Damage Equations for Physically-based Creep Life

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ABSTRACT

Continuum Damage Mechanics (CDM) provides a methodology for assessing the remanent creep life of engineering components. CDM can be used either as a computer-based design method or as a diagnostic tool for calcualting remanent life. To discriminate between differences in the responses of materials to service conditions, the damage evolution and creep constitutive equations in CDM need to be based on the physical mechanisms responsible for the damage processes. The appropriate physical mechanism can be diagnosed by measuring certain parameters from the shape of the uniaxial tensile creep curve and applying a set of simple rules, and the results can be used to formulate constitutive equations for multiaxial stress states.

Continuum Damage Mechanics

Creep lifetime in materials is controlled by the evolution with time of either mechanical or microstructural instabilities which manifest themselves as the tertiary stage of creep. The evolution of strain and damage as a function of time can be determined using the formalism of Continuum Damage Mechanics (CDM) (1-4).

$$\dot{\varepsilon} = f(\sigma, T, \omega) \quad (1)$$

$$\dot{\omega} = g(\sigma, T, \omega) \quad (2)$$

where σ is the stress, T the temperature and ω is a parameter which represents the degree of damage. Equations (1) and (2) are of a form that is particularly useful for computer-solutions of complicated mechanical and thermal loading patterns. They are also of a form, as shown later, that occurs naturally from considerations of the various physical mechanisms of tertiary creep. The constitutive equations formed on the basis of physical mechanisms have the advantage that the damage parameter ω has a precise physical interpretation and the limitation and usefulness of extrapolation techniques can be clearly defined. However, such equations are generally unable to model precisely the results of mechanical testing and it is at this stage that a "tuning" or renormalization of the equations based on physical models is required. An additional advantage of the physical models is that they provide strong insight into the likely behavior of materials when subjected to multi-axial stress states. Multi-axial stress mechanical testing which is difficult and expensive can be significantly reduced or even eliminated by making use of the understanding of the physical mechanisms.

Mechanisms and Mechanics of Tertiary Creep

There is an extensive literature concerned with the physical causes of tertiary creep in metallic alloys: most of it, particularly that which is quantitative, relates to one mechanism, grain boundary cavitation - which also causes intergranular fracture, and less attention has been paid to identifying other mechanisms although some progress has been made. Other causes of tertiary creep have been recently quantified (6,7) and also put within the framework of CDM (8,9). Table 1 is a state-of-the-art summary of the mechanisms and mechanics of tertiary creep, set within the framework of CDM: it is based on the work of Ashby and Dyson (7), but contains a number of differences. The physical mechanisms of tertiary creep have been placed under three categories to reflect the fact that a Strain-Softening mechanism depends on σ, T and ω, whereas a Thermal-Softening mechanism depends on time and temperature only. The Environmental-Softening category contains both a Strain- and Thermal-Softening mechanism but has been identified separately because the damage evolution rate depends inversely on section size, in contrast to all other mechanisms. The symbols used in Table 1 can be found in Ashby and Dyson (7) except for those used to describe the dislocation-substructure mechanism and the two cavitation mechanisms. These additional symbols are listed in Table 2.

The dislocation-substructure mechanism was first described by Dyson and McLean (6) and the particular form given in Table 1 is different from that used in previous publications (6,9) because the damage parameter has been redefined so that it falls within the range $0 < \omega < 1$, rather than being unbounded. The different definitions of damage and damage rate are related by the transformations:

$$\omega = 1 - \exp(-s)$$
$$\dot{\omega} = \exp(-s)\,\dot{s}$$

where $s = \ln(\rho/\rho_i)$ is the previous definition of damage.

Table 1. Creep Damage Categories and Mechanisms

CREEP DAMAGE CATEGORY	MECHANISM	DAMAGE PARAMETER ω	DAMAGE EVOLUTION RATE $\dot{\omega}$	STRAIN RATE $\dot{\varepsilon}$	$\omega_f = 1$: $C_m = \dot{\varepsilon}_m t_f$	$\omega_f = 1$: ε_f	$\omega_f = 1$: λ
STRAIN SOFTENING	Loss of section under constant load	$1 - \frac{A}{A_i}$	$\dot{\varepsilon}_i[\frac{1}{1-\omega}]^{n-1}$	$\dot{\varepsilon}_i[\frac{1}{1-\omega}]^{n}$	$\frac{1}{n}$	-	-
	Dislocation substructure	$1 - \frac{\rho_i}{\rho}$	$C\dot{\varepsilon}_i$	$\dot{\varepsilon}_i[\frac{1}{1-\omega}]$	$\frac{1}{C}$	-	-
	Constrained Cavitation: Growth controlled	$\omega_1 = F$ $\omega_2 = (\frac{r}{h})^3$	$\dot{\omega}_1 = 0$ $\dot{\omega}_2 = \frac{3d\dot{\varepsilon}}{4h}$	$\dot{\varepsilon}_i[\frac{1}{1-\phi\omega_1}]^{n}$	$\frac{4h}{3d}$	$\frac{4h}{3d}$	1
	Constrained Cavitation: Nucleation controlled	F	$K\dot{\varepsilon}$	$\dot{\varepsilon}_i[\frac{1}{1-\phi\omega}]^{n}$	$\approx \frac{1}{K\phi(n+1)}$	$\frac{1}{K}$	$\approx\phi(n+1)$
THERMAL SOFTENING	Particle coarsening	$1 - \frac{S_i}{S}$	$(1-\omega)^4 \frac{\kappa'}{3}$	$\dot{\varepsilon}_{i\rho}(1+\kappa''\omega)^n$	-	-	-
ENVIRONMENTAL SOFTENING	Fracture of corrosion-product	$\frac{2x}{R}$	$\dot{\omega}_{ic}[\frac{1}{1-\omega}]^{n}$	$\dot{\varepsilon}_i[\frac{1}{1-\omega}]^{n}$	Damage parameter only applicable when $2x \ll R$		
	Internal oxidation	$(\frac{2x}{R})^2$	$\frac{4\kappa}{R^2}$	$\dot{\varepsilon}_i[\frac{1}{1-f\omega}]^{n}$	In practice, other mechanisms will intervene long before $\omega = 1$		

Table 2. Symbols used in Table 1 and not identified previously

ρ, ρ_i:	dislocation density; subscript i denotes initial value
C:	empirical constant
F:	fraction of grains containing a cavitated boundary
ϕ:	fractional area occupied by transverse boundaries
d:	grain size
r:	cavity radius
2h:	cavity spacing
κ:	empirical rate constant

There are many different cavitation mechanisms and these can be found in the CDM format in previous publications (8,9,10). The two included in Table 1 are for creep-constrained cavitation, which is believed to be more appropriate to the conditions experienced in service viz, low levels of stress and, often, triaxial states of tensile stress.

When $\dot{\omega}$ and $\dot{\varepsilon}$ are integrated as a coupled pair under constant temperature and loading conditions, expressions are obtained for: (i) the Monkman-Grant parameter, $C_m = \dot{\varepsilon}_i t_f$; (ii) the strain at failure ε_f and therefore, (iii) the creep damage tolerance parameter (12), $\lambda = \varepsilon_f/C_m$. In Table 1, "failure" is always taken to be at $\omega = 1$ and so these are upper bound values for each mechanism acting alone.

In practice, it is unlikely that one damage mechanism will operate alone. For example, although the Hoff (13) limit of $C_m = 1/n$ is predicted correctly in the first mechanism in Table 1, the value of $\varepsilon_f = \infty$ is never found. The simplest procedure is to assume that a second damage mechanism affects only ε_f and does not influence the primary damage evolution rate nor the strain rate. Ashby and Dyson (7) assumed that this happened for the necking instability and gave:

$$\varepsilon_f > \frac{2}{n-1} \tag{4}$$

and therefore

$$\lambda > \frac{2n}{n-1} \tag{5}$$

while still retaining $C_m = 1/n$.

In a similar manner, for dislocation strain-softening materials, $C_m = 1/C$ where C is, as yet, an empirical parameter that characterises material-class. It lies between 30 and 50 for D.S. nickel-base superalloys and appears to be as high as 200 for the aluminum alloy R-R 58. Dyson, Leckie, Shawki (14) have shown that materials obeying the creep constitutive law for dislocation strain-softening are susceptible to the necking instability, and that the condition for instability, is

$$\varepsilon_f > \frac{2}{C-1} \tag{6}$$

The possibility of synergy occurring between two damage mechanisms appears to have been considered just once (11) and then only approximately.

Identifying the dominant damage process

Ashby and Dyson (7) suggested that the creep damage tolerance factor $|\lambda|$ could be used to diagnose whether or not cavitation was the dominant damage mechanism responsible for tertiary creep. The idea is illustrated in Fig. 1 where creep strain is plotted as a function of the dimensionless parameter $\dot{\varepsilon}_i t$ for three classes of material. The data are compared with tertiary models for caviation which predict that creep curves all lie within the range $1 \le \lambda \le 2\text{-}3$, regardless of the detailed mechanism (Table 1 and Ref. 7). D.S. nickel-base superalloys fall well outside the cavitation regime as they should since they were the seminal example of dislocation strain-softening (6): their behaviour appears to be delineated by $30 \le C \le 50$. These alloys fail in tension after developing noticeable necking at strains $\le 15\%$ and yet Eq. 4 predicts that necking should not begin until $\ge 40\%$ strain. Their behaviour is more consistent with the suggestion that Eqs. 6 may be more appropriate for these materials.

The aluminum alloy R-R 58 also falls outside the cavitation regime - the insert shows this more clearly - even though the fracture strain is only 2%. Failure in this alloy is along planes of maximum shear stress in both uniaxial tension and compression and compressive response is identical to tensile. Contrary to an earlier proposal (10) we believe that fracture in this alloy is due to a mechanical instability because of its very high C value of 200. Interestingly, Eq. 6 predicts that the alloy should develop an instability at a strain of 2%, which is identical to the observed fracture strain.

The Type 316 SS is an example of a poorly cavitating material which damages due to the increasing stress at constant load and fails due to necking at the strain predicted by Eq. 4. It provides an upper bound to life for a power law creeping solid under constant load.

Thermal-softening mechanisms can easily be distinguished from others of high $|\lambda|$ simply by pre-aging the material prior to creep: in contrast to its effect on the dislocation strain-softening mechanism, large decreases in creep resistance will be found.

Continuum Constitutive Equations

The continuum constitution equations should be written in such a way that they reflect the physical damage described in Table 1, be capable of describing the tensorial nature of damage and its growth under multi axial stress states and be in a form convenient for fitting experimental data. Equations which to large measure achieve these objectives have the form

$$\dot{\varepsilon}_{ij} = \frac{\partial \Phi}{\partial \sigma_{ij}} \tag{7}$$

where the potential Φ has the form

$$\Phi = \frac{\dot{\varepsilon}_o}{n+1}\left(\frac{\sigma_e}{\sigma_o(1-\omega)}\right)^{n+1}\left[1+\rho\left(\frac{\sigma_I}{\sigma_o}\right)^2\right] \tag{8}$$

and the damage rate is given by

$$\dot{\omega} = \frac{\dot{\omega}_o}{(1-\omega)^{\psi}}\left[\frac{\Delta(\sigma_{ij})}{\sigma_o}\right]^{\nu} \tag{9}$$

In these expressions $\dot{\omega}_o$, n, ψ and ν are constants which can be determined from uniaxial data and $\Delta(\sigma_{ij})$ describes the shape of the Isochronous Surface in stress space, which is equal to σ_I for a maximum principal stress material and σ_e for an effective stress material. The quantity ρ is a measure of the density of crack-like features and is introduced to allow for volumetric effects although these are generally small and are frequently neglected. The damage parameter is interpreted according to the description given in Table 1. The tensorial description of damage does require some discussion however.

The work of Onat and Leckie (11) has demonstrated that damage can be represented by even order irreducible tensors. In the case of damage which takes the form of change in dislocation substructure the damage is isotropic and can therefore be represented as a scalar in Eq. 9 Furthermore we expect for materials which soften according to this form of damage that the function $\Delta(\sigma_{ij})$ is equal to the effective stress σ_e and that λ shall be large.

By contrast when material damage is that of cavity growth, damage occurs on planes perpendicular to the principal stress and is highly anisotropic. In such circumstances it is necessary to identify damage with a number of directions and methods for dealing with this are described in (12). For such materials we expect that $\Delta(\sigma_{ij}) = \sigma_I$ that λ will be small and that the damage is highly anisotropic. In fact it would appear that the value of λ is helpful not only in identifying the physical damage mechanism but in establishing the tensorial form of damage and the form of constitutive equation. The results of non-proportional multi-axial stress loadings on copper and precipitate hardened aluminum alloys (13) provides some evidence in support of this postulate.

SUMMARY

Creep damage mechanisms are presented in a unified form suitable for the development of constitutive/damage equations appropriate for continuum calculations which retain the physical features of the damaging process. The importance of the creep damage tolerance factor λ in identifying physical mechanisms and establishing constitutive equations is disucssed.

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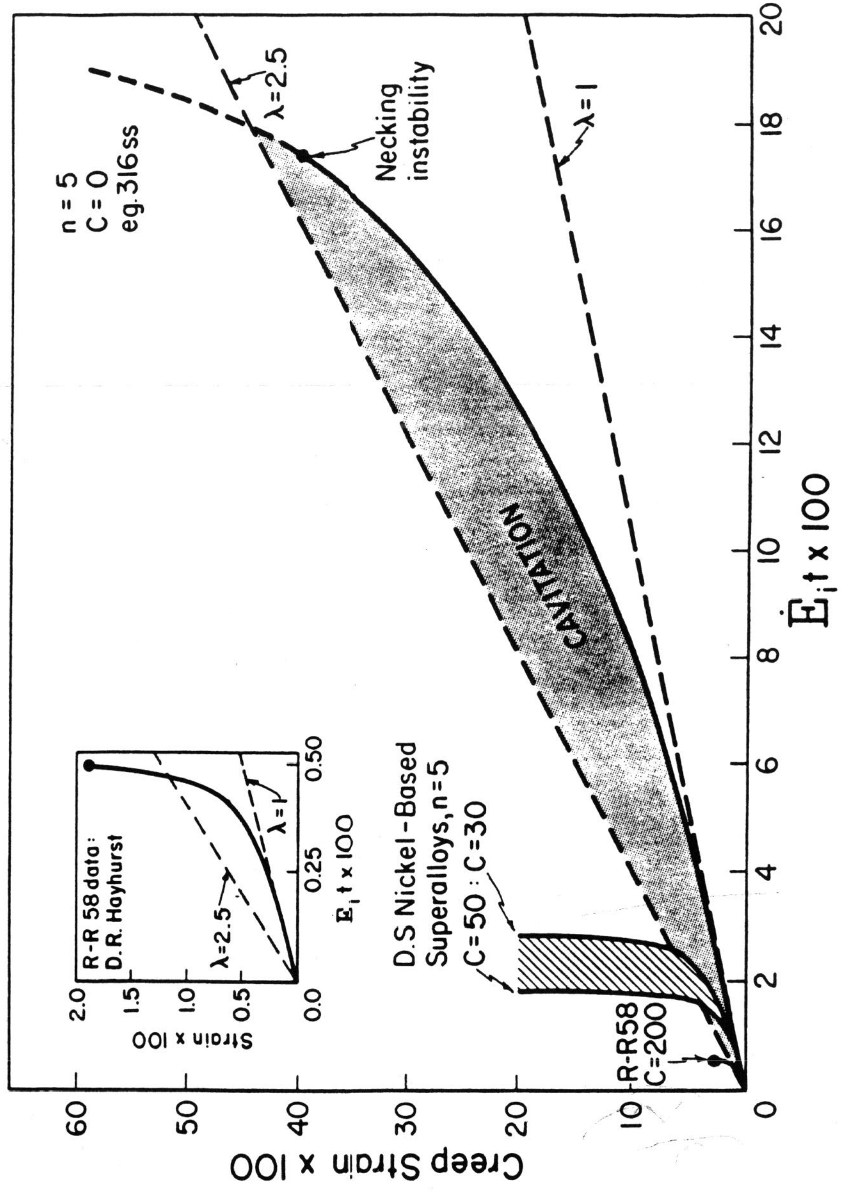

Fig. 1. Diagnostic Diagram