

Plasticity and Thickness Effect

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INTRODUCTION

A careful study of any book dealing with linear fracture mechanics (Broek) shows that much experimental data cannot be explained satisfactory in such a restricted theoretical framework. Among these data are two important phenomena: the stable growth of mode I cracks under monotonic loading and the thickness effect. Considering fracture to be closely related to the constitutive equations chosen for the material and given that elasticity is unable to explain either of the two above phenomena, we undertook this study by means of plasticity (Prager's linear kinematic hardening rule). The main advantage of such a law is its simplicity. It depends on only two parameters: a yield stress σ_0 and a hardening ratio $\alpha = E_t/E$. This rule has the advantage of allowing a smooth transition from purely plastic behavior ($\alpha = 0$) to purely elastic behavior ($\alpha = 1$). This is important since a fracture concept in elastoplasticity is not valid unless it can ultimately account for what occurs in material with very low ductility, i.e. brittle fracture.

PREVIOUS RESULTS

As the analytical study of a crack tip neighborhood is very complex in the elastoplastic case, we investigated the problem through numerical simulation. Such a method is not always simple due to the careful precautions that have to be taken in interpreting the results obtained.

First, we studied the case of a steadily growing crack, focusing on the dissipated energies that can be computed (Anquez, 1985). It appears that in a forced crack length increment Δa , under constant external loading, the overall dissipated energy ΔD_R is the sum of a decohesion energy ΔD_{el} located at the crack tip and a dissipated plastic bulk energy ΔD_p . Approximations of the rates of dissipated energies are obtained by dividing the energy increments by Δa :

$$G_R^{\Delta a} = G_{el}^{\Delta a} + G_p^{\Delta a} \quad (1)$$

In this equation, superscript Δa points out the fact that each term is mesh size dependent. A study of numerical convergence shows that as Δa approaches zero, $G_R^{\Delta a}$ approaches a value close to \mathcal{G} which could be obtained in the pure elasticity case ($K^2 = E\mathcal{G}$ in the plane stress case) and $G_{el}^{\Delta a}$ approaches zero. This brings us back to Rice's paradox: in elastoplasticity, all the energy is dissipated in plasticity during crack growth.

Despite this, we performed numerical simulations of stable crack growth under increasing loading: having found that, for a given mesh (characteristic length Δa), the mechanical state (i.e., stress and strain tensor components) in the finite elements near the crack tip depended on one parameter only, at any step, (i.e. all mechanical quantities are interrelated), we undertook simulations by releasing the tip node whenever a chosen characteristic parameter reached a critical value defined a priori. Each crack length increment Δa required first a load increment characterized for example by ΔK . during which the bulk plastic ΔD_M was dissipated, then a tip node release when the characteristic parameter reached its critical value; during this last process, the energy ΔD_R was dissipated.

The essential result obtained by means of numerical simulations was that the sum $\Delta D_M + \Delta D_R$ divided by Δa remained quasi-constant at any step during a simulation. This result is confirmed by a convergence study. So, going to the limit, we can write:

$$\frac{\partial D}{\partial K} \cdot \frac{dK}{da} + \frac{\partial D}{\partial a} = \frac{dD}{da} = G_c \quad (2)$$

We must point out here that such a result comes from numerical analysis and that it could not be easily established by thermodynamics.

Careful analysis of the numerical results obtained during simulation allows us to write (when small scale yielding hypotheses are satisfied) (Anquez, 1985):

$$\frac{\partial D}{\partial a} = G_c \left(\frac{K}{K_c} \right)^{2\gamma}$$

$$\frac{\partial D}{\partial K} = \frac{\Gamma}{\sigma_0^2 E} K^3$$

where γ and Γ are functions of the hardening ratio α . When $\alpha = 0$ (perfect plasticity), $\gamma = 1$ and $\Gamma = \Gamma_0$ so we have:

$$\frac{dK^2}{da} = 2 \frac{\sigma_0^2}{\Gamma_0} \left(\frac{K_c^2}{K^2} - 1 \right)$$

When α approaches 1 (pure elasticity), γ approaches 0.5 and Γ approaches 0.0, so we obtain the following well-known result: $K = K_c$

These results were obtained in the plane stress case. Similar results are also obtained in the plane strain case.

The question of the thickness effect then arises: if a given critical value G_c is assumed in the plane strain case, what is the value to be taken in the plane stress case? Certainly not the same because, as mentioned above, in the steady state case, G is near G_c which could be calculated in pure elasticity. So taking G_c as a material parameter will not give any thickness effect. We therefore must investigate in another direction.

THICKNESS EFFECT: A SUGGESTION

The starting point of our argument is the following: if a crack can grow in plasticity without any recourse to decohesion, this is because its growth is due only to the accumulation of infinitesimal plastic slips at the tip.

Thus, special attention has to be devoted to the slip lines which can be found by looking for the lines of maximum plastic shear strain (bisectors of the main axes of the incremental plastic strain tensor).

As an example, we present the slip lines obtained for the same α value ($\alpha = 0.1$) and the same K value, in the plane stress case for a stationary crack (Fig. 1) and for a steadily growing crack (Fig. 2), and in the plane strain case for a stationary crack (Fig. 3) and for a steadily growing crack (Fig. 4).

Three zones can be distinguished in these above cases:

- Zone 1, located ahead of the crack, where the slip lines do not reach the crack tip but cross the crack propagation axis at an angle of approximately $\pi/4$. This zone, whose relative size increases as a decreases in plane stress, is negligible in plane strain.
- Zone 2, where the slip lines reach the crack tip forming an angle less than or equal to $\pi/2$ with respect to the crack propagation axis.
- Zone 3 where the slip lines reach the crack tip from behind, forming an angle greater than $\pi/2$ with respect to the crack propagation axis.

As only the slips at the tip (which result from what takes place all along the corresponding slip line) which are at an angle less than or equal to $\pi/2$ can combine to produce a crack tip advance, we concentrated our attention on zone 2.

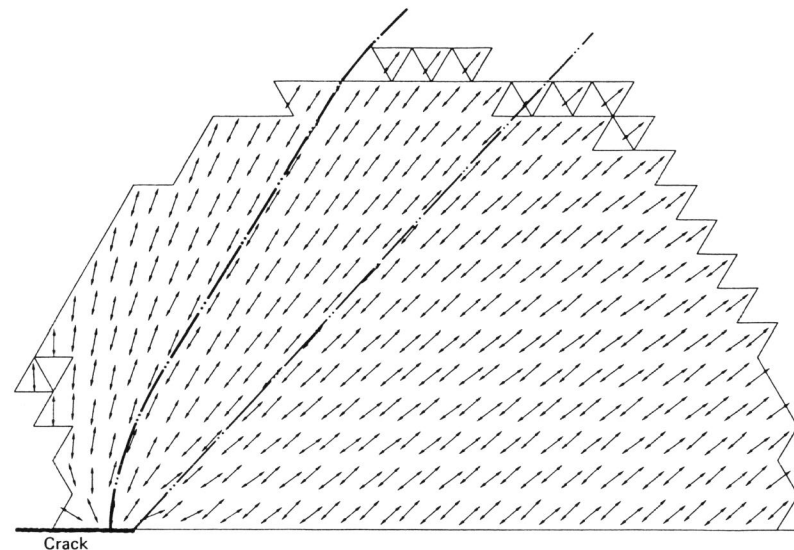


Fig. 1 - Plane stress. Stationary crack. Slip line field at K_D .

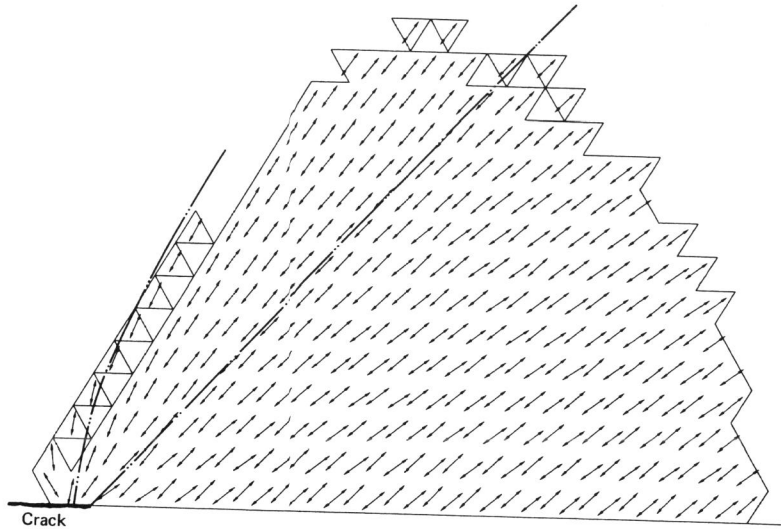


Fig. 2 — Plane stress Steadily growing crack. Slip line field at K_D .

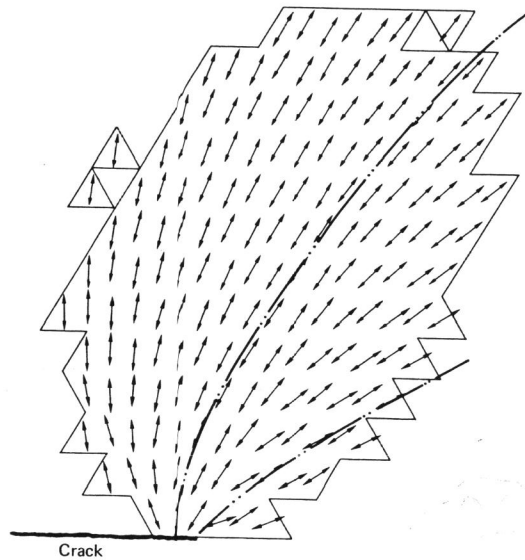


Fig. 3 — Plane strain. Stationary crack. Slip line field at K_D .

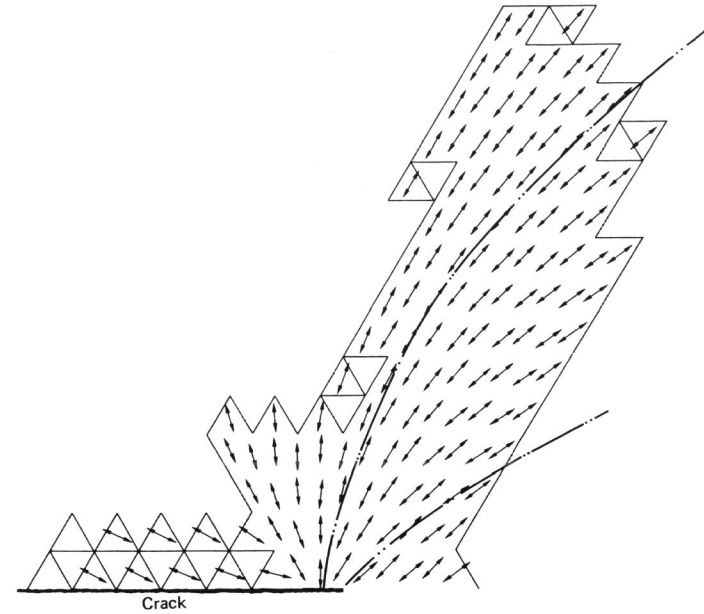


Fig. 4 — Plane strain. Steadily growing crack. Slip line field at K_D .

A good measure of the plastic activity in this zone, which causes the slip concentration at the tip, is the plastic energy increment Δd dissipated either during the load increment ΔK , the crack being fixed, or during a crack propagation increment Δa , the load being fixed.

The last step is to determine as well as possible the spatial limits of zone 2 and to calculate the ratio η of Δd (increment of energy dissipated in this zone) to ΔD (increment of bulk energy dissipated) in the two following cases: loading without crack growth and crack growth under constant load (in plane stress and in plane strain, for different values of the hardening ratio α).

The results obtained are shown in Fig. 5 and Fig. 6.

The essential feature observed is that the ratios η_{CP} and η_{DP} have almost the same value in the stationary and the steady state case. So we can write:

$$\frac{\partial d}{\partial K} = \eta \frac{\partial D}{\partial K} \quad \frac{\partial d}{\partial a} = \eta \frac{\partial D}{\partial a}$$

Multiplying equation (2) by η then gives:

$$\frac{\partial d}{\partial K} \cdot \frac{dK}{da} + \frac{\partial d}{\partial a} = \eta G_c = g_c$$

So there exists a critical g_c defined in zone 2. This energy release rate is more meaningful than G_c as far as the crack propagation is concerned.

We now assume that this value is intrinsic to the material; so we decide that g_c is the same in plane strain and in plane stress.

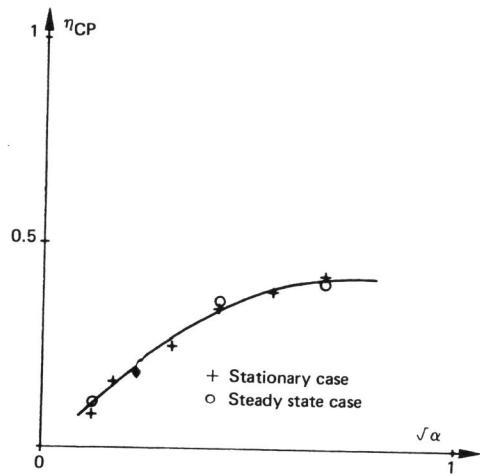


Fig. 5 - Plane stress. Evolution of the ratio η of the useful plastic energy increment to the overall plastic energy versus $\sqrt{\alpha}$ ($\alpha = E_T/E$ being the hardening ratio).

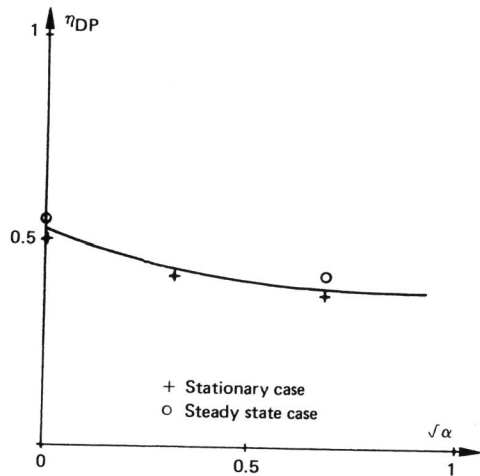


Fig. 6 - Plane strain. Evolution of η versus $\sqrt{\alpha}$.

Given this, suppose that we know K_{Ic} for a given material. Then:

$$\mathcal{G}_c^{DP} = \frac{(1 - \nu^2)}{E} K_{Ic}^2$$

Having pointed out that G and \mathcal{G}_c have approximately the same value, we could write:

$$G_c^{DP} \approx \frac{(1 - \nu^2)}{E} K_{Ic}^2$$

Knowing the material, we can find its hardening ratio α and consequently (see Fig. 6) its appropriate η value in plane strain. So:

$$g_c = \eta_{DP} G_c^{DP} \approx \eta_{DP} \frac{(1 - \nu^2)}{E} K_{Ic}^2$$

Then to determine K_{Ic} , whose ratio to K_{Ic} is a characteristic of the thickness effect, we write:

$$g_c = \eta_{CP} G_c^{CP} \approx \eta_{CP} \frac{K_{Ic}^2}{E}$$

And finally:

$$\frac{K_{Ic}^2}{K_{Ic}^2} \approx (1 - \nu^2) \frac{\eta_{DP}}{\eta_{CP}}$$

The evolution of the ratio K_{Ic}/K_{Ic} versus α (Fig. 7) shows that the theory suggested seems to be able to properly describe the thickness effect.

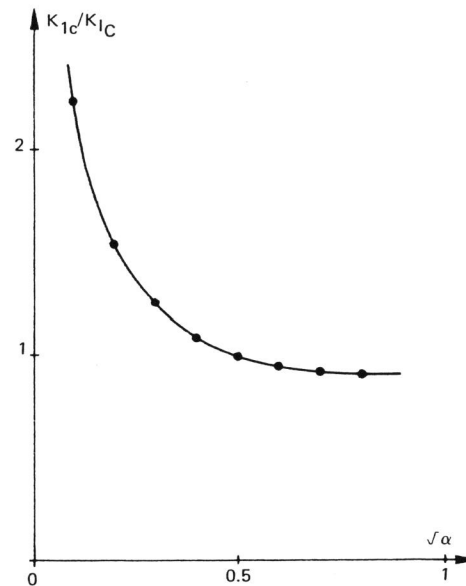


Fig. 7 - Thickness effect versus $\sqrt{\alpha}$.

CONCLUSION

The final result presented in Fig. 7 is encouraging but requires further convergence study.

The first interest of g_c is its physical meaning which allows us to consider it as intrinsic to the material (and consequently usable in a larger framework than small scale yielding).

The second interest of g_c is that it is independent of the mesh size, provided the finite element mesh is sufficiently refined.

Finally, the use of g_c allows us to use the same "simple" theory to consider stable crack growth, the thickness effect and also fatigue crack growth, since we have shown (Anquez, 1988) that in this last case, an energy balance similar to (2) can be found, as well as perhaps even crack initiation (through analysis of microcrack growth as suggested by Griffith in the framework of elasticity).

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