Some Aspects of Fatigue Crack Growth Under Mixed Mode Loading

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ABSTRACT

Starting from a fatigue crack mixed mode cyclic loading above a certain threshold value causes crack propagation in a new direction. After kinking the crack often proceeds in a curved path. Experiments show that despite a constant stress intensity the crack growth rate under mixed mode condition is higher than under mode I. The elevation of the crack propagation rate increases with increasing mode II share of the load. In case of mode I fatigue crack growth mixed mode overloads induce a lesser retardation than a comparative mode I overload does. According to prevailing findings a mixed mode overload does not have any crack deviation as a consequence.

KEYWORDS

Fatigue crack growth, mixed mode loading, crack deflection, cyclic stress intensity factor, CTS-specimen, overload effects.

INTRODUCTION

Despite careful design and manufacturing, damage occurs in machines, plant components and vehicles. The reasons for this kind of damage are in general cracks, small defects which grow as a result of operating stresses and consequently lead to failure of these components. Cracks of this kind often are subjected to complex static or dynamic stresses (Richard, 1985). Especially fracture mechanics have developed concepts for describing the propagation of cracks which are submitted to pure normal loading (mode I), but up to now the influence of overlapping normal and shear stresses (mixed mode) on propagation of fatigue cracks is still largely unknown. However several theoretical assessments (Sih and Barthelemy, 1980; Fischer, 1985) exist, but the present experimental results concerning this problem are not sufficient to judge the applicability of these hypotheses, respectively to be able to predict the crack path and the crack growth rate.
The submitted report deals with the investigation of the propagation behavior of fatigue cracks subjected to mixed mode cyclic loading. Further-

more the influence of mixed mode and mode II overloads on subsequent mode I fatigue crack growth is investigated.

CRACK GROWTH UNDER MIXED MODE LOADING

In case of mixed mode loading (overlapping mode I and mode II) the stress distribution at the crack tip is given with the relation

$$\sigma_{k,l} = \frac{K_I}{\sqrt{2\pi r}} f^{I}_{k,l}(\phi) + \frac{K_{II}}{\sqrt{2\pi r}} f^{II}_{k,l}(\phi); \qquad k,l = x,y \qquad (1)$$

whereas r and ϕ are polar-coordinates at the crack tip and $f^{I}_{k,l}(\phi)$ and $f^{II}_{k,l}(\phi)$ are dimensionless functions. The stress intensity factors K_I (mode I) and K_{II} (mode II) are constant under static stress and time dependent under fatigue stress according to the temporal lapse of the load.

With static or quasi-static strain unstable crack growth starts when the comparative stress intensity factor K_v (calculated from K_I and K_{II}) reaches the fracture toughness K_{Ic} (obtained according to ASTM E 399 for mode I), (Richard, 1987 a,b).

$$K_v(K_I, K_{II}) = K_{Ic} \qquad (2)$$

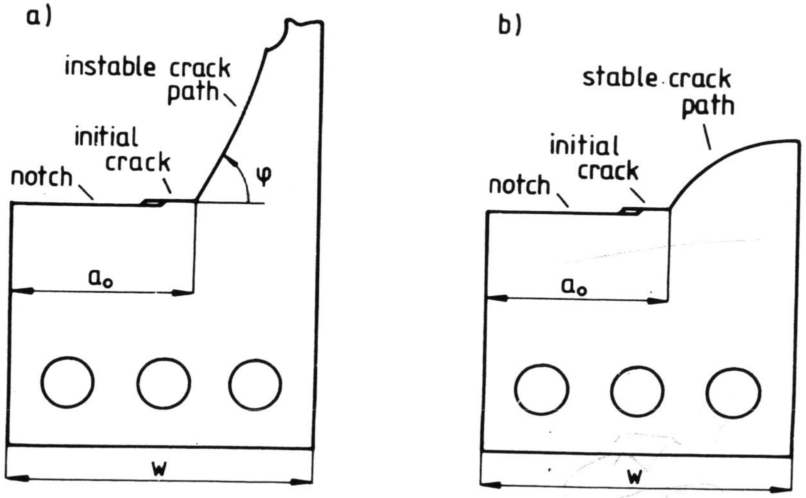


Fig.1. Comparison of the crack paths in CTS-specimens (Richard, 1985) generated under quasi-static (a) and cyclic (b) mixed mode loading $K_{II}/K_I = 1.49$ at the initial crack, material: plexiglas

In case of mixed mode fatigue loading the crack does not necessarily follow the direction of the initial crack. Above a certain threshold value of the cyclic stress intensities ΔK_I and ΔK_{II} the crack will be deflected (Miller and Brown, 1984; Hua et al., 1982). The crack deviation angle ϕ_o averages in general between 0 degrees (mode I) and approximately 72 degrees (at pure mode II). Concerning isotropic materials it depends only on the ratio K_{II}/K_I of the stress intensity factors at the initial crack.

The stable crack propagation takes place at a high speed where the crack almost follows a straight path after kinking. Under fatigue loading crack deviation generally also occurs. But now the developing crack grows stably (slow and controlled). The crack deflection angle is influenced, like under quasi-static loading by the average of the present mode I and mode II components. At the beginning of the crack growth the deviation angle is approximately as large as in the unstable case. The crack path however is considerably more curved (fig. 1). The momentary crack length is dependent on the number of the load cycles, the crack growth rate can for example be determined according to the Paris Law (Henn et al., 1987a)

$$\frac{da}{dN} = C' \, (\Delta K_v)^{m'} \qquad (3)$$

C' and m' are material constants which in some criteria (Sih and Barthelemy, 1980; Fisher, 1985; Henn et al., 1987a) are compared with the constants C and m for mode I fatigue crack growth, ΔK_v is the cyclic comparative stress intensity factor which yields from (Henn et al., 1987a)

$$\Delta K_v = \Delta K_v(\Delta K_I, \Delta K_{II}) \qquad (4)$$

with

$$\Delta K_I = K_{Imax} - K_{Imin} \qquad (5)$$

and

$$\Delta K_{II} = K_{IImax} - K_{IImin} \qquad (6)$$

ΔK_v is varying continuously with increasing crack length while the $K_I(t)$- and the $K_{II}(t)$-curves change suddenly when the crack begins to kink, (fig. 2).

The K_I-share grows while the K_{II} factor decreases to a small value. On the contrary K_{vmax} remains constant. This phenomenon can be explained with the help of the findings about the stress field at the short kinked crack (Henn et al., 1986; Tenhaeff, 1987)

EXPERIMENTAL INVESTIGATIONS AND RESULTS

In order to obtain further knowledge concerning fatigue cracks which are induced under combined mode loading tests were carried out on 7075 (AlZnMgCu 1.5). CTS-specimens in combination with a special loading device, fig. 3, (Richard, 1985) were used. The crack length was measured with the DC potential drop method which was modified for kinked cracks (Henn et al., 1986; Tenhaeff, 1987). The specimens were pre-cracked under fatigue loading until they reached a crack length of a_o = 54 mm. The stress intensity factor ΔK_I

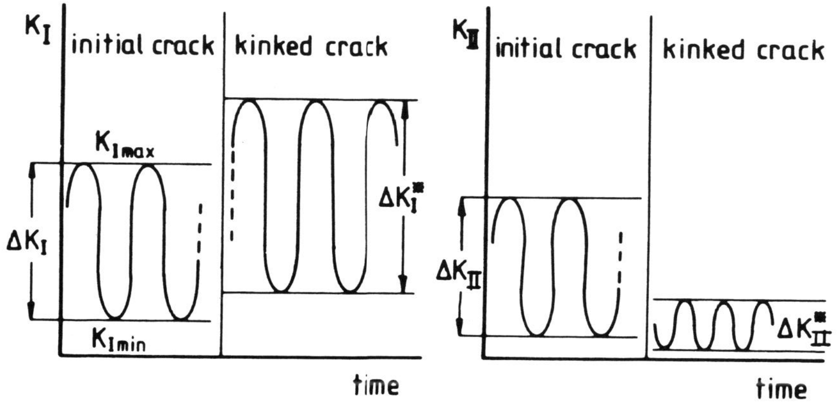


Fig. 2. Change of $K_I(t)$ and $K_{II}(t)$ at the time of kinking

at the initial crack was held constant at 7 MPa√m, the R-ratio was 0.5 and the frequency amounted 40 Hz.
After generation of the mode I pre-crack the specimens were subjected to pure mode I, mixed mode or mode II fatigue loading. For this purpose the loading device was turned (load angle $\alpha = 0^o, 15^o, \ldots, 90^o$), (Richard, 1985). In order to get easy comparable results the upper and the lower limit of the stresses (F_{max} and F_{min}) of the cyclic loading were continuously modified so that ΔK_v and R were constant at the crack tip during the entire procedure:

$$\Delta K_v = 7 \text{ MPa}\sqrt{m} \quad R = 0.5, \quad \omega = 40 \text{ Hz}$$

As a result of the cyclic mixed mode or mode II loading a kinked crack is formed which grows with a constant speed along a curved path.
After 20 to 25 mm crack growth the test was interrupted and the specimen was ruptured under static load. The tests supplied the following results:

- Under pure mode I condition the crack growth rate equals $1.5 \cdot 10^{-4}$ mm/cycle.

- In the case of mixed mode and mode II loading a higher crack speed was measured as under mode I (fig. 4, table 1). The propagation rate increases with rising mode II share and equals about $2.67 \cdot 10^{-4}$ mm/cycle under pure mode II condition.

- With increasing K_{II}/K_I-ratio the crack deviation angles ϕ_o become higher (table 1), but they are however a little bit smaller compared to static loading.

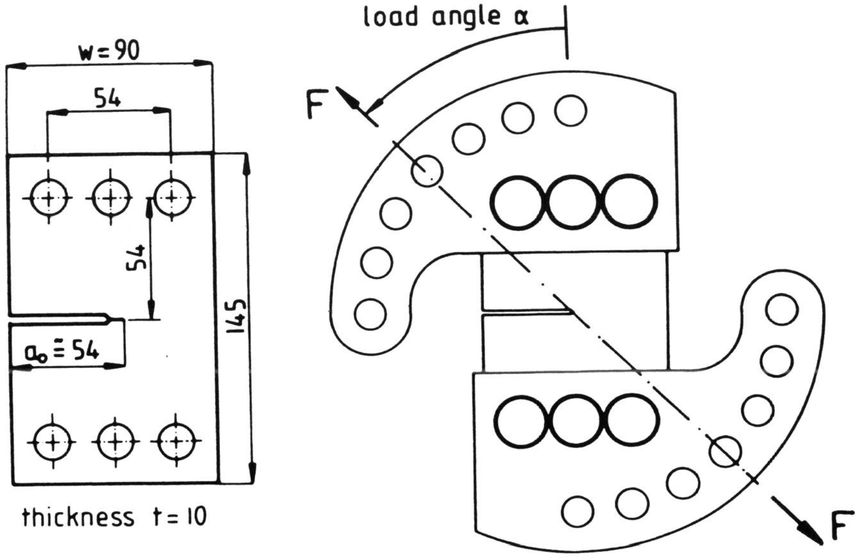
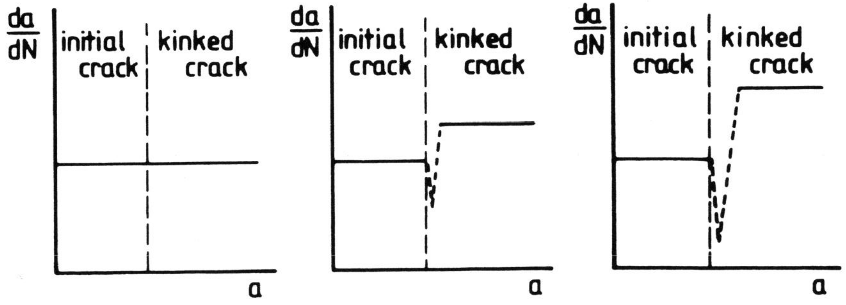


Fig. 3. CTS-specimen and loading device



a) Mode I
b) Mixed Mode
c) Mode II

Fig. 4. Change of the crack propagation rate caused by deviation of the crack in dependence of different loading conditions at the initial crack

The elevation of the crack growth rate at higher K_{II}/K_I-ratios was also measured by DAI and ZHENG (1987).
The short retardation of the crack growth directly after turning the device (see fig. 4) can be attributed to friction effects at the crack surface which however vanish suddenly as soon as the kinked crack begins to grow.

Table 1. Crack growth rates and deflection angles in 7075 aluminum alloy for different K_{II}/K_I-ratios

load angle α (see fig. 3) in (°)	K_{II}/K_I at the initial crack -	crack deviation angle ϕ_o in (°)	crack growth rate da/dN at the kinked crack in (MPa$\sqrt{m}$)
0	0 (mode I)	0	$1.50 \cdot 10^{-4}$
15	0.10	13.4 ± 2.8	$1.52 \cdot 10^{-4}$
30	0.22	23.9 ± 3.5	$1.76 \cdot 10^{-4}$
45	0.39	37.0 ± 3.5	$2.15 \cdot 10^{-4}$
60	0.68	45.6 ± 2.0	$2.28 \cdot 10^{-4}$
75	1.49	55.8 ± 3.2	$2.53 \cdot 10^{-4}$
90	∞ (mode II)	62.8 ± 3.2	$2.67 \cdot 10^{-4}$

- Fatigue loading
  - ·cyclic stress intensity factor $\Delta K_I = 7$ MPa$\sqrt{m}$ $= \Delta K_v$
  - ·ratio K_{min}/K_{max} : $R = 0.5$
  - ·load frequendy : $\omega = 40$ Hz
- Crack propagation rate before kinking: da/dN = $1.5 \cdot 10^{-4}$ mm/cycle
- Crack length at the time of kinking : $a_o = 54$ mm
- Specimen width: w = 90 mm

EFFECTS OF MIXED MODE AND MODE II OVERLOADS ON MODE I FATIGUE CRACK GROWTH

The effects of mixed mode or mode II overloads on subsequent mode I fatigue crack growth in 7075 are investigated by means of CTS-specimens and an appropriate loading device, fig. 3. Experiments are performed at a constant $\Delta K_I = 8$ MPa$\sqrt{m}$, at R-ratios of R = 0.05 and R = 0.25 and a cyclic frequency of $\omega = 60$ Hz.
After a fatigue crack growth of about 5 to 10 mm under mode I condition the experimental procedure is interrupted, then the corresponding mixed mode or mode II overload is applied by turning the device and subsequently oscillation is continued under mode I conditions. The experiments are carried out with overloads of $\Delta K_{vü} = 150$ % ΔK_I and 200 % ΔK_I at seven different K_{II}/K_I-ratios (load application angle $\alpha = 0^o, 15^o, \dots, 90^o$).

To determine the overload value for mixed mode or mode II, the concept of the comparative stress intensity factor developed by RICHARD (1985) is used.

$$K_{vü} = K_{Imax} + \Delta K_{vü} = 0.5\, K_{Iü} + 0.5\sqrt{K_{Iü}^2 + 6\, K_{IIü}^2} \quad (7)$$

With the aid of

$$K_I = F\,/\,(w \cdot t)\ \sqrt{\pi a}\ Y_I$$
$$K_{II} = F\,/\,(w \cdot t)\ \sqrt{\pi a}\ Y_{II} \quad (8)$$

one then obtains the equation

$$K_{vü} = \frac{1}{2}\sqrt{\pi a}\,\frac{F_ü}{w \cdot t}\,(Y_I + \sqrt{Y_I^2 + 6\, Y_{II}^2}) \quad (9)$$

where Y_I and Y_{II} designate the non dimensional geometric functions for the CTS-specimen. From eqn. (9) the required load $F_ü$ is obtained which is to be applied to the CTS-specimen. During the entire experimental procedure the crack length is constantly measured with the aid of the DC potential method and the cracking process at the crack tip is observed by an optical device. The stress cycles endured are counted by the traction machine. The measured values are recorded by an IBM personal computer, which memorizes the crack lengths and the corresponding stress cycles and displays them in a diagram on the screen. The fatigue tests are made at room temperature using a servo-hydraulic traction machine, type PSA by SCHENK AG.
The tests showed the following results:

- The delay of the crack growth after application of a mixed mode overload is the smaller, the higher the K_{II}-share of the overload (fig. 5).

  In contrast to a mode I overload a pure mode II overload of equal size delays crack growth only insignificantly (Henn et al., 1987b).
- The retardation effects measured at $\Delta K_{vü} = 150$ % ΔK_I are more pronounced at $\Delta K_{vü} = 200$ % ΔK_I (Henn et al., 1987b). An increasing overload ratio $R_ü = K_{vü}/K_{Imax}$ effects a greater crack growth delay.
- The effects of mixed mode overloads on mode I crack growth are considerably less at a higher R-ratio (R = 0.25). The findings concerning the effects of a higher K_{II}-share of the overload and the increasing overload ratio are the same as for the experiments made with R = 0.05.
- With all kinds of overloads no crack deviation was found during the subsequent mode I fatigue crack growth.

CONCLUSIONS

Investigations on 7075 (AlZnMgCu 1.5) show that under mixed mode and mode II fatigue loading a higher crack growth rate appears than under a comparative mode I fatigue loading, whereas the crack propagation rate increases with growing mode II-share.
Furthermore after a mixed mode or mode II overload the retardation effects are considerably smaller than after a mode I overload. The retardation effect decreases with increasing $K_{IIü}$-share and becomes nearly zero after pure

mode II overloads.

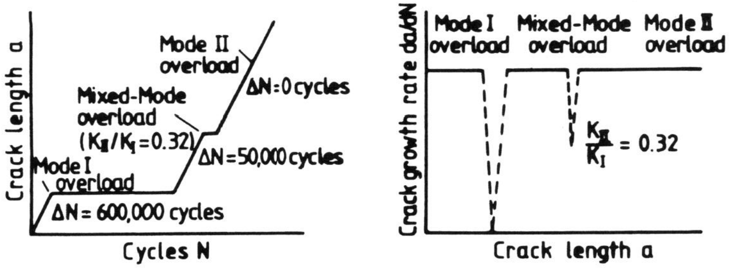


Fig. 5. Influence of different kinds of overloads on mode I fatigue crack growth. $\Delta K_I = 8$ MPa$\sqrt{m}$, R = 0.05, $K_{vü} = 24.4$ MPa$\sqrt{m}$

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