Mesh Generation by Conformal Mapping in Two-dimensional Fracture Problems

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ABSTRACT

Conformal mapping techniques may be used for the automatic generation of finite element meshes in two-dimensional fracture mechanical calculations. The conformal mapping results in relatively small, nearly quadratic elements at the crack tip and a continuous transition to larger elements with increasing distance from the crack tip. By means of elastic-plastic calculations of a compact tension specimen the results of meshes generated by conformal mapping are compared to the results of meshes usually used.

KEYWORDS

Conformal mapping, mesh generation, two-dimensional, elastic-plastic fracture, J-integral, compact tension specimen.

INTRODUCTION

Using the finite element method for problems with large stress gradients, especially fracture mechanical problems, it is necessary to model the stress gradient region with relatively small elements. On the other hand it is reasonable to model areas, where the stresses do not vary very much, with larger elements to save computing time.

For two-dimensional problems the application of conformal mapping techniques as described by Tsamasphyros et al. (1986) automatically delivers a so called orthogonal mesh with small elements at the crack tip and with a gradually increase of the element size with increasing distance from the crack tip. Except of the crack tip element all quadrilateral elements generated are rectangular at their corners, therefore an optimum numerical performance can be expected.

Tsamasphyros et al. (1986) present some numerical examples concerning linear elastic calculations of center and edge cracked plates with meshes built up by conformal mapping. The stress intensity factor results gained there show a satisfactory coincidence with values from literature. Furthermore some comparisons are given between the so called optimum mesh, generated by conformal mapping, and an arbitrary mesh, built up in a conventional manner, where for optimum meshes with about the same number of nodes significantly better results are stated. For example for a center cracked plate with uniform tension loading the maximum crack opening displacement for the arbitrary mesh is reported to be only two third of the optimum mesh.

As the application of the conformal mapping seemed to be very promising to us, we did some test calculations of a compact tension specimen primarily aimed to compare the elastic-plastic behaviour of models with orthogonal and usual meshes, respectively.

FINITE ELEMENT MODELS, MATERIAL DATA AND LOADING CASES

The geometrical data of the compact tension specimen considered are shown in Fig. 1. For symmetry reasons only one half of the compact tension specimen has to be modelled, as indicated in Fig. 1. Fig. 2 shows the finite element meshes used. As there should be no significant influence on the behaviour of the model, the pinhole and the exact shape of the notch of the compact tension specimen were neglected.

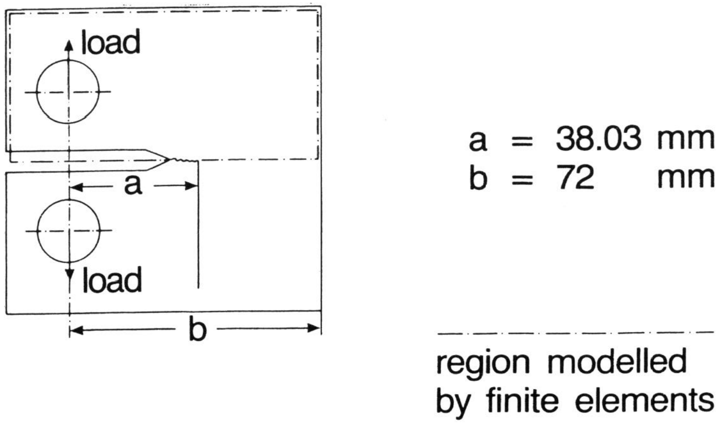


Fig. 1. Geometrical data of compact tension specimen

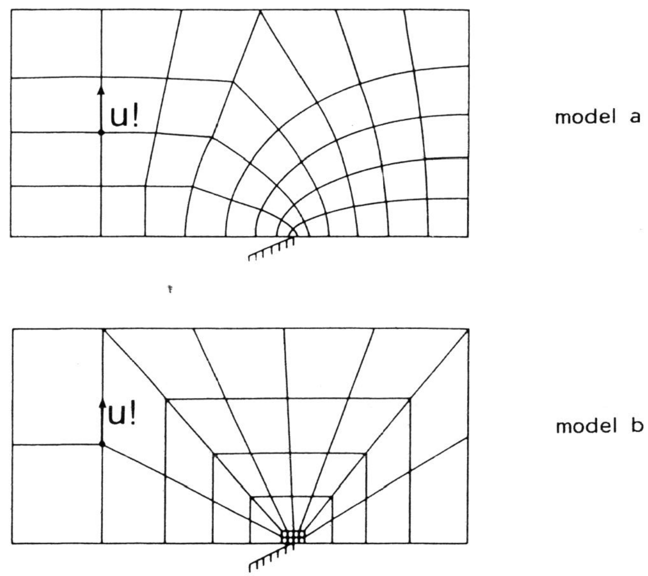


Fig. 2. Finite element meshes of compact tension specimen

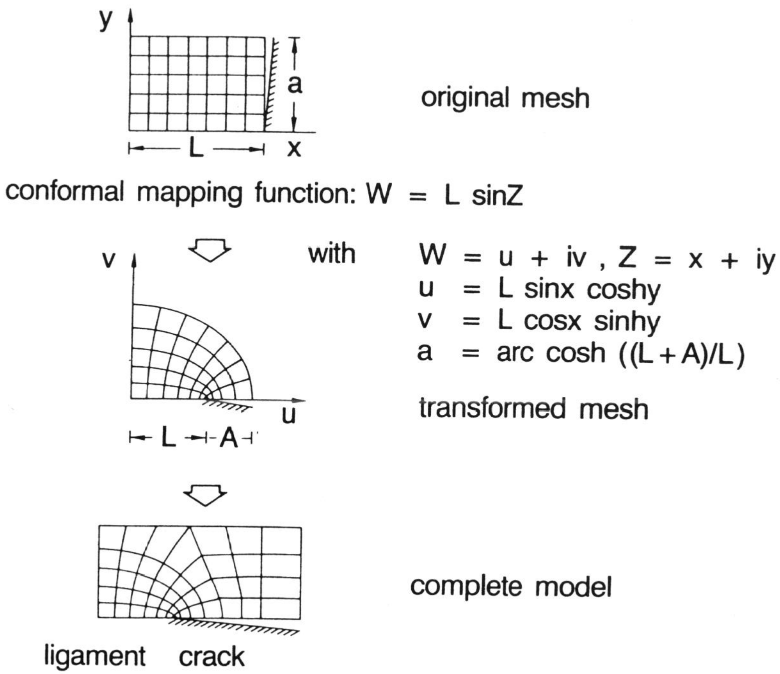


Fig. 3. Procedure for the generation of orthogonal meshes

The part of the orthogonal mesh (Fig. 2a) surrounding the crack tip is generated by means of the conformal mapping, while the rest of the model is added using a usual mesh generator. The procedure is shown schematically in Fig. 3, together with the conformal mapping function used. Both FE-models consist of 8 node isoparametric plane elements.

The specimen considered is made of the very ductile German steel 20 MnMoNi 55. The multi-linear approach of the stress-strain-curve at 300° C shown in Fig. 4 is used for the calculations.

The loading is applied as a prescribed displacement u! in global z-direction of the node in the center of the pinhole of the specimen, as indicated in Fig. 2. The displacement u! is increased to 1.375 mm. Plane stress conditions have been used for the calculations.

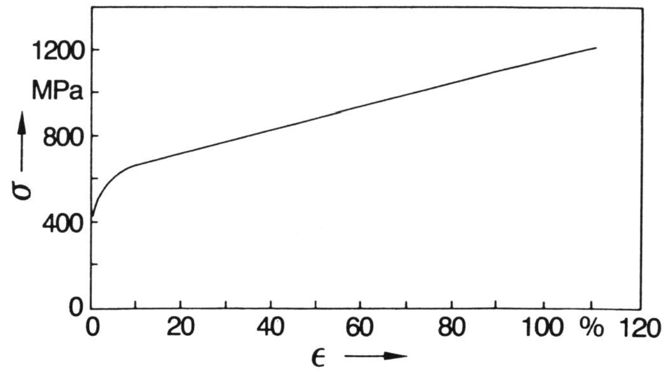


Fig. 4. True stress-strain curve of 20 MnMoNi 55 (at 300°C)

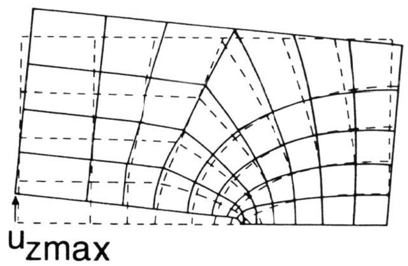
J-INTEGRAL-EVALUATION

In elastic-plastic fracture mechanical problems we mostly use the finite element program ADINA. In our version this program includes some fracture mechanical extensions, e.g. the possibility to calculate J-integral-values. The J-integral may be calculated either as line-integral, as originally introduced by Rice (1968), or as surface or volume integral, respectively, following a proposal by DeLorenzi (1982). In this case the virtual crack extension technique as introduced by Parks (1977) is used. The implementation of these options in ADINA is based on proposals by Schmitt et al. (1983).

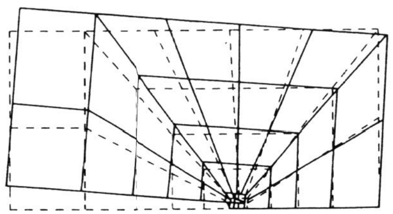
RESULTS FOR THE COMPACT TENSION SPECIMEN

For the calculations of the two meshes shown in Fig. 2 the same timestep increments were used. The elastic-plastic behaviour of the specimen was modelled by the material-nonlinear-only option in ADINA. As far as possible the paths for the J-integral-evaluation are chosen similarily for the different meshes, to get comparable results.

The Figs. 5 and 6 present some structure-mechanical results of the calculations. Fig. 6 shows the maximum crack opening (as specified in Fig. 5) u_{zmax} as function of the prescribed displacement u! for model b. The relative difference of u_{zmax} for the two meshes is smaller than one percent for all load steps considered. Similar results are obtained for the crack tip profile, where larger deviations are found only in the element adjacent to the crack tip. This is due to the different modelling of the crack tip region.



model a



model b

crack tip region of model b

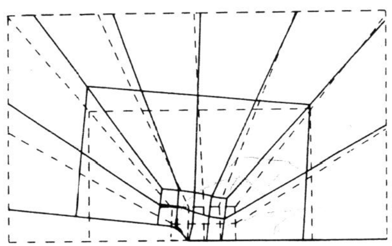


Fig. 5. Deformation of the compact tension specimen at maximum load (model a and b)

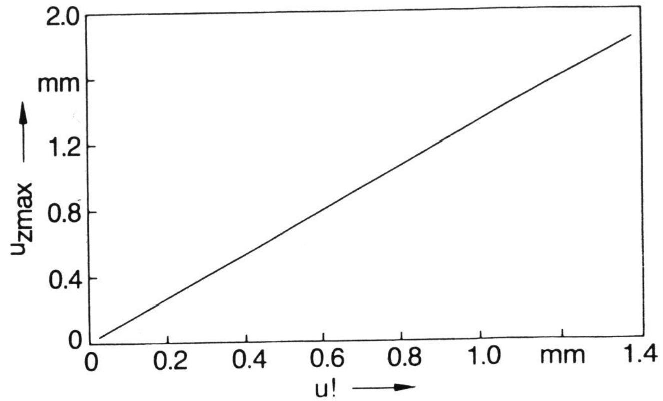


Fig. 6. Maximum crack opening u_{zmax} for model b as function of prescribed displacement

Fig. 7 presents some results of the J-integral-calculations. The J-integral as function of the prescribed displacement is shown for the orthogonal mesh and for the largest integration path considered. The relative difference of the J-values of the corresponding largest paths is less than 1.5 percent for the two models considered.

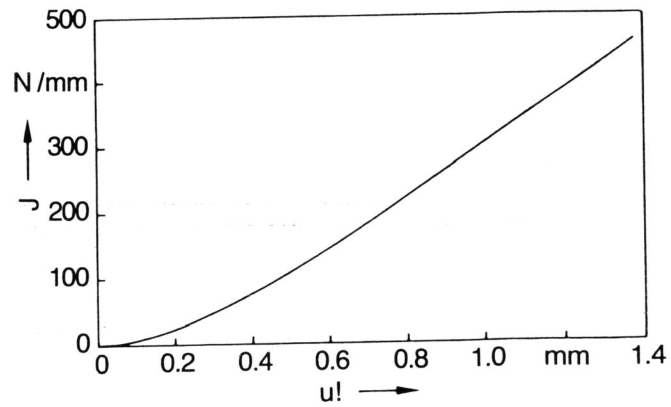


Fig. 7. J-Integral for model a as function of prescribed displacement

DISCUSSIONS OF THE RESULTS, CONCLUSIONS

In contrast to the foundings of Tsamasphyros et al. (1986) the structure-mechanical behaviour of the test model does not vary much with the different meshes. So the relative deviation of the maximum crack opening is smaller than one one percent for the compact tension specimen considered.

The fracture mechanical results also do not differ very much, when integration paths are considered, which are comparable in magnitude. For the two models of the compact tension specimen relativ differences of the J-integral values of less than 2.5 percent are found.

Nevertheless, the following advantages of the orthogonal meshes may be confirmed :

a) It is possible to model a continuous increase of element size with increasing distance from the crack tip almost automatically. The elements generated are rectangular at their corners.

b) The evaluation of stresses in the crack tip region is more exact than in the case of a usual mesh, when the same or even a smaller number of elements is used.

c) Caused by the better evaluation of stresses the integration paths near the crack tip give better results for the J-integral than in the case of a usual net.

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