Mixed Modes of the Crack Propagation Under Biaxial Cyclic Load

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ABSTRACT

The investigation of characteristics of the aluminium allows cyclic crack-stability under biaxial load of arbitrary direction was carried out. The dependence of parameters to be determined on the properties of material, the stressed state form and the initial crack orientation was obtained.

KEYWORDS

Crack growth rate; strain energy density; biaxial load; initial crack orientation.

METHODS OF INVESTIGATION

The method for investigation of the characteristics of cyclic crack-stability for the mixed modes of the crack propagation under biaxial load of arbitrary direction has been elaborated. The following parameters are determined: the crack growth rate; the strain energy density factor; the angle, determining the further direction of crack propagation as a function of its initial orientation; the trajectiry of crack growth and the introduced T-parameter characterizing the cyclic crack-stability. The processing of the test results was carried out in accordance with the (Shlyannikov and Dolgorukov, 1987) procedure. The experimental data interpretation is based on the parameter of the strain energy density S (Sih, 1974) and on the introduced concept of an equivalent straight crack which allows to obtain the computational-experimental and theoretical trajectories of the crack growth under a complex stressed state as well as the fatigue fracture diagram (Shlyannikov and Dolgorukov, 1987; Shlyannikov and Ivanyshin, 1983). For the mixed modes of fatigue loading the following criterion has been proposed (Shlyannikov and Shkanov, 1982):

$$\frac{da}{dN} = \left(\frac{da}{dN}\right)^* \left(\frac{S_{max}}{S^*}\right)^n \quad (1)$$

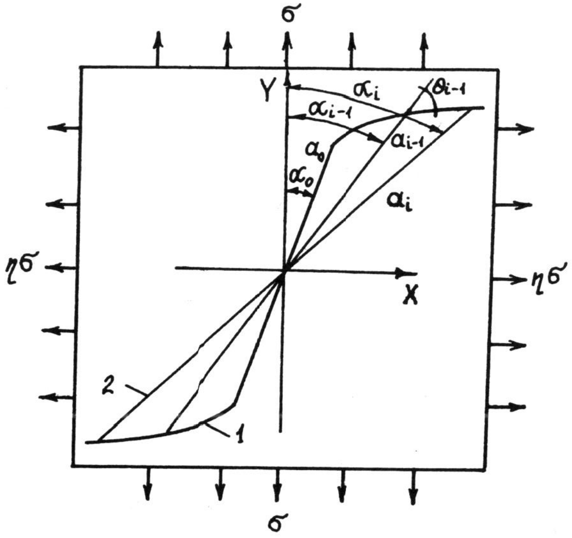

Fig. 1. Geometry of the real (1) and equivalent (2) angled cracks.

in which $(da/dN)=10^{-7}$ m/cycle, S^* and n are the empirically determined constants.

The principal peculiarity of the tests of materials with an initial angled crack is that the crack does not propagate at initial direction and that it has the curvilinear trajectory (Fig. 1). The purpose of investigations of the cyclic crack-stability characteristics is to obtain the dependence of the crack growth rate on the range of the stress intensity factor. In order to obtain such a dependence one must put the values of the stress intensity factor (SIF) into the conformity with each crack length (under an accumulated quantity of load cycles). In the angled crack vertex under the plane stressed state there are realized two forms of fracture described by the corresponding SIF of the normal separation and the pure shear (Panasyik, 1968)

$$K_I = \frac{\sigma\sqrt{\pi a}}{2}\left[(1+\eta)-(1-\eta)\cos 2\alpha\right]$$
$$K_{II} = \frac{\sigma\sqrt{\pi a}}{2}\left[(1-\eta)\sin 2\alpha\right] \qquad (2)$$

in which σ is the nominal normal stress in the axis OY direction; a is the crack length; α is the angle of the crack orientation about the axis OY; $\eta=\sigma_x^\infty/\sigma_y^\infty$ is the relationship of nominal biaxial stresses; Y_I and Y_{II} are the K-taring functions. Thus, in the case of mixed modes of load one must use the equivalent value of SIF which is the function of K_I and K_{II}. As such an equivalent we propose to use the parameter Sih

$$S = b_{11}K_I^2 + 2b_{12}K_I K_{II} + b_{22}K_{II}^2 \qquad (3)$$

$$b_{11} = \frac{1}{16G}\left(\varkappa - \cos\theta^*\right)\left(1+\cos\theta^*\right)$$
$$b_{12} = \frac{2}{16G}\left(\cos\theta^* - \varkappa + 1\right)\sin\theta^* \qquad (4)$$
$$b_{22} = \frac{1}{16G}\left[(\varkappa+1)(1-\cos\theta^*)+(3\cos\theta^*-1)(1+\cos\theta^*)\right]$$

Here θ^* is the angle determining the crack propagation direction as a function of its orientation α taking inti account the stressed state type; G is a shear modulus; $\varkappa=(3-\nu)/(1+\nu)$; ν is a Poisson factor. In order to describe dependences θ^* on α the different criteria have been used in the series of works. However, we have been showed (Shlyannikov and Dolgorukov, 1988) that the existing criteria of the crack growth direction do not spread to all the range of change of the experimental data for materials having the different properties under biaxial tension. Therefore in the procedure of experimental data interpretation we have directly used with a help of the Lagrange polynoms the one-dimensional interpolation of dependences $\theta^*-\alpha$ for each material and stressed state type under the calculation of the current values of b_{11}, b_{12}, b_{22} in (4) as well as the two-dimensional interpolation over the crack length and its inclination angle under the calculation of the current values of K_I and K_{II} (2) and as a result S (3). These interpolation calculations have been carried out for each position of the equivalent straight crack on the real curvilinear trajectory of its propagation. K-taring functions Y_I and Y_{II} from (2) for the rectangular and eight-petal specimens have been determined by the finite element method (FEM) taking into account the singularity.

Peculiarities of SIF calculation by FEM taking into account the singularity

The theory of elastic and elastoplastic versions of the FEM has been elaborated taking into account a singularity for the mixed modes of the crack propagation (Shlyannikov, 1987). Generalization of the method (Hilton and Sih, 1973) on the case of biaxial load of the angled crack with an arbitrary orientation has been given. By the computational values $\sigma_r, \sigma_\theta, \sigma_{r\theta}$ of nondimensional stress functions, the expressions for elastic and elastoplastic strain energy of the domain restricted by the circle with a centre in the crack vertex (Dolgorukov and Shlyannikov, 1988) has been obtained. Potential energy of the whole structure V (Fig. 2) was considered as a sum of contributions from the two care regions V_c and the structure part divided on the finite elements $V^{(\rho)}$

$$V = 2V_c + \sum_{\rho=1}^{N} V^{(\rho)} - \int_{\Gamma} (T_x^n u - T_y^n v)\, d\Gamma \tag{5}$$

$$\sum_{\rho=1}^{N} V^{(\rho)} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} K_{ij} u_i u_j \tag{6}$$

$$V_c = \int_0^{\pi} \int_0^{R} \Phi(\rho, \theta, \alpha, \eta) \rho\, d\rho\, d\theta \tag{7}$$

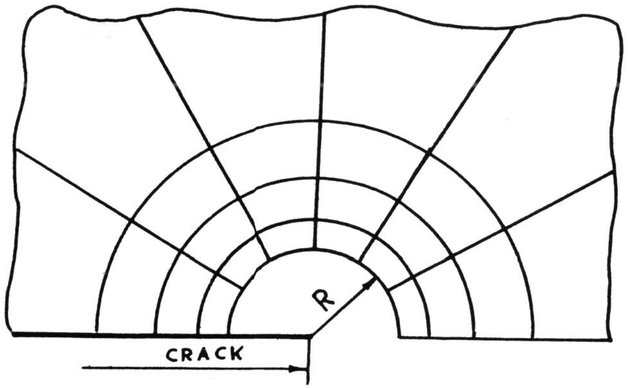

Fig. 2. Care region.

For an elastic versions of the FEM the crack orientation angle α as well as the relationship of biaxial stresses η introduces in (7) at explicit type. By the common energy minimization over the unknown parameters the problem has been reduced to equation systems allowing at the same time as SSS parameters of structure to calculate elastic and elastoplastic SIF. The eight-petal specimens K-taring has been obtained, shown in Fig. 3, which could be used further at the experimental results interpretation.

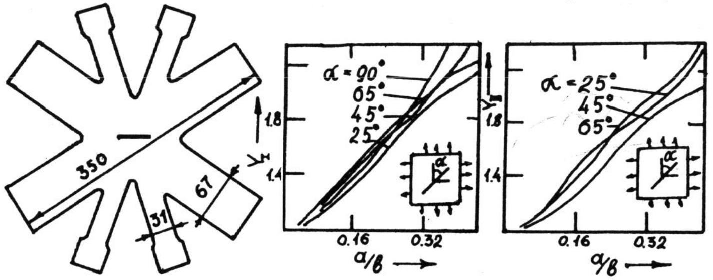

Fig. 3. K-taring function of the plane specimen.

RESULTS AND DISCUSSIONS

On the aluminium alloys having the different properties (whose main mechanical characteristics are presented in table 1, Shlyannikov and Dolgorukov, 1988) the experimental investigation and analysis of the crack growth has been performed taking into account their orientation under uniaxial η =0 and biaxial η =1 and η =0.5 tension on the electrohydraulical stand with antisymmetry factor R = 0.05 at a frequency 3.5 1/s on the rectangular (80x320 mm) and eight-petal specimens by the thickness 3÷5 mm. The fatigue fracture diagrams have been obtained for all the materials at the following stress relationships: η =0;0.5;1 and at the angles of initial orientation α =0;25;45;65;90° of crack with its length a =10 mm, some of them being shown in Fig. 4. The cases (a) and (b) relate to the symmetrical uni- and biaxial tension (α =90°). It was established that for all the materials observed the

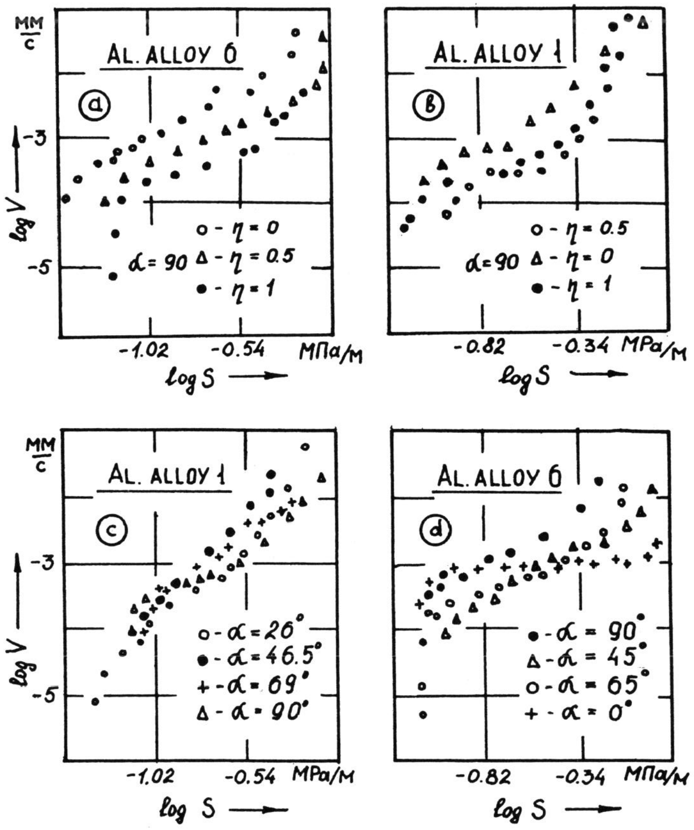

Fig. 4. Diagrams of fatigue fracture.

crack growth rate is greater at uniaxial symmetrical tension than at biaxial one. For the plastic materials the smallest crack growth rate corresponds to biaxial tension η =0.5 while for the more brittle materials η =1. Moreover some difference in shapes of their fatigue fracture diagrams is observed. The cases (c) and (d) in Fig. 4 correspond to mixed modes of the crack propagation at η =0 and η =0.5. At uniaxial cyclic tension the influence of the crack initial orientation angle is not very great. However, as a matter of fact, the greatest crack growth rate corresponds to α =45° at wich the value of K_I is maximum. At biaxial tension the crack initial orientation exerts a greater influence than at uniaxial one. From the common series of diagrams one can distinguish the diagram of fatigue fracture, when the initial crack is directed along the line of the maximum nominal stresses, i.e. α =0°; η =0.5. For the generalized evaluation of characteristics of the cyclic crack-stability at mixed modes of loading the undimensional parameter has been proposed (Shlyannikov and Dolgorukov, 1988), in which the arbitrary values of empirically determined constants of material n and S^* (in equation 2) were divided on their values in the case of equally-biaxial tension (η =1) n_1 and S_1^*

$$T= (n_1/n)/(S^*/S_1^*) \qquad (8)$$

The case of equally-biaxial tension (η =1) is invariant to the crack orientation angle and hence it makes sense to use it as a basic experiment. In Fig. 5 the character of change of the parameter T as a function of α is shown for the different relationships of nominal stresses η. Here one can see that the crack-stability under biaxial load is greater than under uniaxial one at all the regions of the α change. Character of the T-parameter change corresponds to the mutual disposition of the fatigue fracture diagrams. The most dangerous from the point of view of the fracture under biaxial tension are the cases of α =0° and α =90°. In the other cases the deviation of the plane of initial crack orientation from the specimen axis of symmetry leads to decrease of the crack growth rate. It may be seen that the plastic materials have the greater cyclic crack-stability than the brittle ones. The character of dependence of the crack growth rate determined at S_{max}=0.8 MPa m on the T-parameter value (fig. 6) is notable. The indivisible character of this curve for all the materials the types of SSS and the angles of the initial crack orientation (which can be approximated by the m-degree polynom) opens the possibility to predict characteristics of the cyclic crack-stability under mixed modes of fracture. It is obvious that the case of the equally-biaxial tension with η =1 has to appear as a base experiment.

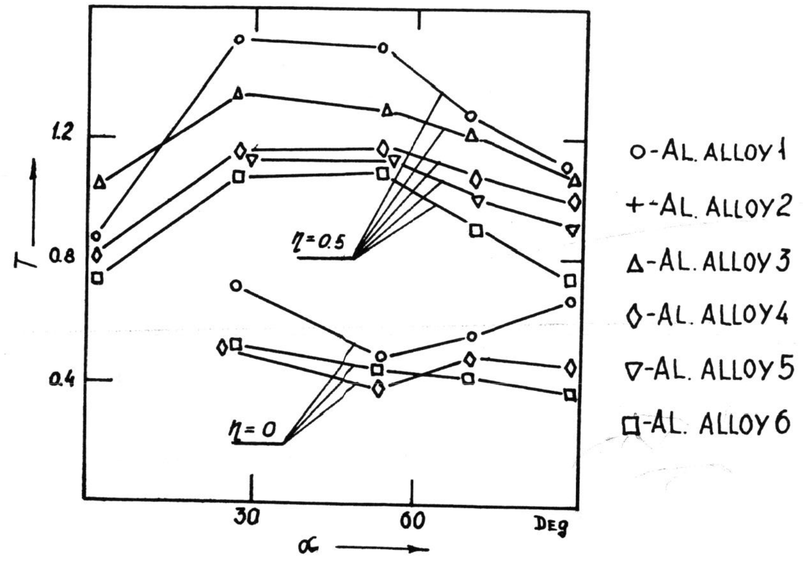

Fig. 5. Dependence T-α.

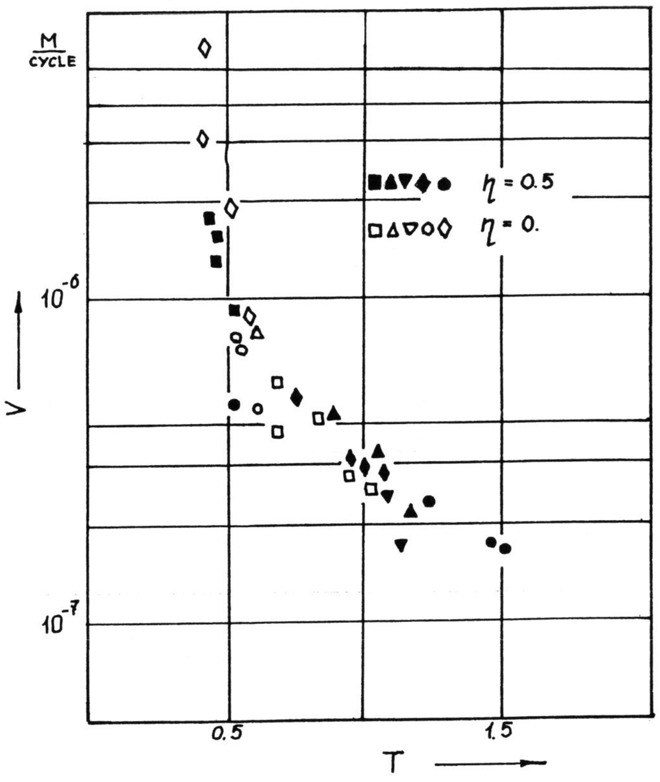

Fig. 6. Dependence of the crack growth rate on the T-parameter.

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