Crack Tip Parameters for a Central Crack in an Elastic Plastic Plate Under Mixed Mode and Repetitive Loading

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ABSTRACT

The repair or replacement of defective structures can be avoided if it can be shown that the defects will not grow in service. Methods are required for assessing the initiation of crack growth in service conditions in terms of the initiation of crack growth in test conditions. Of the parameters which characterise conditions at the crack tip, those which have been calculated most accurately have been contour integrals around the tip. Hellen (1988) has recently developed an infinitesimal virtual crack extension method which calculates what, for a suitable choice of contour, would be an equivalent parameter for a non linear elastic material.

The present note compares these parameters for an elastic plastic material under mixed mode and repetitive loading to obtain some of the information, on the basis of which, guidance can be given as to when the new method and when the contour integral method should be used to correlate test and service conditions.

KEYWORDS

Elastic Plastic Fracture; Contour integrals; Virtual crack extension.

INTRODUCTION

Hellen and Blackburn (1987) have recently reviewed the determination of parameters characterising conditions at crack tips, and have noted 81 cases depending on whether: a) the loading is tearing, sliding, mixed mode; b) the crack is stationary, growing quasi statically, growing dynamically; c) the geometry is two dimensional, axisymmetric, three dimensional; d) the material response may be treated as elastic, time independent, time dependent. In some cases the choice of most appropriate parameter may also depend on whether the near tip loading is radial, monotonic or non monotonic.

In numerical investigations, as when comparing test and service conditions, it is highly desirable that the selected parameters should be computationally robust. Two general techniques produce parameters with this property, integration of a contour integral around the crack tip (especially if the result is approximately path independent), and variation of an expression such as the energy, with respect to the coordinates of the crack tip position. In the simplest case, i.e. the first condition for a, b, c and d above, the contour integral method, Rice (1968), and the virtual crack extension method, Hellen (1975) and Parks (1977), give rise to the same parameter, and this parameter can be evaluated also by taking the difference in energy for two slightly different sizes of crack, de Lorenzi (1982). The usual contour integral used for this simplest case is

$$J^*_{w1} = \int\{W N_1 - t_{ij} N_j \frac{\partial u_i}{\partial x_1}\} ds$$

for a contour around the tip, where W is the strain energy density, N_i are direction cosines of the normal to the contour, u_i are displacement components, t_{ij} stress components (transpose of first Piola-Kirchoff stress in the case of finite deformations), and x_i are Cartesian coordinates with x_3 in the direction of the crack edge and x_2 in the direction normal to the plane of the crack. Because the area near the crack tip is that in which the stresses are calculated least accurately, the computation should be carried out on a more distant contour, with, if necessary, a contribution over the area in between the contours from an integrand obtained from Green's divergence theorem. An improved technique suggested by Li et al (1985) and extended by Shih et al (1986) is to multiply the integrand by a function g which is unity on the inner contour and zero on the selected outer contour. Thus when determining the integral on the inner contour by evaluating it at a distance, the integral on the outer contour is zero. The penalty ensuing is that the integrand in the area in between is more complex, although it can be evaluated from the values at Gauss points where the integrand is likely to be more accurate than on the outer contour. The physical significance of the contour integral is that as the contour shrinks on to a flat process zone at the crack tip the first term in the integrand does not contribute to the value of the integral, so that the value of the contour integral is $-\int t_{ij}N_j du_i$ around the flat process zone. This is the work done in producing new surface on each face of the crack per unit of crack extension. In the simplest case this contour integral is path independent. For more complex cases this need not be so, and an alternative integral $J^*_1 = \int\{\frac{1}{2} t_{ij} (\frac{\partial u_j}{\partial x_i} - \theta_{ji})N_1 - t_{ij}N_j \frac{\partial u_i}{\partial x_1}\}ds$, which also has the same limit for a flat process zone, has been proposed, Blackburn (1972), where θ_{ji} is the thermal strain. This is equivalent to J^*_{w1} for a linear elastic material, and has been suggested, Blackburn (1985), as being more relevant for complex conditions such as repetitive loading. It can be computed away from the tip with a surface integral over the area in between, either directly or by multiplying by the function g which is unity on the inner contour and zero on the selected outer contour.

These integrals are, as shown by Knowles and Sternberg (1971) for J^*_{w1}, the component in the x_1 direction of vector contour integrals. The component in the x_2 direction can, in principle, be calculated similarly. In practice there is the difficulty that, whereas for the x_1 component, when there is no loading on the crack face, both near and far contours can usually be assumed to start and finish on the crack face, this is not so for the x_2 component, except when it is zero, as for single mode loading.

Hence integration has to be carried out on the part of the contour which is on the crack face.

The alternative approach for elastic materials determines quantities G_1, and G_2 by taking the energy difference for small differences in the coordinates at various points on the crack edge. Hellen (1975) and Parks (1977) showed how this could be done with just the one computation rather than with a separate calculation for each crack edge position. Haber and Koh (1985) and Sussman and Bathe (1985) showed how the loss of accuracy associated with taking small differences can be eliminated by carrying out the differentiation analytically. de Lorenzi (1982), Li et al (1985) and Shih et al (1986) have shown the equivalence of the contour integral and virtual crack extension parameters when the material can be treated as elastic. These techniques have been incorporated into BERSAFE and applied to mode I radial loading of elastic plastic material, Hellen (1988). The next question to be discussed is the extension to elastic plastic materials under a non radial loading.

RELATIONSHIP BETWEEN CONTOUR INTEGRALS AND ENERGY VARIATIONS

It is useful to start with a resumé of the elastic case presented by de Lorenzi (1982). For simplicity this summary of his results is confined here to a two dimensional geometry with no body forces or thermal or internal strains and with infinitesimal deformations, though all these restrictions can be removed. For an increment δx_o of the crack tip, he associates a continuous transformation of the neighbouring material by an amount δx_k. He considers in particular the case when a number of layers of elements around the tip move by δx_o, there is then a transition layer and then the elements beyond are not transformed. Thus only part of the body need be analysed, the displacements being fixed but unspecified on the boundaries which are not free surfaces. For an elastic body, the method attempts to calculate $\iint \frac{\partial}{\partial \delta x_o}$ (WdS) where W is the strain energy density, taken over the transformed parts of the body. He showed that $\frac{\partial}{\partial \delta x_o}$ (WdS) = $(t_{ij}\frac{\partial u_i}{\partial x_k} - W\delta_{jk}) \frac{\partial}{\partial \delta x_o} (\frac{\partial \delta x_k}{\partial x_j})$ dS, where the term $-\delta_{jk}$ arises from the differentiation of dS. Li et al (1985) showed that for a crack tip displacement for which $\delta x_1 = \delta x_o$ at the tip, this gives the same result as the contour integration method with g as $\delta x_1/\delta x_o$ for variable δx_1, as in the particular case of a transition layer (or thick contour). For $\delta x_o = \delta x_2$ at the tip, however, the part of the contour on the crack face cannot be ignored unless W is symmetric across it. The value of the integral may then depend on the size of the transformed zone. In the limit when this is small the value should be the same as J^*_{w2}. On using the path independent property of J^*_{w2}, if the crack face is included in the contour, the previous expression with g as $\frac{\delta x_2}{\delta x_o}$ is recovered along with a contour integral along the faces of the difference in Wg. A thorough discussion of the relationship between the integrands of the contour integral and variational transformations of part of an elastic material has been presented by Hill (1986).

For other materials W may be replaced by $\sum \Delta W$, where $\Delta W = t_{ij}\Delta \frac{\partial u_i}{\partial x_j}$, in both the contour integral and variational methods. In the latter case, one of two alternative assumptions is made to allow further progress, either $\sum \Delta W$

is a functions of $\frac{\partial u_i}{\partial x_j}$ only, or ΔW is a function of $\Delta \frac{\partial u_j}{\partial x_j}$ only, in each case t_{ij} being the partial derivative. When one of these assumptions is valid the integrals transform to either $J^*_{wk} = \int (t_{ij} \frac{\partial u_i}{\partial x_k} N_j - \sum \Delta W\ N_k) ds$ or $\sum \Delta T_{ck} = \int \sum (t_{ij} \frac{\partial \Delta u_i}{\partial x_k} N_j - \Delta W\ N_k) ds$ where the contour integral is around the boundary of the transformed region. The function g can be incorporated into the integrand, and the contour integral, except for the part on the crack face for k = 2, transformed back into a surface integral as in the elastic case. The integral $\sum \Delta T_{ck}$ has been discussed by Stonesifer and Atluri (1982). Alternatively under one of the above assumptions, by the same argument as in the elastic case, $\iint \frac{\partial}{\partial \delta x_o} (\sum \Delta W\ dS)$ becomes either

$$\iint (t_{ij} \frac{\partial u_i}{\partial x_k} - \sum \Delta W\ \delta_{jk}) \frac{\partial}{\partial \delta x_o} (\frac{\partial \delta x_k}{\partial x_j}) dS \text{ or } \iint \sum (t_{ij} \frac{\partial \Delta u_i}{\partial x_k} - \Delta W\ \delta_{jk}) \frac{\partial}{\partial \delta x_j} (\frac{\partial \delta x_k}{\partial x_o}) dS.$$

These equal the surface integral parts of J^*_{wk} and $\sum \Delta T_{ck}$ with g as $\frac{\delta x_k}{\delta x_o}$.

For an elastic plastic material under complex loading, more than one crack tip parameter may be required to characterise crack tip conditions, Blackburn (1985). In the contour integral context it is generally agreed; a) that an important parameter is that identified by Rice and Drucker (1967) as the integral over the boundary of what has subsequently been called the process zone, of $t_{ij} N_j \frac{\partial u_i}{\partial x_1}$; and b) that numerical robustness is required in evaluating this, and hence it should be done away from the process zone. For a homogeneous elastic material, it may be done by the contour integral J^*_{w1}, which, for the simple case referred to above, is path independent and thus may be evaluated well away from the crack tip.

Alternative integrals for more complex conditions can be obtained by replacing $\sum \Delta W$ by other functions, e.g. $\frac{1}{2} t_{ij} \frac{\partial u_i}{\partial x_j}$ so that the integral does not involve past history but only the current stress and strain, Blackburn virtual crack extension context there is a similar variety of possible integrands V. They can be evaluated as $\iint \frac{\partial}{\partial \delta x_o} (V dS)$ or $\iint (t_{ij} \frac{\partial u_i}{\partial x_k} - V \delta_{jk})$ (1972). As noted by Blackburn (1985), the contour integral $\sum \Delta T_{c1}$ is associated with the cumulative history of stress and strain. In the $\frac{\partial}{\partial \delta x_o} (\frac{\partial \delta x_k}{\partial x_j}) dS$ or $\iint \sum (t_{ij} \frac{\partial \Delta u_i}{\partial u_k} - \Delta W\ \delta_{jk}) \frac{\partial}{\partial \delta x_o} (\frac{\partial \delta x_k}{\partial x_j}) dS$, but the results will differ unless t_{ij} is $\frac{\partial V}{\partial (\frac{\partial u_i}{\partial x_j})}$ or $\frac{\partial \Delta V}{\partial (\frac{\partial \Delta u_i}{\partial x_j})}$.

Miyazaki et al (1985) interpret the integral J of Kishimoto et al (1980) for an elastic material in such a way that for this simplified case it would be $\iint t_i \frac{\partial}{\partial \delta x_o} (\varepsilon_{ij}\ dS)$ but they seem to evaluate it as if t_{ij} were the partial derivative of $\sum \Delta W$ with respect to $\frac{\partial u_j}{\partial x_i}$ as is done here.

For three dimensional geometries, the contour integral methods may be straightforwardly generalised to evaluate the integral on another contour

in the plane normal to the crack edge, with a surface area integral over the area in between. This can be done either directly or by using the two dimensional g function to eliminate the contour integral part completely. In both cases this gives a value of J^*_{wi} at a specific point (usually a mid edge node) on the crack edge. Alternatively, a three dimensional function g can be used to give a weighted average value of J^*_{wi} in the vicinity of a point on the edge. This is done by choosing g to be 1 at that point and 0 over a surface within the material. Then the integral over the part of the edge within the surface of J^*_{wi} can be evaluated by a volume integral over the volume within the surface. Usually g would be chosen to be non-zero for only one or two elements along the edge, and independent of distance from the edge as far as the transition layer of elements with nodes on the internal surface on which g is zero. When the crack tip meets a free surface the singularity is such that J^*_{wi} is either zero or infinite. An average value over a length of crack edge can be determined however by this last technique. Its value will depend on the length over which it is averaged. Where the volume meets the surface of the body, the volume integral must be supplemented by a surface integral either if the surface is loaded or for components of J^*_{wi} that do not lie in the surface.

For an elastic material, the infinitesimal virtual crack extension method is analogous to the volume integral method with g as $\frac{\delta x_k}{\delta x_o}$ for a transformation δx_k which has the value δx_o at the point about which the averaging has to be taken. If this point is where the edge meets a loaded surface, the work done by these loads must be taken into account. If δx_k is not in a direction is this surface, the difference between the IVCE method and the volume integral method using g will involve the integral of g W on the projection of this surface in the x_k direction, as well as of that of the surface of the crack as, in the two dimensional case.

These considerations suggest that numerical inaccuracy will be the main cause of any difference in the results from the infinitesimal virtual crack extension method and from contour integration with or without g functions in the case of single mode monotonic loading of an elastic plastic material, provided the contour of integration is in a plane normal to the crack edge. The computer program to be used must be adequate for the loading considered, e.g. body forces are still to be incorporated in PLOPPER (Moyser and Hellen, 1985). For most cases though, a choice is available and the decision on the best method will be on the basis of numerical accuracy. This will now be considered for the cases of mode I and mode II and mixed mode monotonic loading and for mode I cyclic loading.

COMPUTATIONS

Numerical investigations of the various crack tip parameters have been carried out by Hellen (1988) for an elastic plastic material using an improved version of BERSAFE (Hellen and Harper, 1985) and PLOPPER (Moyser and Hellen, 1985), and are here extended to various cases of displacement controlled loading of a plate with a small central crack, the cases being mode I, mode II, modes I and II, and repetitive loading. Further cases involving thermal and mechanical loading of plane and axisymmetric geometries are currently being analysed.

BERSAFE evaluates G_1 and G_2 by analytic differentiation of the shape functions, as $\iint (t_{ij} \frac{\partial u_i}{\partial x_k} - \sum \Delta W\ \delta_{jk}) \frac{\partial}{\partial \delta x_o} (\frac{\partial \delta x_k}{\partial x_j}) dS$ for δx_o at the crack tip in the x_1 and x_2 directions, where the integration is over the crack tip elements.

If this method were to be applied to a time dependent material, BERSAFE would increment G_k over a time increment Δt by Δt times a value analogous to the time derivative of J^*_{wk}. The increment should be $\Delta t \frac{dJ^*_{wk}}{dt}$ when $\sum \Delta W$ is a function of $\frac{\partial u_i}{\partial x_j}$ alone, as for example when elastic strains are dominated throughout the transformed region by strains due to steady state secondary creep. This is likely to be relevant to prediction of initiation of creep crack growth, but the subsequent rate of growth is more likely to be related to integrals analogous to ΔT_{cl}, Blackburn (1988).

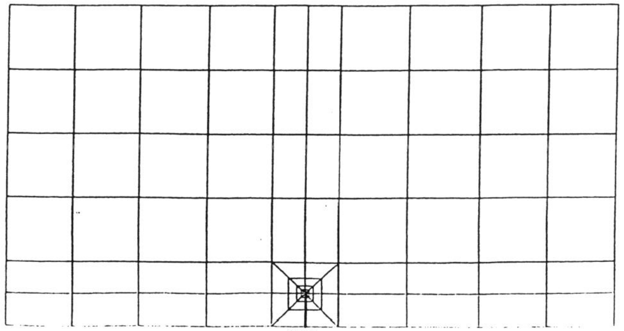

Fig. 1. Mesh representing half of plate

Blackburn (1986) has evaluated contour integrals around the tip of a central crack of size 2a in an approximately square plate for a variety of loadings when the crack size was a tenth of the plate width. A mesh representing half the plate is shown in Fig. 1. It consists of 96 elements with vertex and midedge nodes. The 8 elements at the crack tip are special triangular elements to take account of the elastic square root singularity in the displacement. These elements were 1/16th of the half size of the crack. A condition of antisymmetry was applied to the displacements in the plane which bisects the crack. Displacements were applied to the other boundaries which would produce the following strains in the absence of the crack: A tensile strain normal to the crack plane of 1/600; A shear strain of 0.005; A tensile strain normal to the crack plane of 1/600, and a shear strain of 1/400. With Young's modulus 200,000MPa and Poisson's ratio 0.3 these would have given stresses of 1) $\frac{1,000}{3}$MPa tensile, 2) $\frac{10,000}{13}$MPa shear, 3) $\frac{1,000}{3}$ tensile plus $\frac{10,000}{26}$ shear. The stress plastic strain relationship used by Blackburn (1986) is retained, the yield stress, Y, limit of proportionality, being 240MPa. Contour integration was around the forty elements nearest the tip. Integration along the crack face was included for the mixed mode case.

The results of the contour integration, both in the standard form and (in parenthesis) with the g function method, and of the virtual crack extension method are presented in Tables 1, 2 and 3 for the three cases. For the final case the results are up to when the whole plate becomes plastic when convergence difficulties occurred. For the first case, the results are

Table 1. J^*_{w1} and G_1 for tensile displacement (σ_t stress in uncracked plate)

σ_t/Y	J^*_1	J^*_{w1} (MPamm)	G_1 (MPamm)	$\pi a \sigma_t^2/E$ (MPamm)
0.133	.248	.246 (.254)	.250	.256
0.27	1.042	1.040 (1.073)	1.068	
0.41	2.224	2.385 (2.455)	2.405	
0.55	3.518	4.245 (4.378)	4.232	
0.69	5.033	6.648 (6.906)	6.647	
0.83	7.307	9.874 (10.31)	9.941	
0.97	10.42	14.16 (14.79)	14.80	
1.11	14.96	20.05 (20.91)	20.99	
1.25	19.83	26.26 (27.41)	27.95	
1.39	26.60	33.31 (34.72)	35.54	
1.24	15.49	25.00 (26.39)	22.69	
1.08	4.86	17.30 (18.67)	10.30	
0.93	-5.10	10.25 (11.64)	-1.42	
0.77	-14.23	4.17 (5.60)	-10.96	
0.62	-21.28	0.60 (2.09)	-14.64	
0.46	-26.63	-0.77 (0.81)	-15.96	
0.31	-25.53	1.45 (3.14)	-14.97	
0.16	-24.67	3.63 (5.47)	-13.12	
0	-21.27	6.53 (8.74)	-10.03	
0.16	-14.94	10.75 (12.79)	-1.87	
0.31	-8.01	15.58 (17.47)	6.99	
0.46	-4.08	21.07 (22.83)	16.52	
0.62	7.85	27.20 (28.86)	26.60	
0.77	15.95	32.70 (34.28)	33.19	
0.93	24.07	38.04 (39.55)	38.40	
1.08	28.66	41.12 (42.57)	41.58	
1.24	30.01	44.00 (45.38)	44.53	
1.39	29.60	46.61 (47.89)	46.88	
1.24	18.37	38.32 (39.57)	33.52	
1.08	7.77	30.71 (31.94)	20.71	
0.93	-2.17	23.75 (25.00)	8.56	
0.77	-11.43	17.45 (18.75)	-2.90	
0.62	-19.38	12.90 (14.25)	-9.25	
0.46	-25.39	10.53 (11.97)	-11.67	
0.31	-28.18	11.65 (13.18)	-10.90	
0.16	-24.95	14.57 (16.22)	-8.84	
0	-22.06	17.54 (19.50)	-6.02	
0.16	-15.84	21.50 (23.29)	2.38	
0.31	-8.95	26.09 (27.74)	11.52	
0.46	-1.39	31.34 (32.86)	21.33	
0.62	6.83	37.25 (38.67)	31.81	
0.77	15.45	43.25 (44.60)	40.44	
0.93	23.61	48.54 (49.83)	46.75	
1.08	30.61	52.54 (53.78)	50.38	
1.24	30.79	54.78 (55.98)	53.03	
1.39	31.95	57.79 (58.90)	55.40	

Figures in parentheses are obtained using g function also given when the displacements on the boundary are reduced to zero, then reimposed, reduced to zero and then brought back to their original value.

The analyses were carried out with 10 or 9 steps with 5% tolerance. For each step a partial (80%) tangent stiffness method was used for the second iteration (otherwise initial stress method). For the linear elastic case, i.e. when the first Gauss point becomes plastic, the values are included in Tables 1 to 3 of G_1 (and for Table 3 of G_2), viz. $\pi a(\sigma_t^2 + \sigma_s^2)/E$ and $-2\pi a \sigma_s \sigma_t/E$ where σ_s and σ_t are the shear and tensile stresses in the uncracked plate. The half crack size a was taken to be 16mm.

DISCUSSION

The close agreement between the values obtained for G_1 and J^*_{1w} under monotonic loading is encouraging. For an elastic material, the results of both of these methods, and of explicitly taking an energy difference for a small increment in crack size should be the same, the small differences being due to numerical inaccuracies. The same should be true for an elastic plastic material under radial loading. If the loading is radial only near the crack tip, but not elsewhere, the energy differencing method will not be appropriate.

For radial loading near the tip, sources of numerical inaccuracy for the three methods are as follows, for an elastic plastic material: Energy differencing loses many significant figures. If the value is required at n points on the crack, n+1 computer runs are required, all with an adjustment to the onset of first yield to ensure that for the different crack shapes and sizes, initial yield occurs at the same proportion of total load; Analytic energy differentiation gives rise to a large contribution to G from those elements where some nodes are fixed and some are not. If, as in the present investigation, these include elements near the crack tip, this is the area where numerical accuracy is poorest. In particular, special account of singularities should be taken by using special elements at the crack tip; For contour integration in three dimensions, care is required during mesh generation so to ensure contours can be specified in planes approximately normal to the points of interest on the crack edge. If only an average value is required the difficulty can be reduced by using the volume integral approach; The contour is usually defined through nodes, where the stresses are usually less accurate than at Gauss points. This problem can be avoided by use of a g function with integration at Gauss points, The volume integral method of Li et al (1985), Shih et al (1986), is equivalent to the virtual crack extension method for an elastic material or for an elastic plastic material with radial loading over the area of integration and any area within. Numerical difficulty has also been experienced for approximately isochoric deformation near the crack tip, when a surface integral adjustment is needed, as for thermal strains or axisymmetric deformations when the errors in the hydrostatic component of stress, which has little effect on the energy and hence can have greater numerical inaccuracy than the other components, are becoming significant.

There is a greater discrepancy between G_2 and J^*_{w2}, which neglects the contribution from the crack faces in the crack tip elements. Fortunately they are always less than G_1 and hence less important. In fact the CEGB defect assessment procedure, Milne et al (1988), suggests that in some practical cases of importance these may be ignored.

Table 2. J^*_{w1} and G_1 for shear displacement (σ_s stress in uncracked plate)

σ_s/Y	J^*_{w1} (MPamm)	G_1 (MPamm)	$\pi a \sigma_s^2/E$ (MPamm)
.0855	0.1028 (0.1057)	0.1046	0.107
.398	2.544 (2.668)	2.224	
.710	7.988 (8.655)	7.832	
1.022	15.299 (16.643)	15.675	
1.334	23.02 (24.98)	23.41	
1.646	29.76 (32.23)	30.12	
1.958	36.07 (39.06)	36.62	
2.270	42.07 (45.51)	42.94	
2.581	47.93 (51.86)	49.21	
2.893	54.16 (58.52)	55.30	
3.205	60.22 (64.96)	61.43	

Table 3. J^*_{w1}, J^*_{w2}, G_1 and G_2 for combined displacements (σ_s and σ_t shear and tensile stress in uncracked plate)

σ_t/Y	σ_s/Y	J^*_{w1} (MPamm)	J^*_{w2} (MPamm)	G_1 (MPamm)	G_2 (MPamm)	$\pi a(\sigma_s^2+\sigma_t^2)/E$ (MPamm)	$2\pi a\sigma_s\sigma_t/E$ (MPamm)
.0708	.0818	0.1556 (0.1605)	0.1068 (0.1096)	0.1608	0.1344	0.169	0.167
.203	.234	1.434 (1.471)	0.869 (0.892)	1.277	0.921		
.334	.386	4.029 (4.201)	1.825 (1.890)	3.667	2.284		
.466	.538	8.687 (9.226)	3.486 (3.624)	8.101	4.743		
.598	.690	14.265 (15.182)	5.848 (5.965)	13.53	7.884		

Figures in parentheses are obtained using g function

For reversed loading, there is a greater discrepancy between G_1 and J^*_{w1} which need no longer be identical. On repeated loading, the peak values of G_1 and J^*_{w1} are in agreement, but the difference over the cycle in G_1 is much greater than that in J^*_{w1}. There is less variation from cycle to cycle of the maximum and the minimum of the values of J^*_1 than those of G_1 or of J^*_{w1}.

These results are in general conformity with those of previous investigations of contour integrals for repeated loading of a stationary crack (Blackburn et al, 1977, Blackburn, 1987a and Wong and Jones, 1987), but the virtual crack extension results of Miyazaki et al (1985) did not show a hysteresis loop on reloading. Extensions to repetitive loading of a growing crack have been reported by Brust et al (1986) and by Blackburn (1987a,b).

CONCLUSIONS

The infinitesimal virtual crack extension (IVCE) method is preferable to taking energy differences, particularly for three dimensional geometries or non linear materials.

For materials which may be treated as elastic (e.g. no unloading of an elastic plastic material), the infinitesimal virtual crack extension method and the g method will be preferable to the standard contour integration method, for those three dimensional cases where it is difficult to obtain a mesh suitable for defining a contour around the tip, so that the contour of integration is not in a plane normal to the crack edge, and also in mixed mode I and II situations as far as the out of plane component is of interest. For mode I or II loading of an elastic plastic material under monotonic conditions, the results are comparable and would, for an elastic material, differ only by numerical errors if the g function is unity for the elements within the contour except those touching the contour, and the IVCE transformation was over all the elements within the contour over a one element wide region.

Neglect of the integral over the face of the crack tip elements can significantly underestimate the contribution to that component of the contour integral which is normal to the crack plane in the mixed mode case.

For elastic plastic materials where cycling has occurred, there is little difference between the results of contour integration and the variational technique with the same replacement of W by $\sum \Delta W$, as far as peak values are concerned. However the variation in the values on unloading is significantly different and is likely to be underestimated by the variational technique.

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