

Damage Analysis of Materials with Microvoids

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INTRODUCTION

Generally practical materials have some microflaws or macroflaws. It is an important to analyze the macroscopic behavior of these materials with Damage Mechanics.

Kachanov⁽¹⁾, Lemaitre and chaboche⁽²⁾ introduced the effective stress and described the effect of microflaws on the macroscopic behavior of materials. Rousselier⁽³⁾ assumed that damage variable is associated with material density. McClintock⁽⁴⁾, Rice and Tracey⁽⁵⁾ analyzed the damage materials with microscopic model. An anisotropic damage variable which has more general sense is given with associative method of macroscopic and microscopic models in this paper. The differential equation of state function for the anisotropic damage with microvoids is given and its approximate solution is obtained. For the special case of spherical voids, the final result is compared with Rousselier's result. It is also pointed out that plastic potential is associated with the stress state and growth way of microvoids.

ANISOTROPIC DAMAGE VARIABLE

Lemaitre⁽⁶⁾ considered the free energy function as follows

$$\psi = \psi_e(\underline{\epsilon}^e, D) + \psi_p(p) \quad (1)$$

where ψ_e is elastic energy density associated with damage; ψ_p is the free energy associated with accumulative plastic deformation. ψ_e is given as

$$\psi_e = \frac{1}{2\bar{f}} \underline{\epsilon}^e : \underline{c} : \underline{\epsilon}^e (1-D) \quad (2)$$

let $\underline{\tilde{c}} = \underline{c}(1-D)$, then equation (2) becomes

$$\psi_e = \frac{1}{2\bar{c}} \underline{\varepsilon}^e : \underline{\tilde{c}} : \underline{\varepsilon}^e \quad (3)$$

$\underline{\tilde{c}}$ is the effective elastic constant. Damage is the state variable which has an effect on the elastic constant according to equation (3). It is also obtained as

$$\underline{\sigma} = \underline{c} : (\underline{I} - \underline{D}) : \underline{\varepsilon}^e = \underline{\tilde{c}} : \underline{\varepsilon}^e \quad (4)$$

The damage variable \underline{D} can be determined from $\underline{\tilde{c}}$.

Consider an infinite body including the finite inhomogeneous region with the ellipsoidal voids of same orientation and the matrix of the body is an isotropic medium as shown in Fig.1.

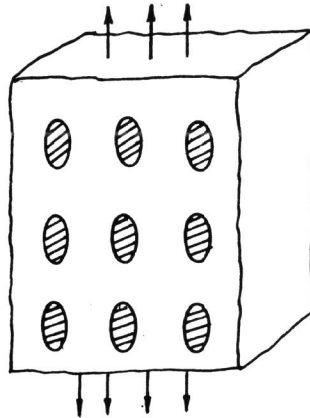


Fig.1

Refs. [7] and [8] were given the solutions of problems of single and more inclusions, respectively. Let Σ_{ij} and E_{ij} be the global stress and strain of the body, then the strain energy is as [7, 8]

$$W = \frac{1}{2} \tilde{C}_{ijkl} \Sigma_{ij} \Sigma_{kl} \quad (5)$$

\tilde{C}_{ijkl} indicates the effective elastic modulus. It is as

$$\tilde{C}_{ijkl}^{-1} = C_{ijkl}^{-1} + f A_{ijmn} C_{mnlk}^{-1} \quad (6)$$

C_{ijkl}^{-1} , f and A_{ijmn} are elastic modulus of matrix, ellipsoidal inclusion volume fraction and coefficient associated with the size and volume fraction of ellipsoidal voids [9]. In the form of tensor,

$$\underline{\tilde{c}}^{-1} = \underline{c}^{-1} + f \underline{A} : \underline{c}^{-1} \quad (7)$$

From equation (4), the anisotropic damage variable is as

$$\underline{D} = \underline{I} - \underline{c}^{-1} : (\underline{c}^{-1} + f \underline{A} : \underline{c}^{-1})^{-1} \quad (8)$$

It is noticed that the damage variable depends on the volume fraction and size of ellipsoidal voids. So \underline{D} can be assumed as follows

$$\underline{D} = \underline{D}(\gamma, f) \quad (9)$$

where γ is the ratio between the long and short axis. \underline{D} can be assumed the simple function of γ and f for some special problems.

DIFFERENTIAL EQUATION OF THE STATE FUNCTION

In this paper, the following assumption are made:

- a) The flaws are the forms of microvoids after yielding;
- b) The material with damage is of the general standard material;
- c) The damage parameter is an independent internal variable of irreversible thermodynamics and satisfies continuance conditions.

If the growth of microvoids is isotropic, damage dissipation of energy consists of two parts: a) work done by external force for the growth of microvoids; b) the dissipation of energy forming new surface. Let W_g indicate the sum of two parts.

For an infinite homogeneous medium with an ellipsoidal void, W_g is expressed as

$$W_g = \frac{\sigma_m}{f} \frac{\dot{f}}{1-f} + G \cdot \dot{S} \quad (10)$$

where σ_m , G and S are the average stress, surface energy of unit area and area of microvoids. In fact, $G \ll \sigma_m$ in quantity, so

$$W_g = \frac{\sigma_m}{f} \frac{\dot{f}}{1-f} \quad (11)$$

The state function can be divided into two parts. One is free energy shown in equation (3). Another one is potential of dissipation which can be assumed as

$$\psi = \psi_1(p) + \psi_2(\gamma, f, \varepsilon_{ij}^e) \quad (12)$$

From equation (12), we have

$$B_\gamma^{(d)} = \frac{\partial \psi_2}{\partial \gamma}, B_f^{(d)} = \frac{\partial \psi_2}{\partial f}, A = \frac{\partial \psi_1}{\partial p} \quad (13)$$

Substituting eqn. (13) into (11), one obtains

$$B_\gamma^{(d)} \dot{\gamma} + B_f^{(d)} \dot{f} = \frac{\sigma_m}{f} \frac{\dot{f}}{1-f} \quad (14)$$

$\gamma=0$ when the growth of ellipsoidal voids is isotropic. Then

$$\frac{\partial \psi_2}{\partial f} = \frac{1}{f} \sigma_m \frac{1}{1-f} = \left(\frac{\partial \psi_1}{\partial \varepsilon_{ij}^e} + \frac{\partial \psi_2}{\partial \varepsilon_{ij}^e} \right) \frac{1}{3(1-f)} \quad (15)$$

According to eqn. (3), one gets

$$\frac{\partial \psi_e}{\partial \epsilon_{mn}^e} = \frac{1}{3} \tilde{C}_{mnkl} \epsilon_{kl} \quad (16)$$

Let $\tilde{C}_{mnkl} = K_{kl}(\gamma, f)$ be known function, take approximate solution of ψ_2

$$\psi_2 = f_1(\gamma, f) + f_{ij}(\gamma, f) \epsilon_{ij}^e \quad (17)$$

Substituting eqn. (17) into (15), one obtains

$$\frac{\partial f_i}{\partial f} = \frac{1}{1-f} f_{ii}(\gamma, f), \quad \frac{\partial f_{ij}}{\partial f} = \frac{1}{(1-f)} K_{ij}(\gamma, f) \quad (18)$$

In the case of determined γ and f , ψ_2 can be obtained by equ. (18).

PROBLEM OF SPHERICAL VOIDS

Assume spherical voids does not affect on elastic modulus of material, then

$$\psi_e = \frac{1}{2f} \epsilon_{ij}^e \epsilon_{ijkl}^e \epsilon_{ij}^e, \quad \psi = \psi_1(\rho) + \psi_2(\beta, \epsilon_m^e) \quad (19)$$

β is damage variable of microvoids. When $\gamma=1$, then

$$\beta = \beta(f) \quad (20)$$

According to the constitutive relation, we have

$$\sigma_{ij} = \frac{\partial \psi_e}{\partial \epsilon_{ij}^e} + \frac{\partial \psi_2}{\partial \epsilon_{ij}^e}, \quad B = \frac{\partial \psi_2}{\partial \beta}, \quad A = \frac{\partial \psi_1}{\partial \rho} \quad (21)$$

From eqn. (19) and (21), one obtains

$$\frac{\sigma_m}{f} = K \epsilon_m^e + \frac{1}{3} \frac{\partial \psi_2}{\partial \epsilon_m^e} \quad (22)$$

where $K = E/3(1-\mu)$. Substituting eqn. (22) into eqn. (15), the following equation is given

$$f \frac{\partial \psi_2}{\partial \beta} \dot{\beta} = \left(\frac{1}{3} \frac{\partial \psi_2}{\partial \epsilon_m^e} + K \epsilon_m^e \right) \frac{1}{1-f} \quad (23)$$

Generality is not lost, let $\beta = \beta(f)$, then

$$\psi_2 = -\frac{1}{f} K \beta \epsilon_m^e + \frac{K}{6} \beta^2 \quad (24)$$

Equation of $\psi = \psi_1(\rho) + \psi_2(\beta)$ is chosen and eqn. (14) is not mentioned in Ref. [3].

The discussion of plastic potential is based on semianalytical assumption. It is reasonable that ψ_2 is function of β and ϵ_m^e .

Take $\beta = \beta(f)$, one obtains

$$B'(\rho) \psi_2(\beta) = -\frac{\sigma_m}{f} \quad (25)$$

The difference between eqn. (26) and result of Ref. [3] is that the right term of Ref. [3] is σ_0/c . In fact, the result of Ref. [3] is the approximate result in the case of perfectly plasticity.

Take plastic potential as general form as

$$F\left(\frac{\sigma}{f}, \psi_1(\rho), \psi_2(\beta), \beta\right) = \left[J_2\left(\frac{\sigma}{f}\right) \right]^{\frac{1}{2}} + \frac{\psi_1(\rho)}{3} + F_1(\psi_2, \beta) \quad (26)$$

The plastic potential of isotropic growth of spherical voids can be obtained with similar method of Ref. 3 and it is as follows

$${}^1F = \left[J_2\left(\frac{\sigma}{f}\right) \right]^{\frac{1}{2}} + \frac{\psi_1(\rho)}{\sqrt{3}} + 0.327 f \exp\left(\frac{3\psi_2}{2\sigma_0}\right) \quad (27)$$

Actually, the growth of microvoids depends on the stress state. The plastic potential of spherical voids in the case of uniaxial tension is as

$${}^3F_1 = \frac{3f}{(3-5f)} \left[\frac{25B^2}{16} + 0.644\sigma_0 \cosh\left(\frac{3B}{2\sigma_0}\right) + 0.009\sigma_0 \sinh\left(\frac{3B}{2\sigma_0}\right) \right] \quad (28)$$

where $A = \psi_1(\rho)$, $B = \psi_2(\beta)$, $\sqrt{J_2} = \sigma_0/3$ if the matrix is perfectly plastic medium. When the last term of eqn. (28) can be neglected, plastic potential is as

$${}^3F = \left[J_2\left(\frac{\sigma}{f}\right) \right]^{\frac{1}{2}} + A/\sqrt{3} + \frac{3f}{(3-5f)} \left[\frac{25}{16} B^2 + 0.644\sigma_0 \cosh\left(\frac{3B}{2\sigma_0}\right) \right] \quad (29)$$

CONCLUSION AND DISCUSSION

An anisotropic damage variable which has more general sense was obtained by the associative method of macro-microscopical models. Present damage variable is associated with the size of microvoids. The state function taken in this paper consists of reversible and irreversible parts. The differential equation of state function was obtained. The plastic potential of spherical voids in the case of isotropic growth was given. The plastic potential of anisotropic growth of spherical voids was also given in this paper.

It is noticed that the solution of differential equation for multivariable damage problem is not closed. The general analytical solution of ψ should be given in next study.

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