), and V_{Dc} is the critical damage parameter and may be considered as a material constant.

For proportional loading, the damage criterion is very simple;

$$V_D = f(\sigma_m/\bar{\sigma}) \cdot (\bar{\varepsilon}_p - \bar{\varepsilon}_0) = V_{Dc} \quad (5)$$

It is easy to see that the present damage criterion has obvious

micro- and macroscopic physical meaning and is comparable with other local criteria for ductile fracture.

From macroscopic viewpoint, it is equivalent to the plastic strain criterion:

$$\bar{\varepsilon}_p = \bar{\varepsilon}_f \quad (6)$$

From microscopic viewpoint, it is equivalent to the damage criterion proposed by Lemaitre(1985):

$$D = D_c \quad (7)$$

or to the critical void growth criterion by Beremin(1981):

$$R/R_0 = (R/R_0)_c \quad (8)$$

It is similar to the criterion given by Oyane(1972):

$$\int_{\bar{\varepsilon}_0}^{\bar{\varepsilon}_p} \left(1 + \frac{\sigma_m}{A\bar{\sigma}}\right) \cdot d\bar{\varepsilon}_p = C \quad (9)$$

where A and C are constants.

Under condition of proportional loading, let $\bar{\varepsilon}_0 = 0$, then it is similar to the criterion given by Zheng and Radon(1985).

$$V_G = \exp\left(1.5\frac{\sigma_m}{\bar{\sigma}}\right) \cdot \bar{\varepsilon}_p = V_{Gc} \qquad \ln\left(\frac{R}{R_0}\right) = \ln\left(\frac{R_c}{R_0}\right) \quad (10)$$

$$V_D = f\left(\frac{\sigma_m}{\bar{\sigma}}\right) \cdot \bar{\varepsilon}_p = V_{Dc} \qquad \ln\left(\frac{D}{D_0}\right) = \ln\left(\frac{D_c}{D_0}\right) \quad (11)$$

Damage Criterion for Macrocrack Initiation

For a ductile solid containing a macrocrack, the similar damage processes take place in the intensely deformed nonlinear region called "Damage Zone" at the crack tip, and the internal damage plays a significant role in ductile fracture and crack initiation. In fact, many theoretical and experimental investigations have shown that the coalescence of a void with the blunted crack tip is similar to the process of void coalescence which takes place in the center of bulk specimens. So the present damage criterion may be applicable to cracked body. Considering the fact that the damage state varies near the crack tip greatly and the micro-mechanisms which initiate failure cannot operate at a "point" but require a certain minimum volume of material, a modified failure initiation condition is to assume that the blunted crack initiation occurs when the damage parameter V_D reaches its critical value V_{Dc} within the damage zone at the crack tip, that is:

$$V_D(\Omega) = V_{Dc} \quad (12)$$

where Ω is the damage zone size parameter and V_D depends on the stress and strain distribution at the crack tip.

Damage Parameter V_{Dc}

In order to determine the critical damage parameter V_{Dc} experimentally both circumferentially notched and unnotched tensile specimens are used to study the effects of stress state on the ductile failure of the specimens and on the parameter V_{Dc}, and

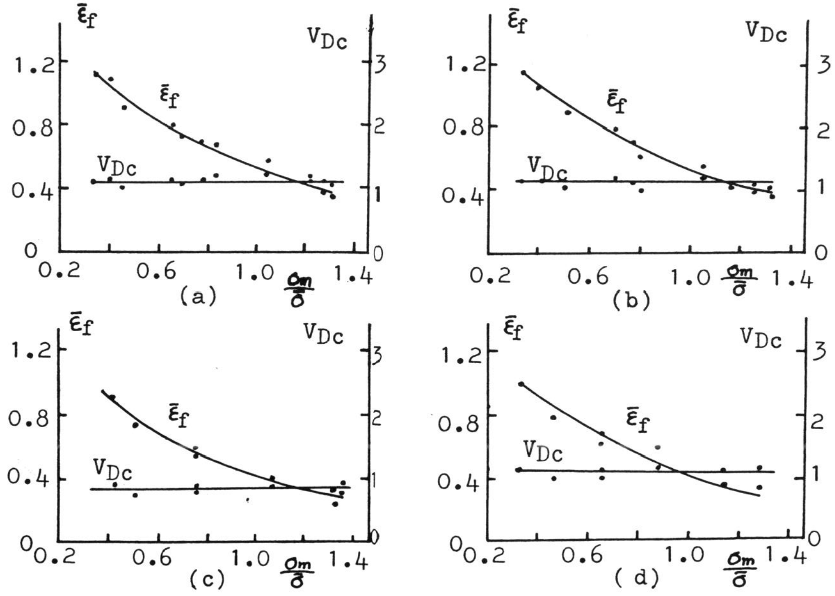

Fig. 1. Effects of the stress triaxiality $\sigma_m/\bar{\sigma}$ the effective plastic strain $\bar{\varepsilon}_f$ and V_{Dc} for(a) 20, (b) A3, (c) 30CrMnSiA and (d) 16Mn.

a relationship between the stress triaxiality ratio $\sigma_m/\bar{\sigma}$ and the effective plastic strain to failure $\bar{\varepsilon}_f$ is obtained for chinese steels 20, A3, 30CrMnSiA and 16Mn. The results show(see Fig.1a-d):

$$V_{Dc}=f(\sigma_m/\bar{\sigma})\cdot \bar{\varepsilon}_f \approx \varepsilon_f \tag{13}$$

where ε_f is the uniaxial tension strain at failure and easy to determined by uniaxial tension test of smooth bar.

The examination of the experimental data given by Zheng and Radon(1985) and Lautridou and Pineau(1981) also suggests that V_{Dc} may be approximately regarded as a material constant independent of stress state.

From the viewpoint of continuum damage mechanics, it is also reasonable to consider V_{Dc} as a material property:

$$V_{Dc}= C\cdot \ln(D_c/D_0) \tag{14}$$

Some related experiments have been done for low-carbon steel 20 and A3, by using quantitative microscopy methods(Tai, 1988). The results show that (D_c/D_0) is weakly dependent on the stress triaxiality and is almost constant in the corresponding range of stress triaxiality.

The Relationships Between V_{Dc} and Other Parameters

From the HRR singularity fields near the crack tip we have

$$\bar{\varepsilon}_p= C_0\cdot\left(\frac{J}{\sigma_y\cdot X}\right)^{n/n+1} \tag{15}$$

where n is the strain hardening exponent, X the distance from the crack tip and σ_y the yield stress. Then from the damage criterion one has

$$V_{Dc}=C_1\cdot\left(\frac{J_{Ic}}{\sigma_y\cdot X_0}\right)^{n/n+1} \tag{16}$$

or

$$J_{Ic}=C_2\cdot \sigma_y\cdot X_0\cdot V_{Dc}^{n+1/n} \tag{17}$$

in which X_0 is the damage zone size that may be considered as the spacing of void nucleating particles.

For ductile material, we approximately let $n=\infty$, then

$$J_{Ic}=C_2\cdot \sigma_y\cdot X_0\cdot V_{Dc} \tag{18}$$

This is similar to the equation obtained from the Amar and Pineau investigation(Tai and Yang, 1987)

$$J_{Ic}=C_3\cdot \sigma_y\, \Delta a_c\cdot V_{Dc} \tag{19}$$

where Δa_c is related to the inclusion spacing.

APPLICATIONS

Ductile Failure Under Proportional Loading

Under proportional loading, the stress triaxiality ratio can be considered as constant with respect to time and the damage criterion is simply given by eqn.(5), from which a relationship of $\sigma_m/\bar{\sigma}$ and $\bar{\varepsilon}_f$ can be derived

$$\bar{\varepsilon}_f= \varepsilon_f\cdot f^{-1}(\sigma_m/\bar{\sigma}) \tag{20}$$

This gives a good calculated results in excellent agreement with the experimental data for chinese steels 20, A3, 30CrMnSiA and 16Mn, as shown in Fig.1(a-d).

Ductile Failure Under Nonproportional Loading

In general, the stress triaxiality ratio $\sigma_m/\bar{\sigma}$ varies with plastic strain under nonproportional loading condition. In this case only numerical calculations can be made integrating eqn.(2) as follows:

$$V_D=\int_{\bar{\varepsilon}_0}^{\bar{\varepsilon}_p} f\left[\left(\frac{\sigma_m}{\bar{\sigma}}\right)(x)\right]\cdot dx=V_{Dc} \tag{21}$$

For multi-step loading the damage criterion takes the following form:

$$V_D= \sum_{i=1}^{N}\int_{\bar{\varepsilon}_{p_{i-1}}}^{\bar{\varepsilon}_{p_i}} f(\sigma_m/\bar{\sigma})_i\cdot d\bar{\varepsilon}_p =V_{Dc} \tag{22}$$

If the stress triaxiality remains constant in each loading stage, then the damage criterion is given by

$$V_D=\sum_{i=1}^{N} f(\sigma_m/\bar{\sigma})_i\ (\bar{\varepsilon}_{pi}-\bar{\varepsilon}_{pi-1}) \qquad (23)$$

For the two-step test finished by Marini et al.(1985) the damage criterion gives a good prediction. Fig. 2(a,b) show the comparison of the results calculated from the damage criterion by setting $(\sigma_m/\bar{\sigma})_i$=const. (i=1,2) and from the (R_c/R_0) criterion with the experimental data(Marini et al, 1985). Fig. 3(a,b) show the comparison of the results calculated from the damage criterion by using $(\sigma_m/\bar{\sigma})=c_i\cdot \bar{\varepsilon}_p^{\ ti}$ (i=1,2) and from the (R_c/R_0) criterion with the experimental results. From the figures one can see that the theoretical prediction are in agreement with the experimental results and the calculated results are better by taking into consideration the effect of stress state on the damage processes.

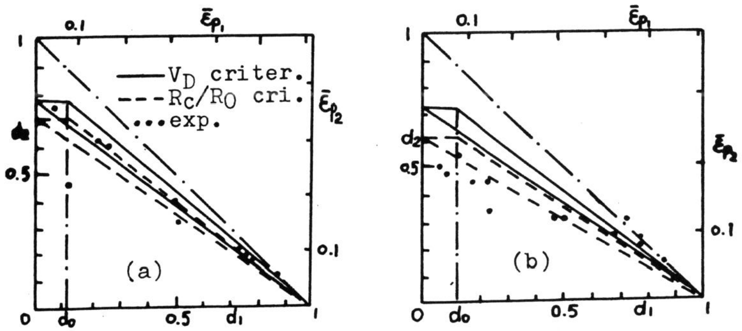

Fig. 2. Ductile failure under two-step loading (a) NR1-NR0.4 path (b) NR1-NR0.2 path.

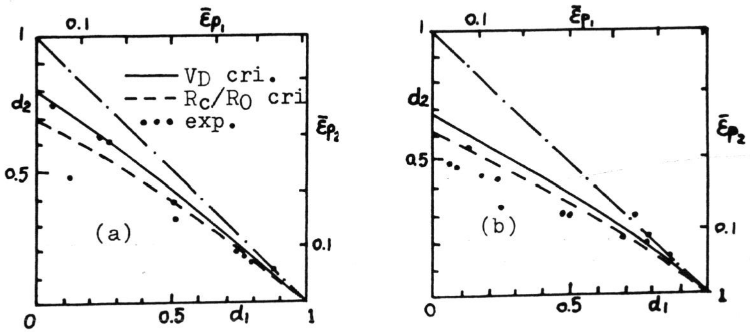

Fig. 3. Ductile failure under two-step loading (a) NR1-NR0.4 path (b) NR1-NR0.2 path.

Ductile Failure Initiation at Macrocrack Tip

For a ductile solid containing a crack the application of the damage criterion(12) requires the knowledge of stress and strain distributions in the damage zone near the crack tip. Firstly, it is assumed that the stress triaxiality remains constant in the damage zone during loading, then based on the experimentally

determined effective plastic strain distribution in the damage zone by means of recrystallization technique (Tai, 1988) and the damage criterion (12), the COD at fracture initiation can be expressed as follows:

$$\delta_{tc}=C_t\cdot X_0\cdot \exp(\sqrt{3}/2\cdot\bar{\varepsilon}_f)(-\bar{\varepsilon}_f/\bar{\varepsilon}_r)^{1/m} \qquad (24)$$

where $\bar{\varepsilon}_r$ is the critical effective plastic strain below which recrystallization cannot occur, C_t is a proportional factor and m=0.6-0.7 is a material constant. Secondly, we assume that the stress triaxiality in the damage zone at a blunted crack tip is given by

$$\sigma_m/\bar{\sigma}=1/\sqrt{3}\,[1+2\ln(1+2X/\delta t)] \qquad (25)$$

Using eqn.(25) and the damage criterion (12) one obtain

$$\delta_{tc}=\frac{2X_0}{\exp\left[\frac{1}{2}\left(\frac{(\bar{\varepsilon}_f/\varepsilon_f)^{-1}-2/3(1+\nu)}{1-2\nu}\right)^{\frac{1}{2}}-\frac{1}{2}\right]-1} \qquad (26)$$

A comparison of the values δ_{tc} calculated from eqn.(24) and from eqn.(26) with the experimental data for steel 20 and steel 4340 (Shoji, et al, 1978) is shown in Fig.4(a,b). It is easy to see that the critical crack opening displacement calculated from eqn.(26)agree the experimental results very well.

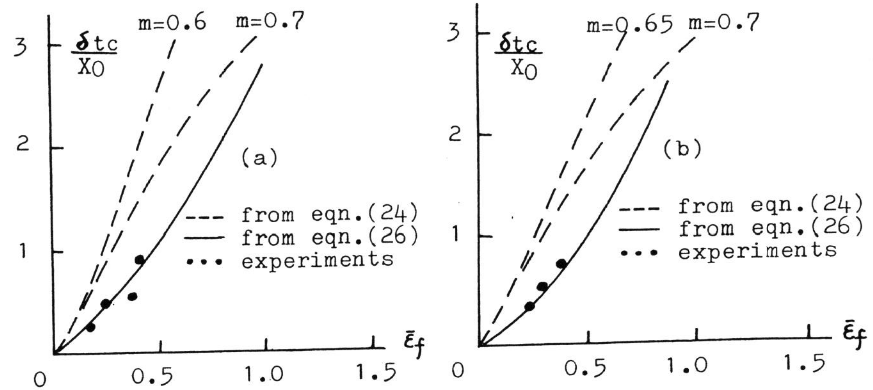

Fig. 4. Calculated and experimentally obtained COD at fracture as function of fracture strain for (a) steel 20 and (b) 4340.

Fracture Toughness of Ductile Metals

Fracture toughness K_{Ic} and fracture ductility $\bar{\varepsilon}_f$ are two important material properties. It is significant to study the relationships of the two parameters with other mechanical properties and some microstructural properties of the material. As far as the fracture toughness is concerned, many investigations have been conducted to correlate it with the uniaxial tensile properties. These are all based on the same assumption, that is, failure would occur when the critical value for stress or strain has to be reached or exceeded over a certain distance or volume.

Table 1. Measured and calculated K_{Ic} values

Material	σ_y (MPa)	f	n	X_0 (μm)	δ_{tc}/X_0	K_{Ic} (MPa$\sqrt{m}$) exp	calc1	calc2
0.45CNiCrMo								
0.008S	1450	0.65	.06	6.1	1.5	71.44	64.87	72.09
0.016S	1453	0.51	.06	5.4	1.3	60.83	53.72	57.71
0.025S	1472	0.59	.06	4.4	1.3	55.79	52.70	55.64
0.048S	1496	0.54	.06	3.7	1.1	46.82	46.44	46.88
X2NiCoMo1885								
T=-20 C	1854	0.965	.100	5.2		90.52	80.11	
T=-50 C	1952	0.994	.066	4.6		84.94	77.11	
T=-100 C	2040	0.915	.057	4.2		71.18	72.18	
T=-150 C	2148	0.803	.082	3.7		64.80	62.51	
T=-196 C	2276	0.795	.096	3.3		47.12	53.63	
30SiMnCrMo	980	1.00	.12	25		165.4	157.1	
34CrNiMo	1225	0.40	.10	28	.64	87.0	106.23	88.23
	1450	0.86	.085	12	.80	94.6	113.88	97.31
CrNiMo	1450	0.66	.09	6.7		72.9	72.44	
	1460	0.52	.09	6.0		62.3	60.38	
	1479	0.59	.09	4.8		57.5	58.21	
	1499	0.54	.085	4.1		47.9	51.12	
18Ni	1332	0.75	.09	38	.60	126.7	180.31	143.34
Monix-3R								
T=100 C	1589	0.757	.162	1.9	4.5	77.8	55.16	78.62
T=200 C	1570	0.855	.147	2.4	6.6	102	63.74	98.69
T=300 C	1413	0.839	.108	1.6	8.0	89.6	43.78	75.47
T=400 C	1329	0.808	.104	1.7	8.0	98.6	41.77	71.61

Here, from the damage criterion (12) and eqn.(25) it is easy to express the fracture ductility at the blunted crack tip in terms of the critical COD and the damage zone length X_0;

$$\bar{\varepsilon}_f = \varepsilon_f\left\{\frac{2}{3}(1+\nu)+(1-2\nu)\cdot(1+2\ln(1+2X_0/\delta_{tc}))^2\right]^{-1} \qquad (27)$$

Next, we assume that the distribution of strain in mode I plastic zone is similar to the shear strain distribution of mode III then application of the damage criterion yields(Schwalbe, 1977):

$$K_{Ic}=\frac{\sigma_y}{1-2\nu}\left[\pi\cdot(1+n)\cdot X_0\cdot\left(\frac{E\cdot\varepsilon_f'}{\sigma_y}\right)^{1+n}\right]^{\frac{1}{2}} \qquad (28)$$

where

$$\varepsilon_f'=\frac{\sqrt{3}}{2}\bar{\varepsilon}_f \qquad (29)$$

In Table 1, the measured and calculated K_{Ic} values are listed with the material properties necessary for calculation. The calc. 1 represents the calculated K_{Ic} values by setting $X_0/\delta_{tc}=1$ in eqn.(27), which exhibit a better coincidence with the experimental data for three steels: 0.45CNiCrMo, X2NiCoMo1885 and CrNiMo than the calculated results given by Schwalbe(1977) and Chen(1978). But for other materials with larger or smaller ratios X_0/δ_{tc}, it is not reasonable to let $X_0/\delta_{tc}=1$ and the calculated values are not so good. So we let X_0/δ_{tc} takes its experimental value or its approximately estimated value, and the calculated values are listed in Table 1 marked calc.2 which are in agreement with the measured values.

CONCLUSIONS

1. The damage criterion has obvious micro- and macroscopic physical meaning and is easy to use. It is applicable for both proportional and nonproportional loading conditions, and can also be used to predict the ductile failure initiation of a cracked solid.

2. The critical damage parameter V_{Dc} is experimentally verified as a material constant independent of the stress state and easy to be determined by a macroscopic method. It may be used as a characteristic parameter describing macrocrack initiation and can be related to J_{Ic}.

3. The present analysis shows that both stress state and plastic deformation have great effects on ductile failure of the materials.

4. The calculated fracture toughness K_{Ic} are in good agreement with experimental data by using the damage criterion and by taking into account the effect of the stress triaxiality.

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