Energy Criteria for Stable Crack Growth Simulation Under Biaxial Loading Using Finite Element Method

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ABSTRACT

A central notched specimen under biaxial loading has been analysed by the finite element method using quadratic isoparametric elements. Critical crack tip opening angle (CTOA) has been adopted as the criterion for crack initiation and stable crack growth (SCG). Plastic energy, non linear strain energy release rate ($\widetilde{G}$) and separation energy rate (G^{Δ}) have been determined during SCG. The G^{Δ} has been found to be fairly constant during SCG and could be a good engineering parameter for simulation of SCG. Energy parameters are sensitive to mesh discretization and stress-strain representation and have pronounced effect of higher biaxial (tension-compression) load factor.

KEYWORDS

Biaxial loading, stable crack growth, separation energy release rate, non-linear strain energy release rate, plastic energy, isoparametric finite element, crack tip opening angle.

INTRODUCTION

Triaxial state of stress in the vicinity of crack tip due to externally applied biaxial load becomes conspicuous in respect of shape, size and orientation of plastic zone and governing strains which cause the crack initiation and stable crack growth. Biaxial load effects on fracture parameters for stationary crack have been studied by several researchers (Kfouri and Miller, 1977; Lee and Liebowitz, 1977; Liebowitz et al., 1979). Raju and Dash (1979) incorporated biaxial load effects on stable crack growth for infinite geometry specimen. Their approximate analytical study is based on evaluation of plastic energy dissipation rate in the plastic zone excluding the zone of unloading. Based on linear relation of plastic energy and crack size, Liebowitz et al. (1979) have suggested a method of obtaining crack growth curves theoretically and then predicting the point of unstable fracture. The work of Abou-Sayed et al. (1981) is also worthy of mention in this context. Singh and Ramakrishnan (1984a, 1984b) studied the biaxial load effects on J-integral, plastic zone, wake, incremental CTOD, incremental J and stress distribution in ligament during

SCG. In their work they have found that critical CTOD was affected by less than 7.5% in the range of biaxial load factor (k) = -2 to 3 and σ / σ_y ratio of 0.3 and could be used as useful fracture parameter for crack initiation and SCG under biaxial loading. Eftis (1984) has studied fracture characteristics of collinear cracks under biaxial loading.

This paper presents variation of plastic energy and energy parameters with crack length under varying externally applied biaxial loads on a central notched (CN) specimen. Staircase type of loading has been adopted. A series of small load steps have been chosen for this purpose. The crack is propagated on attainment of the critical CTOA. Biaxial load factors (k) 1, 0, -1, -2 have been considered.

CRITERION FOR CRACK INITIATION AND STABLE CRACK GROWTH

The main problem associated with the simulation of stable crack growth is the adoption of a suitable criterion for crack initiation and advance. Among the several criteria proposed two important ones for SCG are i) linear plastic energy crack length relation and ii) critical crack tip opening angle. The authors have felt that plastic energy being extremely sensitive to yield stress and the stress-strain representation, is not suitable for a unified approach attempted in this investigation. On the other hand, the critical CTOA is straight forward and an attractive engineering parameter for simulating SCG. It assumes that crack initiates on the achievement of critical value of CTOD one element behind the advancing crack tip. The elements, through which the crack has to propagate, are of uniform length. Thus, the latter criterion, essentially implies the constant CTOA as criterion for stable crack growth. Critical semi CTOD obtained in an earlier work of the authors (1984a) has been made use of. The critical CTOD for the present investigation has been calculated in the above work by making use of the experimental crack growth resistance curve for a centre notched specimen under uniaxial loading conditions by Liebowitz.

The present analysis predicts the crack growth behaviour of the same specimen under biaxial loading.

MATERIAL PROPERTIES, SPECIMEN GEOMETRY, DISCRETIZATION AND NODAL RELEASE TECHNIQUE

The aluminium alloy (2024-T3) characterised by the Ramberg-Osgood type of stress-strain relation has been chosen for analysis as a typical example of light weight high strength material. The constitutive relation in simple tension is given by the relation

$$\varepsilon^* = \sigma^* + \alpha^* \sigma^{*n} \tag{1}$$

where ε^* and σ^* are non-dimensionalised strain and stress. Equation (1) can be written as

$$\varepsilon = (\sigma/E) + \alpha^*(\sigma_y/E)\,(\sigma/\sigma_y)^n \tag{2}$$

Here α^* and n are the strain hardening coefficient and exponent and equal to 2.207 and 6.025 respectively. The α^* is taken as zero for $\sigma \leq \sigma_y$. The initial yield stress, σ_y, and modulus of elasticity E, have been taken as 3850 kg/cm^2 and 724200 kg/cm^2 respectively. The stress-plastic strain curve has been represented by multilinear segments. von-Mises yield criterion has been

assumed to be valid. Incremental theory of plasticity has been used to generalise the stress-strain relation to multiaxial stress state in the vicinity of crack tip.

The CN specimen considered, consists of a rectangular plate (30 cm x 80 cm) having a thickness of 0.1575 cm with a central sharp line crack 15 cm. in length. Because of symmetry only the first quadrant of the specimen has been analysed. The analysis has been made by using the FEM. Numerically integrated eight noded isoparametric elements have been used. The uncracked ligament ahead of crack tip has been discretized into 12 fine elements of uniform length, and coarser ones after these. The crack tip elements are 0.1 cm. long in extended crack plane which is 1.33 per cent of the original crack length. The ratio of the element size and crack length decreases with crack propagation. Three fine elements of the same size have been provided before the original crack tip. The vicinity of crack tip has been discretized into sufficiently fine elements, keeping in view the development of elastic plastic boundary with load increase and crack propagation, depending upon loading pattern described later. The first quadrant of the specimen has been discretized into 98 eight-noded isoparametric elements and 337 nodes. The fine crack tip elements have specific advantages particularly in releasing the nodes to effect crack advance.

On the satisfaction of the condition for crack propagation, the crack tip node/nodes are released by debonding them, thereby bringing the cohesive force at the released nodes to zero. Since the present investigation uses quadratic isoparametric elements, two nodes, one at the crack tip and another the midside one just ahead, are released to effect a crack, advance by one element.

LOADING PROCEDURE

Four loading cases considered are uniaxial (k=0); tension-tension (k=1); tension-compression (k=-1) and tension-compression (k=-2).

In all the four cases the load normal to crack has been applied in 28 incremental steps. At a particular load step, on the satisfaction of the criterion for crack initiation i.e. on the attainment of the critical CTOA, the crack propagates by one element at constant load. On the other hand, if the critical CTOA has not been reached, the next load increment is applied at the same crack length. This is continued until instability sets in. However, it has been observed that the crack advances even consecutively by two elements, one after the other, at a particular load step, indicating unstable fracture.

ENERGY PARAMETERS

Following energy parameters are computed in this investigation.

Plastic Energy (P)

It is obtained by computing incremental work hardening for all gaussian points of all yielded elements and summing that up for all time steps till convergence such that

$$P = \int \underset{\sim}{\sigma}^T \, \Delta \underset{\sim}{\varepsilon}^{vp} |J| \, W_i W_j \tag{3}$$

where $\underset{\sim}{\sigma}$ is the stress vector, $\Delta \underset{\sim}{\varepsilon}^{vp}$ is the incremental viscoplastic strain vector, $|J|$ is the determinant of Jacobian matrix of the element and W_i and W_j are the Gaussian weights and abscissa respectively.

Non-linear Strain Energy Release Rate ($\tilde{G}$)

The total strain eneregy, per unit thickness, U, of the plate is obtained as

$$U = \int_\Omega \int_0^{\varepsilon^e} \underset{\sim}{\sigma}^T \, d\underset{\sim}{\varepsilon}^e \, d\Omega + \int_\Omega \int_0^{\varepsilon^{vp}} \underset{\sim}{\sigma}^T \, d\underset{\sim}{\varepsilon}^{vp} \, d\Omega \qquad (4)$$

where $d\underset{\sim}{\varepsilon}^e$ and $d\underset{\sim}{\varepsilon}^{vp}$ represent the incremental elastic and visco-plastic strain vectors respectively. The complimentary energy (V) is calculated from the expression

$$V = \int_\Gamma \underset{\sim}{T} \, \underset{\sim}{u} \, d\Gamma - U \qquad (5)$$

where $\underset{\sim}{T}$ represents the traction vector along the loaded boundary and $\underset{\sim}{u}$ the corresponding displacement vector. Here Ω and Γ represent the domain and boundary of the specimen. Thence, $\tilde{G}$ is obtained in fixed load condition as

$$\tilde{G} = 2 \frac{dv}{da} \qquad (6)$$

where da is the crack extension.

Separation Energy Release Rate (G^Δ)

This is computed from the nodal reactions (R) and the corresponding displacements (v) for the nodes being released to effect the crack advance by one element. The G^Δ is equivalent to the work done by the external forces required to cancel the nodal reactions and is given by

$$G^\Delta = \sum_{i=1}^{n} \frac{1}{2} R_i v_i \qquad (7)$$

where n = number of nodes released per crack advance step. In this investigation the crack is propagated by one element and so n = 2. R_i represents nodal reaction just before crack initiation and v_i the corresponding vertical nodal displacement after crack advance is complete.

Evans et al. (1980) computed the separation energy rate (Q) as work of separation which equals released potential energy minus plastic energy dissipation and is given in the form of work rate as

$$Q = \frac{du^e}{da} - \frac{du^{vp}}{da} \qquad (8)$$

where du^e and du^{vp} are obtained from the difference in elastic and visco plastic energies just before and after crack advance step.

RESULTS AND DISCUSSION

The results are discussed as follows.

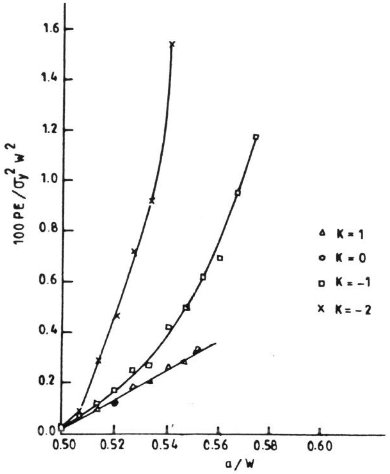

Fig.1. Plastic energy-crack length relation.

Plastic Energy

Fig.1. presents plastic energy-crack size relation for various k's. Plastic energy varies linearly with crack size and is coincident for k = 0 and 1. For k = -1, it is linear up to a/w = 0.525. With further growth of crack, a/w > 0.525, it becomes non-linear. For k = -2, this relation is rather complicated. It is linear for 0.507 < a/w < 0.53. The non-linear relation for k = -1 and -2 with a/w > 0.525 and 0.53 respectively is attributed to the coarse discretization of the finite element mesh and the large scale yielding. Biaxial loading has pronounced effect on plastic energy. While linear plastic energy-crack size relation can be used to simulate SCG for k = 0 and 1, its use for k = -1 and -2 is not established.

The difference between the authors' work and the works of Lee and Liebowitz (1978) and Liebowitz et al. (1979a) is attributed to the multilinear segment representation of stress-strain curve adopted in this investigation. It is noteworthy that the use of higher order isoparametric elements over triangular elements of Lee and Liebowitz has the advantage of better accuracy.

Separation Energy Rate

Variation of $\tilde{G}$, G^Δ and Q with crack length and biaxiality is shown in fig.2. While large scatter is indicated in $\tilde{G}$ and Q at later crack advance steps with biaxiality, G^Δ is fairly constant during stable crack growth. It is not much affected by biaxial loading in the range investigated. Thus G^Δ can be regarded as a better material parameter for stable crack growth simulation under biaxial loading. Lotsberg (1978) observed larger scatter in the global separation energy rate than in the local one. It is to be noted that the value of G^Δ is very reliable since all the precautions recommended by, Bleakley and Luxmoore (1978) have been taken into account.

Fig.3. shows the correlation between J_{av}/G vs. G/G_o and G^Δ/G vs G/G_o for various biaxiality ratios. Here G is the elastic strain energy release rate for various crack length and appropriate σ/σ_y ratio and G_o is the elastic strain energy release rate at first load step. The four curves above show variation of J_{av}/G for various values of G/G_o. The variations are all very smooth. The value of J at any particular level of G/G_o is the same for the two biaxiality ratio, k = 0 and k = 1. Thus the two curves corresponding to k = 0 and 1 coalesce while the curves for k = -1 and -2 are shown above. These observations are in agreement with those of Kfouri and Miller (1977).

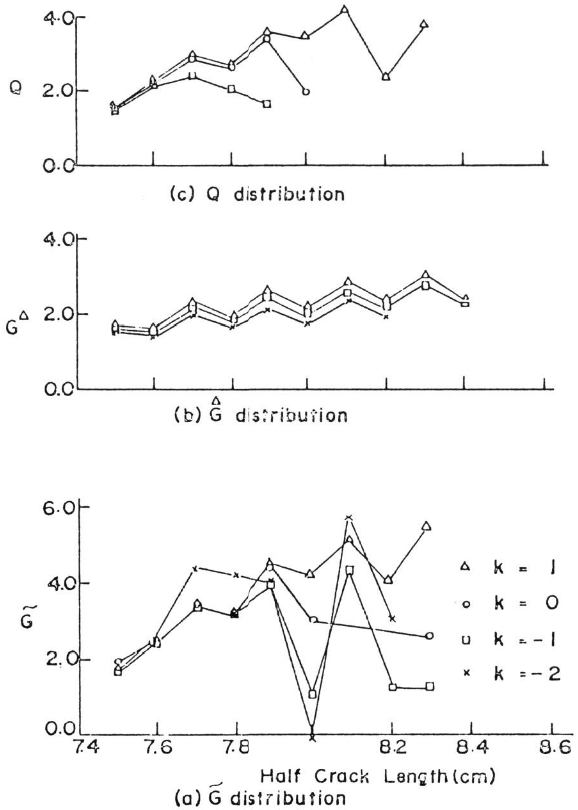

Fig.2. Energy rate-crack length relation during SCG.

The plot of G^{Δ}/G vs. G/G_o is shown by the three curves at the bottom half of thefigure-3. The curves for k=0 and 1 are coincident. There is some scatter in the values but however only the trend is represented by the graphs. There is certainly a decreasing trend for G^{Δ}/G as G/G_o increases. The rate of decrease for high value of k is smaller.

Because of inherent limitations in the computation of various energy rates, no conclusion could be obtained so far. Further numerical trials are necessary before any definite energy rate is identified as a single characterising parameter for crack growth.

Comparison of Parameters

The critical values of energy rates ($\tilde{G}_c$, G_c^{Δ}, Q_c) and J_c at crack initiation are in good agreement. With maximum difference of the order of 6 per cent in G_c^{Δ} from J_c, G_c^{Δ} can be regarded as good engineering parameter for identifying the initiation of SCG. $\tilde{G}_c$ though fairly close to J_c, except at k

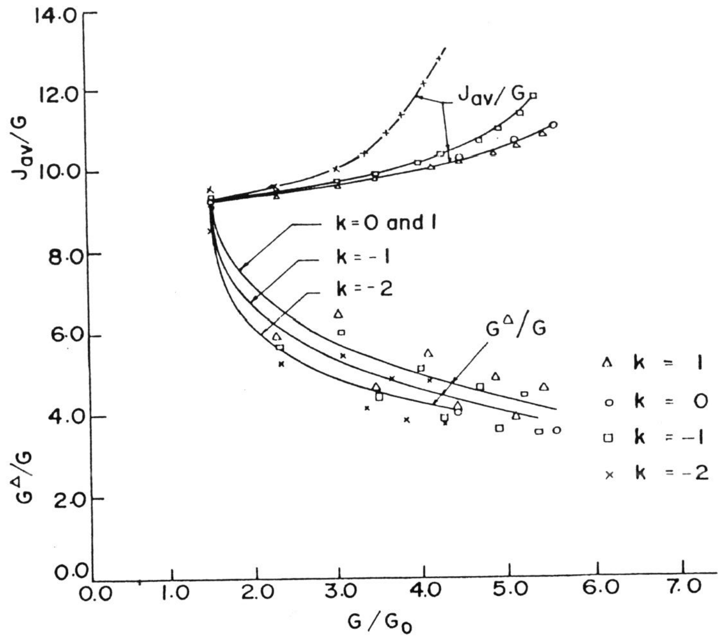

Fig.3. Correlation between G^{Δ}, J and G during SCG.

= 0, shows wide scatter with crack advance. Evans et al. (1980) report significant difference in the Q_c and J_c values at initiation of SCG.

CONCLUSIONS

Following are the conclusions of the investigation

(i) Plastic energy-crack length relation is linear for k=0 and 1.
(ii) The separation energy rate (G^{Δ}) is fairly constant for k=1, 0,-1,-2 and can be a good parameter for SCG simulation.
(iii) The critical values of the J integral, the $\tilde{G}$, G^{Δ} and Q are unaffected by load biaxility at initiation of SCG.
(iv) The $\tilde{G}$ and Q have similar behaviour in the range of biaxial load factor investigated during SCG by 5 elements.
(v) Biaxial loading has pronounced effects on $\tilde{G}$ and Q as SCG approaches onset of unstable fracture.

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An Integral Equation Method Based on Resultant Forces on a Piece-wise Smooth Crack in a Finite Plate

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ABSTRACT

Integral equations for the resultant forces on a piece-wise smooth crack line are formulated and coupled to the standard BEM equations for the outer boundary of a finite plane. The resulting equations are a generalization of the equations for infinite geometries (Cheung and Chen, 1987). The integrals along the crack line, with the dislocation densities as unknowns, contain only a weak logarithmic singularity. An improvement of the numerical formulation at a kink of the crack line is introduced. Two numerical experiments are presented and compared with alternative numerical calculations.

KEYWORDS

Integral equation method; BEM; resultant forces; kinked crack; finite geometries.

INTRODUCTION

Singular integral equations are widely applied for the solution of fracture mechanics problems. For cracks in two dimensional infinite geometries and elastic materials, these equations can for example be derived either by applying the integral transform method, or by using complex potentials (Erdogan, 1983). The resulting integral equations, with dislocation densities as unknowns, are expressions for the tractions along the crack line and contain a Cauchy-type singularity. More general integral equations for finite geometries, also for the tractions along the crack line, can be derived by performing partial integrations of the standard BEM equations (Zang and Gudmundson, 1988a). The numerical formulation of the integral equations is based on a suitable numerical evaluation of the singular integrals. A collocation method is then applied to derive an approximate solution of the equations.

It was however shown by Zang and Gudmundson (1988a) that if a piece-wise smooth crack is considered, the expressions for the tractions on the crack line are not valid at a kink. This fact can give rise to numerical difficulties. Lo (1978) applied a Green's function for a point dislocation which satisfies the traction free boundary conditions on the main crack surfaces. An integral equation for the branched portion of the crack