Energy Analysis of Crack-Damage Interaction

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ABSTRACT

The energy release rates associated with a main crack propagating into a surrounding damage zone, and a damage zone translation relative to the main crack, as well as an energy of interaction between the two are analyzed. The displacement and stress fields for the crack-damage interaction problem are reconstructed employing a semi-empirical stress analysis and experimental evaluation of the average craze density in the crazed zone.

KEYWORDS

Damage Zone; Energy Release Rate; Eshelby's Energy Momentum Tensor; Interaction Energy.

INTRODUCTION

Evaluation of the energy release rates faces the difficult problem of crack damage interaction. The elastic interaction of multiple cracks has been addressed by various authors (Hoagland,1980; Gross, 1982; Chudnovsky,1983; Kachanov at al., 1984; Rose,1986; Rubinstein,1986). We consider a case when damage consists of an array of microcracks or crazes. Solution of such problem for a large number of randomly distributed microcracks is unavailable at present. A Semi-Empirical Stress Analysis (SESA) has been recently proposed to study such a crack-microcrack array interaction (Chudnovsky and Ouezdou,1988)

In this paper we employ SESA to solve the elastic crack-damage interaction problem as a basis for evaluating energy release rates. Evolution of damage in a vicinity of the fatigue crack in polystyrene (an amorphous polymer) is considered to illustrate a methodology of energy analysis. The damage zone for this material consists of an array of crazes parallel to the main crack. The craze zone is observed as two subzones. The first subzone is a core of crazing, adjacent to the crack tip. The second one consists of peripheral crazes, which are much less dense and can be measured. The average craze density within a core of crazing is estimated through the assumption that the stress intensity factor vanishes. This assumption is based on the observed shape of crack opening (Fig.1 taken from Botsis et al., 1988). Then, the energy release rates J_1^C due to crack extension with the damage zone being stationary, and J_1^A due to craze array advance with the crack being stationary are evaluated.

SEMI-EMPIRICAL STRESS ANALYSIS

Following Chudnovsky (1988), the displacement vector $\mathbf{u}^A$ at a point, $\mathbf{x}$, generated by the craze array can be expressed as:

$$\mathbf{u}^A(\mathbf{x}) = \int_{V_A} \mathbf{c}(\boldsymbol{\xi}) \cdot \boldsymbol{\Phi}(\boldsymbol{\xi},\mathbf{x})\, d\boldsymbol{\xi} \tag{1}$$

where $\boldsymbol{\Phi}(\xi,x)$ is the second Green's tensor (Sternberg, 1952) and $\mathbf{c}$ is craze opening density which is the product of a craze opening $\mathbf{b}$, and the craze density, ρ; V_A is the area of damage zone. Therefore, the total displacement field for a crack damage interaction problem which consists of the displacement $\mathbf{u}^0(\mathbf{x})$ due to remotely applied load and displacement caused by the crack and the damage zone can be presented as:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}^0(\mathbf{x}) + \int_{-L}^{0} \boldsymbol{\delta}(\boldsymbol{\xi}) \cdot \boldsymbol{\Phi}(\boldsymbol{\xi},\mathbf{x})\, d\boldsymbol{\xi} + \int_{V_A} \mathbf{c}(\boldsymbol{\xi}) \cdot \boldsymbol{\Phi}(\boldsymbol{\xi},\mathbf{x})\, d\boldsymbol{\xi} \tag{2}$$

The stress field follows from (2) and Hook's law:

$$\boldsymbol{\sigma}(\mathbf{x}) = \boldsymbol{\sigma}^0(\mathbf{x}) + \int_{-L}^{0} \boldsymbol{\delta}(\boldsymbol{\xi}) \cdot \mathbf{F}(\boldsymbol{\xi},\mathbf{x})\, d\boldsymbol{\xi} + \int_{V_A} \mathbf{c}(\boldsymbol{\xi}) \cdot \mathbf{F}(\boldsymbol{\xi},\mathbf{x})\, d\boldsymbol{\xi} \tag{3}$$

The components F_{ijk} of the third rank tensor $\mathbf{F}$ can be obtained directly from $\boldsymbol{\Phi}$.

The crack opening displacement $\boldsymbol{\delta}(\boldsymbol{\xi})$ also can be decomposed into the COD $\boldsymbol{\delta}^0(\boldsymbol{\xi})$ due to remotly applied load and COD $\boldsymbol{\delta}^A(\boldsymbol{\xi})$ resulting the interaction with the damage zone as:

$$\delta^0(x) = \frac{4}{E} \int_{x}^{L} [\, \pi(l^2 - x^2)/l\,]^{-1/2} K_{I0}(l)\, dl \tag{4}$$

$$\delta^A(x) = \frac{4}{E} \int_{x}^{L} \{ [\, \pi(l^2 - x^2)/l\,]^{-1/2} \int_{V_A} \mathbf{c}(\xi)\, G_{SIF}(\boldsymbol{\xi},l)\, d\boldsymbol{\xi} \}\, dl \tag{5}$$

where $K_{I0}(l)$ is the stress intensity factor (SIF) for a slit with no damage and G_{SIF} is the SIF Green's Function (Chudnovsky and Ouezdou, 1988). Then the displacement and stress fields associated with the crack can be decomposed into the elastic field $\mathbf{u}^c$, $\boldsymbol{\sigma}^{co}$ induced by the main crack in the absence of damage and $\mathbf{u}^{CA}$, $\boldsymbol{\sigma}^{CA}$ reflecting the crack-damage interaction.

It should be noted that crack-damage interaction problem is solved once the vector $\mathbf{c}(\boldsymbol{\xi})$ of craze opening density is determined. In the semi-empirical analysis $\mathbf{c}(\boldsymbol{\xi})$ is assumed to be evaluated experimentlly.

As it is mensioned above $\mathbf{c}(\xi)$ is a product of the craze density $\rho(\xi)$ which is reported by Botsis (1988) and the craze opening $\mathbf{b}(\xi)$ which directly reflects the elastic interaction. Thus, $\mathbf{b}(\boldsymbol{\xi})$, should be observed in the actual configuration under the load. Reliable experimental data have been obtained for peripheral craze opening, however, this experimental technique employed by Botsis (1988) does not resolve $\mathbf{b}(\boldsymbol{\xi})$ in the core of damage where

craze density, ρ, is quite high. Hence we use another means to estimate $\mathbf{b}(\boldsymbol{\xi})$ in the core.

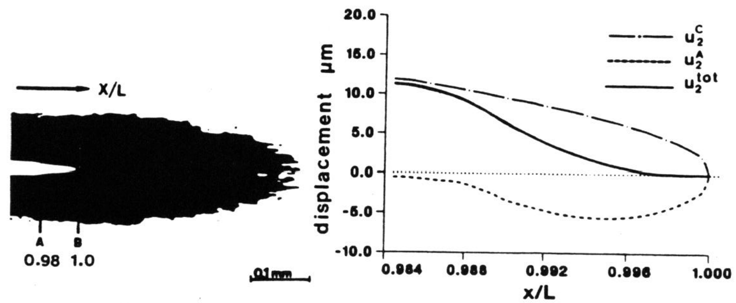
The micrograph in Fig.1a taken from Botsis (1988) shows a crack opening displacement at the crack tip surrounded by the damage (craze) zone. It resembles the COD of the crack tip in Dugdale-Barenblatt model and thus suggests the absence of stress singularity. In addition, studies on craze phenomena (Kambour, 1973; Chudnovsky et al., 1981) propose a stress limitation within the craze zone similar to Von Mises or Tresca criteria in plasticity. Based on this we assume that K^{tot} resulting from the remotely applied load K_{I0} and the traction at the crack line induced by the damage zone K_{IA} vanishes.

K_{I0} for given load specimen configuration is well known (Tada at al., 1973). K_{IA} can be obtained using Green's function for SIF $G_{SIF}(x)$. In the case under consideration it is observed that crazes are parallel to the main crack, ($\mathbf{n}$=(0,1)), and the vector of craze opening density $\mathbf{c}$ has also only one nonzero component, i.e., $\mathbf{c}=(0,c_2)$. The form of $G_{SIF}(\mathbf{x})$ suggests that only the part of the damage zone immediately adjacent to the crack tip, i.e., core of crazing affects the stress intensity factor K.

Thus a core average component $\langle c_2 \rangle$ of craze opening density $\mathbf{c}$ resulting from the condition $K^{tot} = 0$:

$$\langle c_2 \rangle = K_{I0} [\int_{V_A} G_{SIF}(\boldsymbol{\xi},\mathbf{n})\, d\boldsymbol{\xi}]^{-1} \tag{6}$$

To examine the assumption of $K^{tot} = 0$, we compare the calculated COD in a vicinity of the crack tip with that observed experimentally as shown in Fig. 1b. It agree well with the experimentally observed COD shape and dimensions.



(a) (b)

Fig.1 (a) the optical microgarphs displaying damage zone and the COD in the vicinity of the propagating crack tip. (b) profiles of crack face displacement due to remote loading, damage zone and the total crack face displacement near the crack tip for crack length L=1.55 mm.

TRANSLATIONAL ENERGY RELEASE RATES

We introduce a "center of gravity" $\underset{\sim}{x}_c(x_{1c}, x_{2c})$ with respect to the damage density ρ and consider the crack length l and $\underset{\sim}{x}_c$ rates as independent kinematic variables. The energy analysis associated with these variables is the aim of the section.

Energy Release Rate Due to the Main Crack Extension

To evaluate energy release rate due to crack extension under the condition that the damage density is stationary $\rho(x,t) = \rho(\underset{\sim}{x})$, we model the crack-damage interaction by the traction induced by the damage zone on the crack line.

Since we consider a continuous craze opening density, $\underset{\sim}{c}(x)$, there is no contour which separates crack faces from distributed crazing. Thus, the energy release rate J_1^c can be expressed as the conventional contour integral over the contour which includes the crack faces similar to that in Dugdale-Barenblatt model:

$$J_1^c = \int_{\Gamma_c} [\, f\, n_1 - \sigma_{ij} n_j u_{i,1}]\, d\Gamma \tag{7}$$

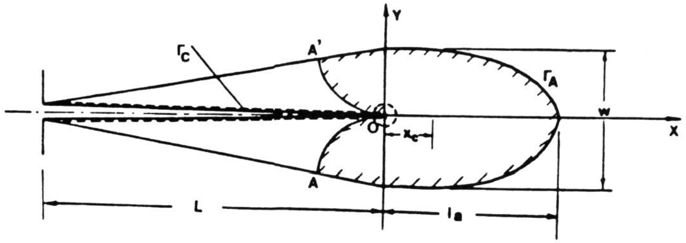


Fig.2 Sketch of damage zone near the crack tip and the integration pathes.

where the integration path Γ_c is shown in Fig.2. The strain energy density f can be decomposed into that in the absence of damage, f^0, and that due to the crack-damage interaction, f^{int}.

$$f^0 = \frac{1}{2E}\left[(\sigma_{11}^{c0})^2 + (\sigma_{22}^{c0})^2 - 2\nu\sigma_{11}^{c0}\sigma_{22}^{c0} + 2(1+\nu)(\sigma_{12}^{c0})^2\right] \tag{8}$$

$$f^{int} = \frac{1}{2E}\Big\{\left[(\sigma_{11}^{cI})^2 + (\sigma_{22}^{cI})^2 - 2\nu\sigma_{11}^{cI}\sigma_{22}^{cI} + 2(1+\nu)(\sigma_{12}^{cI})^2\right] +$$

$$+ 2\left[\sigma_{11}^{c0}\sigma_{11}^{cI} + \sigma_{22}^{c0}\sigma_{22}^{cI} - (\sigma_{11}^{c0}\sigma_{22}^{cI} + \sigma_{11}^{cI}\sigma_{22}^{c0}) + 2(1+\nu)\sigma_{12}^{c0}\sigma_{12}^{cI}\right]\Big\}$$

here $\underset{\sim}{\sigma}^{co}$ and σ^{cI} result from the second term in (3) combined with (4) and (5). Then, apparently,

$$J_1^c = J_1^0 + J_1^{int} \tag{9}$$

A "shilding" effect on energy release rate, i.e., negative J_1^{int} normalized by J_1^0 is shown in Fig.3

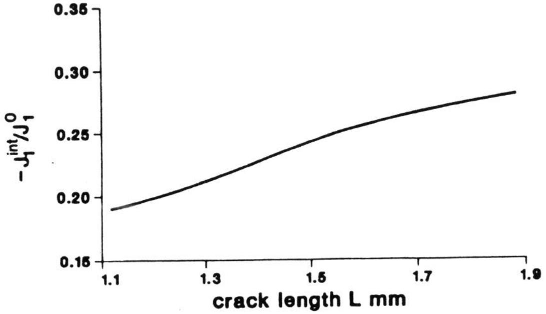


Fig.3 The crack-damage interaction energy release rate J_1^{int} associated with crack extension into the damage zone, normalized by the energy release rate J_1^0 in absence of damage.

Energy Release Rate due to Damage Zone Translation

For an elastic medium with damage, we consider potential energy density π as a function of the conventional state parameters $\underset{\sim}{\sigma}$ and T (absolute temperature) for an elastic solid and an additional damage parameter ρ(Chudnovsky,1984; Chudnovsky,to appear).

$$\pi = \pi(\underset{\sim}{\sigma}, T, \rho) \tag{10}$$

Then the potential energy Π of the solid, the variation of damage density with the damage zone translation δx_1 (along the x-axis, and the corresponding variation of $\delta\Pi$ can be written:

$$\Pi = \int_V \pi\, dv \quad \text{and} \quad \delta\rho = -\delta x_{1c}\partial_1\rho \tag{11}$$

$$\delta\Pi = -\delta x_{1c}\int_V \frac{\partial\pi}{\partial\rho}\partial_1\rho\, dv \tag{12}$$

Isothermal conditions are assumed and only defects, such as microcracks, which do not generate internal stress are considered.

The above integral over a domain $v \supset v_A$ can be reduced to v_A(the area of the damage zone) since $\partial_1\rho \equiv 0$ outside of v_A. The integral (12) is directly related to the Eshelby's energy momentum tensor P_{ik}.

$$\frac{\partial \pi}{\partial \rho}\,\partial_1 \rho = \partial_k P_{1k}, \quad \text{and} \quad P_{ik} = f\delta_{ik} - \sigma_{kj} u_{j,i} \tag{13}$$

where f stands for the elastic strain energy density. Then the energy momentum tensor $\underset{\sim}{P}$ can be decomposed into $\underset{\sim}{P}^A$ associated with the craze array self-action and $\underset{\sim}{P}^{AC}$ reflecting the crack damage interation:

$$P^A_{ij} = f^A \delta_{ij} - \sigma^A_{jk} u^A_{k,i} \tag{14}$$

$$P^{AC}_{ij} = f^{AC}\delta_{ij} - \sigma^C_{jk} u^A_{k,i}$$

$$f^A = \frac{1}{2E}\left[(\sigma^A_{11})^2 + (\sigma^A_{22})^2 - 2\nu\sigma^A_{11}\sigma^A_{22} + 2(1+\nu)(\sigma^A_{12})^2\right] \tag{15}$$

$$f^{AC} = \frac{1}{2E}\left[\sigma^C_{11}\sigma^A_{11} + \sigma^C_{22}\sigma^A_{22} - 2\nu\sigma^C_{11}\sigma^A_{22} + 2(1+\nu)\sigma^C_{12}\sigma^A_{12}\right]$$

Substituting (13), (14) and (15) into (12), we obtain the energy release rate due to craze zone translation:

$$-\frac{\partial \Pi}{\partial x_{1c}} = J^A_1 + J^{AC}_1 \tag{16}$$

where

$$J^A_1 = \int_{\Gamma_A} P^A_{1k}\, n_k \, d\Gamma \tag{17}$$

$$J^{AC}_1 = \int_{\Gamma_A} P^{AC}_{1k}\, n_k \, d\Gamma \tag{18}$$

The integral path Γ_A is chosen as shown in Fig.2, where the line OA and OA′ constitute the trailing edge of the damage zone (Chudnovsky, 1988). An appropriate regularization of the integral is undertaken in the numerical realization. The average craze opening bo= 0.51 (μm) in the peripheral part of the damage zone is evaluated experimentally.

The integral $J_1{}^A$ possesses path invariance with respect to the part of the integration contour outside of V_A. The total energy release rate for crack extension and damage zone translation as a function of crack length is shown in Fig.4.

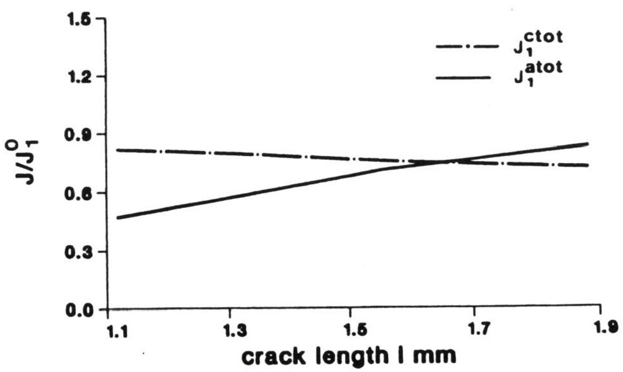


Fig.4. The energy release rates J_1^{ctot}, J_1^{atot}, normalized by J_1^{0}

DISCUSSION AND CONCLUSION

1. This work was initially thought of as energy analysis of a crack-damage interaction based on the semi-empirical stress analysis. However, crazing within the core of damage turns out to be too dense to resolve the craze opening in a relatively thick specimen. Thus, an assumption about the absence of stress singularity at the crack tip was utilized to evaluate an average craze opening density at the core crazing. It yields a crack opening profile resembling that observed experimentally and thereby gives indirect support for the assumption.

2. Damage shields the main crack in the following sense: it reduces the elastic energy release due to main crack extension with a stationary damage zone. In other words, damage reduces the energy available for main crack advance. The shielding effect is produced by a core of crazing.

3. The peripheral part of the damage zone is the main contributor to this energy release rate associated with the damage zone growth with a stationary main crack. The energy release rate due to the craze zone translation can be decomposed into J_1^{AC} a portion exclusively due to the crack-damage interaction and J_1^A, a portion which mainly reflects self-action of the damage zone but also includes the interaction.

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