Computer Modelling of the Evolution of Hydraulic Fracturing Phenomena in Hydrocarbon Reservoirs

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ABSTRACT

In particular referring to hydrocarbon reservoirs, the paper considers the evolution of advanced simulation techniques able to study moving boundary problems concerning the fracture propagation consequent to well stimulation operations of hydraulic fracturing. The FEM modelling keeps being adopted in order to forecast the possible results of the said operations. A new and original Lagrangian approach to the problem has been developed in order to try a study of the crack propagation phenomena in modular ways. The optimized simulators include implementable sections which have been designed for data gathering and statistical approach purpose. The paper analyzes a deal of results obtained both by 2D and by 3D FEM simulations of hydraulic fracturing, and describes the potentiality of a processing and computing system which has been assembled by combining and interfacing different approach techniques to the problem.

KEYWORDS

Fracturing; hydraulics; FEM modelling; Lagrangian analysis; production.

BACKGROUND

The hydraulic fracturing methods have been extensively applied by the oil industry for the secondary recovery of hydrocarbons. Most recent studies have been performed in order to extend the application of the hydraulic fracturing methods to other fields such as the geothermal energy extraction and the radioactive waste disposal. The design of fracturing processes usually includes at least three stages: fracture geometry prediction, fracture cleanup prediction and fractured well performance prediction. Many authors have already published papers describing simulators able to predict all three stages of the hydraulic fracturing process. It is worth mentioning Hagoort et al., 1980, and Settari, 1980, who made use of sequential methods of solution in order to solve the reservoir flow equation, the fracture flow equation and the fracture geometry equation, and - above all - Nghiem et al., 1984, who described a fully implicit hydraulic fracture model able to accomplish the said three stages by simultaneously solving all the equation systems, restricting the discussion to the vertical fracture pattern described by Geertsma & de Klerk' s modelling. Afterwards pseudo-three-dimensional modelling techniques have been developed in order to describe the evolution of geometry of 3D hydraulic fracture systems generated by fluid injection (Settari et al., 1986 above all) and to optimize suitable computation sections for incorporation purpose inside 3D fracturing simulators. During the described evolution the study of the fluid temperature distribution in the fracture systems has been probably given little attention; the importance of this aspect has come to be clear only in the last few years, and will be carefully examined in a forthcoming paper.

INTRODUCTION TO THE PROBLEM

Hydraulic fracturing has been defined as the process of creating fracture systems in porous media by injecting fluids under pressure through a welbore in order to overcome native stresses, as to cause a number of material failures and cracks inside the involved formations.
The existing literature reports many applications of the main theories of the mechanics of failure of rock subjected to internal fluid pressure solicitations. Such theories can be summarized as follows: - maximum stress th.; - maximum shearing th.; - maximum strain th.; - maximum strain/energy th.; - Mohr's th.; - Coulomb-Navier theory; - Griffith's theory; - Walsh-Brace th.; - single plane of weakness th.; - variable cohesive strength th.; - octahedral shearing stress theory. Most authors, although some disagreement among them remains about the orientation of fractures, start from the assumption that - when pressure increases in the formation - breakage occurs in the plane which is perpendicular to the direction of the least compressive stress. Many laboratory studies have been developed during the last few years in order to try to understand the effects of in situ stress gradients, supposed to be potential barriers able to contain the height of hydraulically generated vertical and subvertical fractures (Thiercelin et al., 1986, e.g.). One of the most typical phenomena which are immediately consequent to well stimulation by hydraulic fracturing is the curvilinear growth/interaction of fracture systems. Fracture paths inside reservoir structures are known to be associated and related to changes in stress loading both in time and in space; this non uniformity of the stress field usually causes curved paths. Moreover the fracture growth process through rock inhomogeneities (also having fundamental importance as far as the global reservoir production evaluation is concerned) involves the problem of checking the fracture containment in the pay zone, up to determine whether a fracture intersects an inclusion. In order to maximize the permeable communication of a well with the surrounding media it is desirable to have multiple fractures in the same welbore and to create linked fracture networks; the growth of the fracture system(s) to substantial lengths leads to a kind of 'competition' between corresponding wings.
The crack growth process has already been studied and carefully described by many authors; the main contribution of the paper aims to be the development of an integrated analysis system, according to 2 main requirements:

- dividing the crack propagation problem into sections which can be treated separately, also out of the elasticity field;
- creating suitable sections inside the simulators for data gathering and as well for statistical organization purpose.

The first requirement has been faced by adopting an original pseudo-3D independent Lagrangian formulation, allowing the said separation and also global recompositions by virtual work analyses; the second requirement has led to the development of sections able to introduce the probability distributions of the input variables instead of single valued functions and parameters, and to organize and treat data in real time.

LAGRANGIAN APPROACH TO THE SIMULATION OF HYDRAULIC FRACTURING

The Lagrangian approach to fracture modelling and evaluation allows dividing the study of the crack propagation problem into sections (crack shape, leak off, separation energy, flow inside the crack, etc.) and allows treating and studying both elastic and plastic phenomena. Starting from the original Lagrangian formulation, the fracture propagation being considered to occur only in quasi-static conditions, the general differential equation which governs the phenomenon can be expressed as follows:

$$Q = \partial E / \partial q + \partial D / \partial \dot{q} \qquad (1)$$

being:

∂ = partial derivative operator	E = potential energy
D = dissipation function	q = generalized system coordinate(s)
$\dot{q} = \partial q / \partial t$	t = time coordinate

Q = total of the forces not deriving from either dissipation or potential functions.

The shape of the crack comes to be determined by the distribution of the fluid pressure. The Lagrangian approach does not require the knowledge of the shape of the crack in advance; the present study considers standard shapes only. The variation of the pressure in the crack has been assumed to be linear in space; fluid leakoff has been considered and included into the global Lagrangian formulation by dividing the total flow rate into a first part only contributing to the fracture volume, and into a second part only contributing to the fluid loss in the formation. Leakoff modelling has been performed referring to the displacement of the formation fluid by a quasi-incompressible fracturing fluid, with a moving boundary separating the different fluids. For the wall building fluids it has been assumed that no kind of pressure drop occurs before the spurt loss leaking off; afterwards the pressure drop through the growing wall has been assumed to vary linearly with the group: (leakoff rate * fluid volume passing through the wall). The Lagrangian pseudo-3D formulation yields systems of differential equations which are nonlinear in the geometric fracture parameters (length, width and height); the virtual work analysis allows reducing such systems to much simpler single type differential equations (e.g. in the only fracture length etc.), which can be solved numerically. Then the injection pressure variation in time for a given rate can be immediately obtained.

ANALYSIS OF THE MODELS

A set of fundamental equations which are able to describe: the flow in the nearby of the wells and in the reservoir (Di Molfetta et al., 1980, Cravero et al., 1987); the two-phase non Newtonian flow in the fractures, the heat transfer in the fracture system and in the reservoir (Biot et al., 1987, Lewis et al., 1985); the propagation of the crack in the space, the proppant transport, the welbore hydraulics, and the PVT behavior of the reservoir and of the fracturing fluids (Settari et al., 1984) has been taken as the basis of the modelling process. The turbulence effects, which are not negligible for gas flow at high velocity, have been considered by mean of the nonlinear Forchheimer's equation in the form including one turbulence coefficient. Fracture geometry computation and pressure distribution computation have been performed in a coupled way, directly from the overall force balance and flow equations. Also depending on the adopted fracture geometry modelling, boundary conditions can assume different physical meaning and constitution (imposed flow rate / pressure / volumes) and different forms.
The continuous crack growth process has been approximated by stepwise sequences as far as the FEM approaches are concerned; for any growth step the volume of the fluid in the crack total cavity has been assumed to be constant. The performed modelling has not included the possible additional effects of proppants on the fracture geometry. Fracture width variation in the unpropped part has been studied in conditions of plasticity only by a Lagrangian approach, leading to analytic analyses. It has also been possible to optimize interfaces able to transpose the results of Lagrangian analyses (when selected) into finite and discrete patterns, suitable for FEM restarting. The approximation of the combined modelling has shown to be better than the approximation of the FEM approach used by itself; the possibility exists of running both only FEM and only Lagrangian analyses.
A new point of the research also concerns the development of adequate interfaces able to work in two-way directions, with the aim of processing the data coming from the reservoir in real time. That involves the assembling (with the support of opportune methodologies and instruments, Lacy, 1987 e.g.) of an integrated analysis system which can be used also by the reservoir engineers for further aims. Presently the simulator sections which have been set up for such purpose are able to organize the stored data and to introduce sets of probability distributions. For any of the variables of the problem one can define, inside a chosen range, sets of values generated according to statistical laws (e.g. fixed standard deviation, defined stochastic distribution kind or shape, uncontrolled variation in fixed ranges).

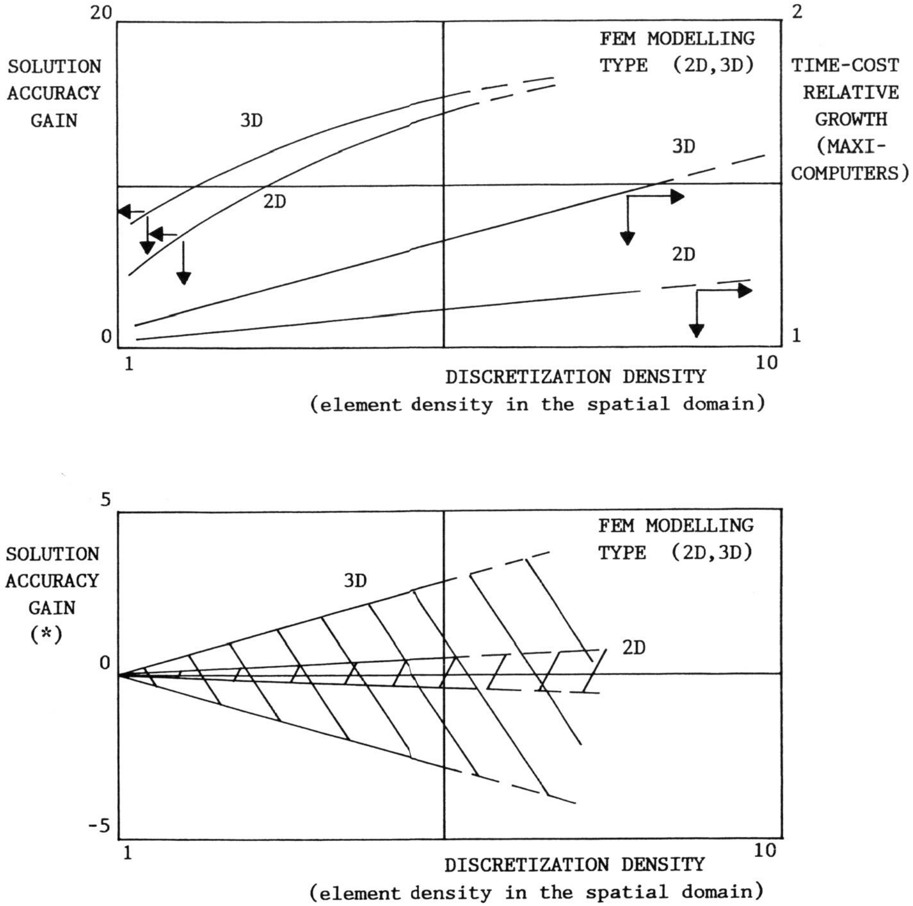
That has been made in order to simulate the uncertainty which always derives from the evaluation process of the considered variable(s) and unknown(s), and in order to quantify the related possible influence upon the results of the modelling and forecasting stages.

INTEGRATED SOLUTION OF THE PROBLEM AND DISCUSSION OF THE RESULTS

The great potentiality of today's data processing systems allows running a deal of computer programmes which can solve highly complex structures (with up to thousands degrees of freedom). The adoption of the FEM modelling also gives the opportunity of assigning the constitutive and behavioral laws of any material, of selecting the most adequate element types inside wide libraries, of discretizing complex domains with the required detail, of solving different kinds of equation systems through adequate storage memories by either iterative, or direct front, or band (etc.) solution methods (Dhatt et al., 1984, Bathe et al., 1976, Christie et al., 1976, Cheung et al., 1967). The Finite Element Method (FEM) has been adopted for the study of the hydraulic fracturing phenomena also in order to perform detailed analyses of the spatial correlations and interactions. The final achievement of the FEM development is a numerical process which includes:

- the formulation of the FEM matrixes;
- numerical integrations for matrix computation purpose;
- the treatment of the single element matrixes up to assemble the global matrixes which represent the discretized domain(s);
- the solution of the systems of equations which describe the physical meaning of the problem.

As far as the hydraulic fracturing simulation is concerned, the FEM discretization has been applied to the spatial description of the continuum, while the variations in the time have been studied by conventional Finite Difference techniques. For the flow problems in particular, the required integral formulation (for FEM application purpose) has been obtained from the differential formulation by weighted residual methods. The solution process has been reconducted to Galerkin's scheme by selecting the weight function(s) on the basis of the function(s) which are able to approximate the field variable behavior.
First of all it is worth pointing out that the 2D and 3D FEM analyses have greatly emphasized the fact that the input data are a much smaller cause of uncertainty and numerical instability in case of fixed-height-geometry models. Fig. 1, referring to zeroD coordinates, compares the results obtained by 2D and 3D FEM simulations of 100 cases of fracture cleanup. In the upper part of the picture lines have been drawn representing the solution accuracy gain and the time-cost relative growth vs the density of the elements that discretize the spatial domain. The computations and the comparisons have been made referring to the theoretical solution of the problem, considering single valued variables only. The lower part of Fig. 1 shows the degrees of uncertainty which come out whenever propped fracture properties vary inside the specified ranges in uncontrolled ways; careful analyses have shown that the regions with negative precision gains of 3D FEM simulations statistical



(*) propped fracture properties vary in their ranges in uncontrolled way

Fig. 1. Comparison of the results obtained by 2D and 3D FEM modelling of 100 different cases of fracture cleanup, referring to the corresponding theoret ical solution, being:

formation permeability	=	1.0 md
fracturing fluid viscosity	=	3.0 cps
oil viscosity	=	1.2 cps
propped fracture permeability	=	600 md +/- 100 md
propped fracture width	=	5 mm +/- 1 mm
bottom hole pressure covers the range		(2500 - 4000) psia
average reservoir depth	=	3000 m
reservoir temperature	=	330 K

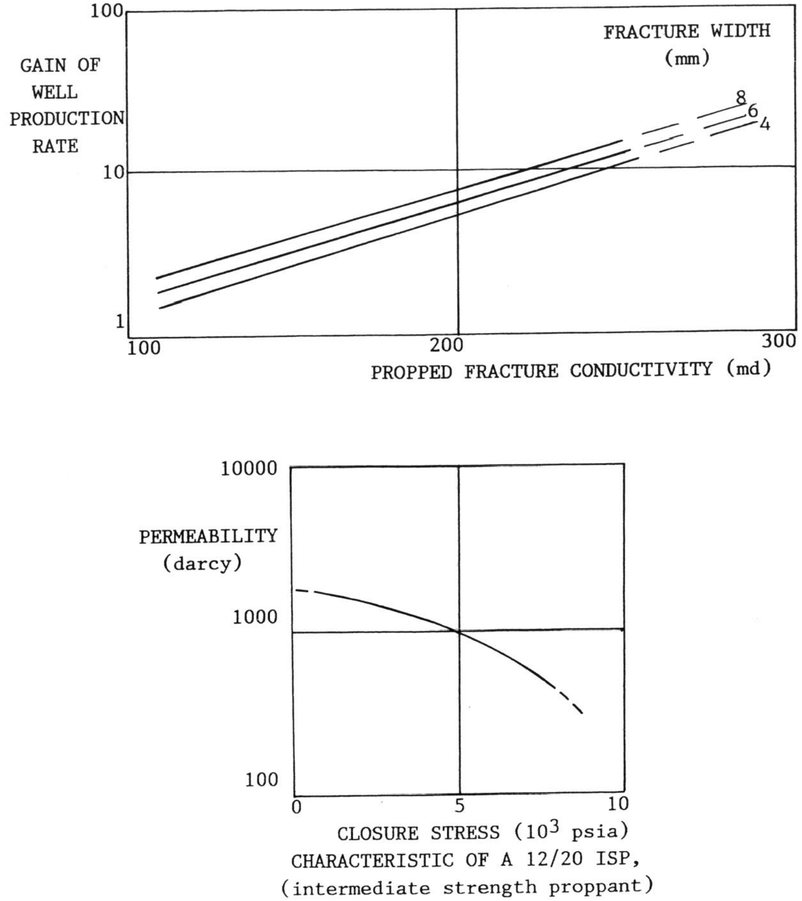


Fig. 2. Results of an analysis of the influence of propp ed fracture conductivity on the increase of well production rate (same data as Fig. 1, with propp ed fracture conductivity of 200 md +/- 100 md).

ly keep almost the same areas as the corresponding positive gain regions even in the case that the propped fracture properties vary inside the specified ranges according to fixed standard deviation values. As far as the 3D FEM modelling is concerned, problems also come from the need of assigning the var iables describing the physical properties of the confining layers and the stress fields acting inside the entire volume (see later on). Moreover the 3D computation of the crack extension requires detailed knowledge of the val ues of the bottom hole pressure during the hydraulic fracturing job. Since the behavior of pressure while fracturing is in progress comes to be directly influenced by the vertical growth of the fractures, the postfracturing evaluation study requires the performance of back-analyses, with the support of adequate data-processing backgrounds. The propped fracture conductivity is the parameter which determines the values of the production rates after the closure; it mainly depends on the history of the simulation job and on the position fracture(s) vs pay, since only the part of the channel which is positioned directly against the pay really contributes to the increase of productivity. Fig. 2 shows that, the data being the same as Fig. 1, an approximation of the value of the propped fracture conductivity in the range: 200 md +/- 100 md can lead to predicted gains of well production rate which differ up to 10 times. The moving boundary treatment has been performed by checking the position of the fracture tip at the end of every time step, by generating new fracture nodes and by repeating the computation step anytime the said tip comes to interest new gridblocks. The stability of the procedure can be reached by zeroing the term of the fluid loss from the fracture section to the new block(s) when the tip retreats to the block boundary, (Nghiem et al., 1984). An extensive stage of grid sensitivity study has been performed in order to check the convergence of the computed solutions to the theoretical ones while refining the grid mesh along different directions. Over a number of 100 two-phase cases, the global injected rate varying in the range 500 - 5000 cubic metres / day at an initial pressure of 3300 psi, the following results have come out:

- the mesh refinement perpendicularly to the fracture propagation directions leads to much better results (ratios can reach up to 10000 and more) than in any other direction;
- the average ratio of precision of the obtained results for the case (perpendicular vs parallel direction of mesh refinement), determined on the basis of the relative errors, is 5 to 1 at least;
- the time step discretization involves negligible errors whenever the time steps are designed according to geometrical series having ratios lower than 1.12 and initial values lower than 0.002 days.

The Lagrangian analysis has repeatedly led to results which point out the non critical dependence of the crack dimensions on the fracture shape modelling (as shown in Fig. 3): that comes to confirm that in this case the knowledge of the exact shape of the crack is not needed. For the conventional fracturing fluids (e.g. water and oil) it has been checked that the Lagrangian approach tends to predict higher leakoff rates (up to 10 times) than the FEM one. For wall building fracturing fluids the rate values predicted by the Lagrangian approach substantially agree with the FEM modelling results. From a merely practical point of view the Lagrangian approach has confirmed to be the most immediate way of evaluating fracture global dimensions only. The development of a section where it is possible to introduce the probability distribution of a variable instead of single deterministic values has also been gone through on behalf of a better characterization of the geomechanical properties of the confining layers (especially for the case of rock layers). This problem is usually by-passed when adopting the 2D FEM modelling, which only requires the selection of the value of the fracture height.

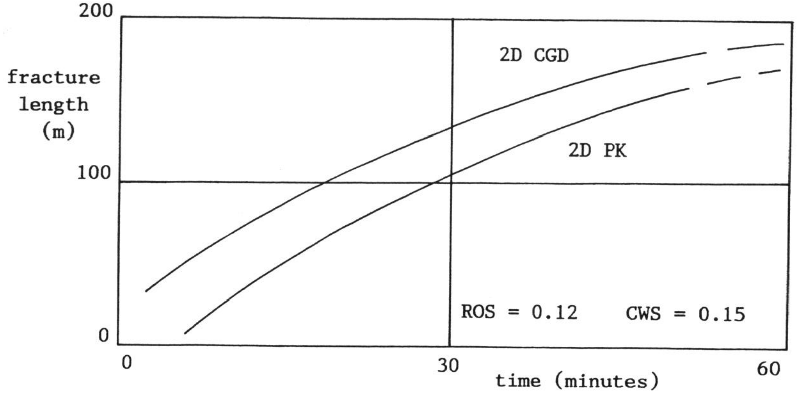


Fig. 3. Results of Lagrangian analyses of the dependence of the fracture dimension variation on the fracture shape modelling (used shape models: 2D PK = Perkins-Kern; 2D CGD = Christianovich-Geertsma-de Klerk), after Settari et al., 1986, being:

injection pressure: 3300 psia		; injected water rate: 100 m^3/day	
Poisson's ratio	: 0.20	; Young's modulus	: 3,000,000 psia
formation depth	: 3000 m	; formation pay	: 15 m
formation porosity	: 0.18	; formation permeability	: 0.05 md
oil viscosity	: 1.5 cps	; water viscosity	: 0.8 cps

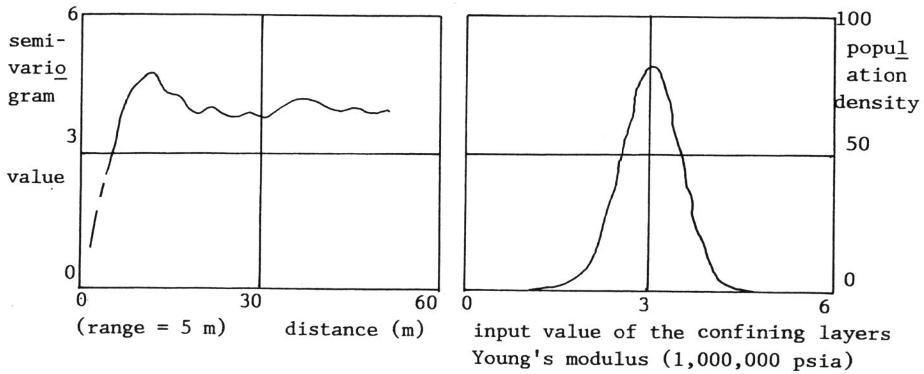


Fig. 4. Frequency hystogram and spatial correlation semi-variogram of the statistical simulation of the confining layers Young's modulus (same data as Fig. 1).

As far as the 3D FEM modelling is concerned, it often happens that the available data are not adequate for deterministic predictive application, because of lacks of information about the layers which confine the reservoir. One possible probabilistic approach to the geomechanical characterization of the upper and lower boundary Young's modulus is summarized in Fig. 4, the spatial correlation spherical-type semi-variogram and the frequency hystogram being referred to a 5 m correlation range, which approximates a Monte Carlo simulation (the range tending to 0). Of course element dimensions greatly smaller than the correlation ranges have to be chosen, not to interfere with the impact of the spatial correlations. The study of a careful approach apt to consider the values of temperature in the fracture as functions of time and position is still in progress; the variational formulation of the convective heat transfer (already examined and applied by Davies et al., 1988) is a fundamental contribution to the optimization of the design of treatments, the thermal aspect of the problem really playing a key role.

CONCLUSION

An integrated simulation system has been assembled, including the presence and the contribution of different approach techniques to the analysis and to the evaluation of the evolution of hydraulic fracturing phenomena consequent to operations of well stimulation in hydrocarbon reservoirs. The FEM modelling has been adopted in order to study the spatial fracturing phenomena, in connection with a FDM discretization scheme of time domains. The Lagrangian approach to the division of the crack propagation problem into separate sections has been tested and inserted into the global system. Last but not least a section has been set up for the storage and the statistical treating of physical and geometric parameters; for this section of the system further optimization stages are in progress. It looks reasonable to think that the combined use of the adopted methodologies enables to face a wide range of requirements, also as far as the analysis degree of accuracy and reliability is concerned.

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