Computational Fracture Mechanics

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ABSTRACT

The field of Computational Fracture Mechanics is reviewed. The paper focuses on the impact of computational methodology on furthering the understanding of fundamental fracture phenomena. The current numerical approaches to the solution of fracture mechanics problems, e.g. finite element methods, finite difference methods and boundary element methods are reviewed. The application of these techniques to the problems of linear elastic fracture problems is discussed. Particular emphasis is placed on three dimensional problems and the issues involved with surface crack geometries and stress intensity factor calculations.

Numerical solutions of two dimensional ductile fracture problems are surveyed. A special focus is placed on the effect of stable crack growth on the field quantities and the implications of numerical solutions for fracture prediction. Creep fracture problems are discussed. The similarities and differences between creep and ductile fracture problems are highlighted. The importance of large strain phenomena and accurate modeling of nonlinear effects are highlighted.

The current state of knowledge of continuum fields for elastostatic cracks, elastodynamic cracks, ductile cracks and viscoplastic cracks is summarized. The range of applicability of asymptotic solutions (especially in the nonlinear regimes) is highlighted.

Major research needs in computational fracture mechanics are detailed. Emphasis is placed on coupled theoretical and numerical approaches. Prospects for future research trends are proffered. Application of fracture mechanics and computational fracture approaches are explored.

KEYWORDS

Crack propagation; failure prediction; finite element; fracture mechanics; nonlinear methods; numerical methods; research needs.

INTRODUCTION

The field of fracture mechanics has progressed significantly over the past thirty years. Fracture mechanics now provides a firm theoretical basis for the prediction of fracture and the fracture-proof design of new structures for many applications (most notably for applications with solely elastic response). For other problems (where ductility or environmental effects are present), fracture mechanics has progressed toward an understanding and theoretical framework for the future. While much additional research is required before fracture mechanics can be considered a mature discipline, it is recognized that significant advancement has been made. Fracture mechanics is based on the assumption of a continuum material behavior of the structural component under analysis. The effect of atomic spacing and material microstructure, therefore, is assumed to be totally represented by the constitutive equations employed in the continuum model. Hence, this assumption is the major limiting factor in the development of a quantitative, cohesive theory of fracture. The ultimate theory of fracture should attempt to couple the microscopic and macroscopic fracture characteristics in a coherent manner. This task is a major requirement of future research in fracture mechanics.

The advent of the digital computer made it possible to solve engineering and scientific problems by using numerical techniques. Many problems which could not be addressed analytically could (at least in theory) be solved numerically. As computers have become faster, cheaper, more powerful and more widely available, the number of problems which are addressed numerically has grown exponentially. The field of fracture mechanics has benefited dramatically from the use of the digital computer. Routine use of Linear Elastic Fracture Mechanics (LEFM) in fracture-proof design can be largely attributed to the ability to solve fracture problems routinely using digital computers. Critical technology problems involving material and geometric nonlinearities have been addressed successfully using numerical solutions. Indeed, many application areas would have been significantly hindered (if not stopped) without the numerical solution of fracture problems. In addition, much fundamental understanding of the behavior of materials containing cracks has been gained through numerical simulation of fracture problems.

The purpose of this paper is to provide a critical examination of the impact of numerical methods on the field of fracture mechanics. For the purposes of this discussion, fracture mechanics problems will be subdivided into three major classes: Linear Elastic Fracture Mechanics (LEFM) problems (both static and dynamic), problems involving composite materials, and ductile fracture problems (including rate dependent problems). These broad topics represent the major areas of challenge and application of the field of fracture mechanics.

The paper starts with a discussion of the major numerical approaches available for the numerical solution of boundary value problems. Emphasis is placed on the Boundary Integral Equation Method (BIEM) and the Finite Element Method (FEM). These approaches are the major methods employed for the solution of fracture mechanics problems. Historical note is made concerning integral equation methods and finite difference methods. Emphasis in this section is on the strengths, weaknesses and successes of the methods to date.

The problem areas of LEFM and ductile fracture problems are then considered in turn. The emphasis in each section is placed on highlighting the impact

of numerical solutions on the understanding of each problem area and the application of the methodology to design considerations. Also considered is the role of asymptotic and analytic ideal problem solutions in the numerical solution of real engineering problems. An important issue is the value of numerical solutions and the delineation of their limitations.

After surveying the major problem areas and their state of the art, the discussion turns to the major needs of fracture mechanics and the role that numerical methods can play in fulfilling these needs. The majority of this centers on the role of computer simulation, visualization and the interpretation of results. Emphasis is on coupling accurate numerical solutions to physical insight and understanding. A very important concern is the consideration of the numerical solution needs in the formulation stage.

The paper concludes with a discussion of the major obstacles and challenges that face researchers in the numerical solution of fracture mechanics problems. Coupling of numerical and theoretical advances and approaches is emphasized. An attempt is made to focus on those issues which can shed important light on the open questions in the field of fracture mechanics.

NUMERICAL METHODS FOR SOLUTION OF FRACTURE PROBLEMS

The problems of fracture mechanics reduce to the solution of boundary value problems (which may be static or dynamic) which have mixed boundary conditions. These mixed boundary conditions can give rise to singularities in the stress and strain fields. The problems may involve both material and geometric nonlinearities which complicate the formulation and render prediction of convergence extremely difficult. Because little can be done with these problems analytically, numerical methodologies are required. The advent of large scale computers coupled with the rapid growth in the field of algorithmic methods render many of the problems of fracture mechanics tractable today.

The finite difference method is the oldest technique for the solution of boundary value problems and was widely employed in the 1960s. The method directly involves the solution of the governing differential system in an approximate manner by subdividing the domain of interest into a connected series of discrete points called nodes. These nodes are the sampling points for the solution and are linked using the finite difference operators to the governing equations. For example, the second order finite difference operator for the second partial derivative of a two dimensional field variable is given by

$$\left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x_i, y_j} = \frac{\psi(x_{i+1}, y_j) - 2\psi(x_i, y_i) + \psi(x_{i-1}, y_j)}{(\Delta x)^2} \tag{1}$$

where ψ is the field variable and x and y are the independent spatial variables. This is a second order difference operator and the error is proportional to the square of the mesh spacing in x ($O(\Delta x^2)$). Employment of the finite difference operators results in a system of algebraic equations for the discrete nodal values of the field variable.

Gradients can be evaluated by employing finite difference operators to the discrete solution.

Finite difference methods can be used to discretize both space and time. In addition, they provide easy error estimation techniques. Unfortunately, finite difference methods are difficult to use for irregularly shaped domains. Often absurd discretization is required for accurate solution. In addition, it is difficult to implement meshes without equal grid spacing. Convergence is difficult to gauge with this characteristic. An excellent discussion of the finite difference approach to the solution of partial differential equations can be found in Lapidus and Pinder (1982).

Finite difference methods have not performed very well for problems involving singularities. One major reason for this is that the fine meshing required near a singularity cannot easily be reduced for the rest of the domain. Special finite difference techniques which directly handle singularities can be developed; however, they have not been very successful for practical applications. Computational requirements for convergence are larger than for finite element and boundary element solutions. The finite difference method is not seriously employed for fracture problems today.

In addition to finite difference methods, integral equation methods are a historic approach to the solution of fracture problems and are still used by some researchers today. The basic approach employed involves an analytic formulation of the elasticity problem to the point of a singular integral equation. The singularity is then extracted and the result is a nonsingular integral equation which can be solved quite accurately with any number of techniques. This approach yields excellent solutions, however, it requires an extensive analytic formulation which is different for each new problem. The method is quite useful, nonetheless, for establishing benchmark solutions to compare with other methods as the degree of accuracy can be guaranteed. The method is only applicable to elasticity problems (no nonlinearities). For three dimensional problems, it is almost impossible to derive the integral equations in a finite period of time. An excellent discussion of the method can be found in Muskhelishvili (1953).

Two major numerical approaches are available for the solution of fracture mechanics problems today: the Boundary Integral Equation Method (BIEM) and the Finite Element Method (FEM). These techniques have been widely researched and developed. For two dimensional Linear Elastic Fracture Mechanics (LEFM) problems, either can be employed with much confidence and accuracy. Both BIEM and FEM are actually a class of approaches with many variants which allow a flexible approach for modeling many areas of application. The discussion of each given below will focus on the methods as they commonly are applied to fracture mechanics problems and the variants employed by some authors for better solution characteristics.

The BIEM method is a numerical approach to the solution of linear boundary value problems with known Green's function solutions. The boundary of the domain of interest is discretized using "elements" which are interconnected at discrete points called nodes. For a three dimensional problem, the mesh is two dimensional; for two dimensional problems, the mesh is one dimensional. The boundary value problem is formulated as an equivalent surface or line integral using the Green's function solution and the governing differential system. For linear elasticity in two dimensions, the formulation is based on Betti's theorem and the resulting system of equations is given by

$$C_{lk}u_k + \int_\Gamma u_k T_{lk}\, d\Gamma = \int_\Gamma t_k\, U_{lk}\, d\Gamma \qquad (l,k = 1,2) \qquad (2)$$

where u_k and t_k are the surface displacement and traction vectors, is the domain boundary, and U_{lk} and T_{lk} are related to the Green's function solutions for displacement and tractions. At each boundary point, either u or t is specified and the other variable is unknown. These relate to the physical field variables in question. A complete discussion of the approach can be found in Banerjee and Butterfield (1981).

The BIEM method is a quickly convergent, highly robust method for the solution of linear boundary value problems. It is relatively easy to employ and general purpose commercial software can be developed around the method (the BEASY code is a widely available example; see BEASY in References). Because the surface of the domain need only be discretized, it is easier to use the BIEM than the FEM (to be discussed subsequently). For static problems, the BIEM method reduces to the solution of a system of dense linear equations which may be nonsymmetric (although methods of symmeterizing the systems recently have been very successful). If surface data is the only quantity required (as is the case in many fracture problems where the only interesting results are the stress intensity factors and the compliance), the BIEM is often computationally superior to the FEM for two dimensional problems. If interior data is required, the method is computationally costly. For three dimensional problems, BIEM solutions are often very expensive as the resulting linear system is dense, unbanded and often nonsymmetric. Ongoing research, however, is addressing this problem rapidly. BIEM solutions often yield excellent results for field quantities and their gradients (e.g. displacements and strains). Primary unknown predictions on par with FEM solutions usually predict better gradients within the BIEM concept.

For applications in fracture mechanics, the BIEM has received a good bit of attention recently. For two dimensional problems, the BIEM can be employed for the solution of fracture problems with much success. Mesh generation is quite simple and users can master the techniques rapidly (much more so than for the FEM). Accurate solutions can be obtained and reasonable error estimates can be predicted. It is certainly competitive with the FEM if not better for these problems. The numerical techniques employed for fracture mechanics problems are summarized in Table 1.

Three dimensional LEFM problems have been solved using the BIEM without great success. These solutions are quite costly and often do not produce good solutions. As an example, consider the problem of an edge cracked rectangular bar subjected to uniaxial uniform tensile stress as shown in Fig. 1. The resulting stress intensity factor distribution is shown in Fig. 2 and is compared with well established finite element results. It can be seen that near the midplane the results agree well. Far from the midplane, however, resolution degrades. Because it is well known that FEM solutions of surface crack problems overestimate the boundary layer effect near the free surface, the BIEM results are in error (Rooke et al., 1987). Interior crack problems have been solved successfully; however, this is not a sufficient test of the method. Ongoing research hopefully will address this problem, although the BIEM is not a current competitor for three dimensional problems.

Table 1. Numerical methods for the solution of fracture problems

Method	Strengths	Weaknesses
Finite Difference	Easy to employ	Slow convergence
	Error estimates available	Uniform mesh requirements
		Cannot model singularities
Finite Elements	Good convergence	Modeling is difficult
	Singularities can be modeled	Few exiting error estimators
Boundary Elements	Modeling is easier	Computationally more expensive for most problems
	Error estimation is easier	Converge slowly for singular problems
Hybrid Approaches	Good for specific problems	Usually developed for restricted problem class
	Generally very accurate	Often difficult to implement

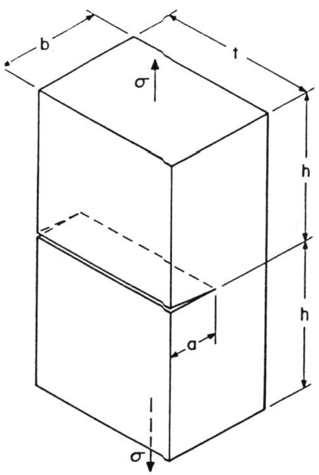


Fig. 1 Edge cracked rectangular bar subjected to uniaxial uniform tensile stress

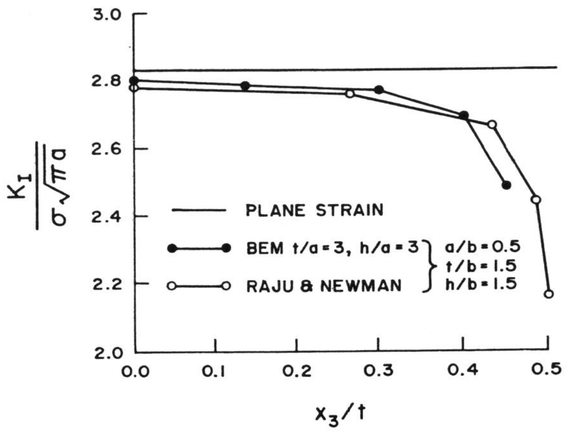


Fig. 2. Variation of the stress intensity factor along the crack front

Much effort has been focused recently on the extension of BIEM to nonlinear problems where known Green's functions do not exist. This work is in its infancy and it is fair to say that the approach has yet to impact the field of fracture mechanics. Indeed, available solutions to problems with extensive nonlinear material behavior are disappointing (e.g. Wilson et al., 1985). Ongoing research may establish BIEM approaches to nonlinear problems which produce reasonable answers. For nonlinear problems, analytical Green's functions are not available. A variational approach with assumed trial and weight functions must be employed. The formulation is similar to that employed by the finite element method. The BIEM, therefore, will have the same approximate formulation as the FEM.

The FEM is the most widely employed numerical method for the solution of fracture mechanics problems. The formulation of the FEM is based on a variational statement of the governing physics. For the problems of linear elasticity, the principle of Virtual Work, given by

$$\int_V \sigma_{ij}\, \delta\varepsilon_{ij}\, dV = \int_S \sigma_{ij}\, n_j\, \delta u_i\, dS \quad , \tag{3}$$

is employed where σ_{ij} is the stress tensor, $\delta\varepsilon_{ij}$ is the virtual strain tensor due to virtual displacements δu_i and n_j is the normal vector to the surface of applied tractions. The domain is discretized into subdomains (elements) which are interconnected through common discrete points (nodes). The primary unknown field variables are nodal values. The formulation reduces the problem to the solution of a system of algebraic equations in terms of the nodal variables (for dynamic problems, the result is a system of ordinary differential equations). Finite element systems tend to be relatively banded and symmetric for most problems. The resulting systems can be solved using a number of techniques. For nonlinear problems, algorithms are also available, however, accuracy and convergence are much larger problems.

For fracture mechanics problems, the finite element method can be employed in the standard manner or modified to account for the singular nature of the near crack fields. A summary of these methods can be found in Liebowitz and Moyer (1987), and they are treated in this paper with specific application areas. As discussed in the next section, calculation of two dimensional stress intensity factors for LEFM problems is commonplace and can be performed using commercial software by the average user (any commercial code containing a quadratic displacement element, e.g. MARC, ABAQUS, NASTRAN, ADINA can be employed). Indeed, this application is the most successful example of the use of LEFM and FEM in design. For three dimensional LEFM problems, it is often more difficult. The next section discusses these issues in more detail.

A major problem with the finite element method is the design of an appropriate mesh. While much experience has been gained in the past twenty years, finite element mesh design (especially in three dimensions) is more of an art than a science. Automated mesh design is still an emerging discipline and all known algorithms produce unrealistic meshes for problems containing cracks. Another major problem with the finite element method is the prediction of error. This is an area where promising research is ongoing, however, the current state of the art is not very accurate.

The FEM has been widely employed for the solution of nonlinear fracture problems. Problems involving ductile crack growth, creep crack growth, fatigue and large deformation can be addressed accurately using the FEM. Often, however, these calculations are extremely time consuming and expensive [for example, a modest amount of creep crack growth was modeled by Moyer and Liebowitz (1987) and required 75 CPU hours of VAX 11-780 time to reach a converged solution]. Understanding the results and establishing converged solutions in the nonlinear regime is also more of an art than a science. While much research is needed and ongoing, the FEM is a useful numerical tool for addressing nonlinear fracture problems.

Of all the numerical approaches available, the FEM is the most widely employed and understood method for the solution of fracture mechanics problems. While the method continually evolves, the current state of the art is sufficient to address many important problems. Much understanding of fracture phenomena has arisen from numerical solutions of fracture problems. The remainder of this paper examines the key areas in fracture mechanics and highlights the impact of numerical methods (especially the FEM) on the field of fracture mechanics.

IMPACT ON LEFM

The ability to predict accurately the stress intensity factors for cracked elastostatic bodies using standard numerical techniques has greatly advanced the use of Linear Elastic Fracture Mechanics concepts in application. Both BIEM and FEM technologies have been developed for the prediction of stress intensity factors for cracked bodies of arbitrary geometry and loading. In two dimensions, this is a mature technology which can be routinely employed. Most commercial FEM and BIEM codes have 2-D elastostatic fracture capabilities built-in and automated. A minimal amount of user knowledge is required.

The Griffith energy release rate theory of LEFM is widely accepted as a design criteria for fracture proof design. The application of this theory reduces to determining the largest possible flaw in a design which can

exist in a subcritical state. If this flaw size is larger than an acceptable flaw size, then the design is considered safe assuming that only elastic deformation is present. The flaw size is determined by assuming the existence of a flaw at a location of stress concentration and calculating the energy release rate associated with a virtual extension of that flaw. In practice, it is not necessary to calculate the energy release rate as this can be related to the stress intensity factor of the crack. It is sufficient, therefore, to obtain accurate numerical stress intensity factors for arbitrary geometries and loadings.

The Griffith approach to fracture proof design is, unfortunately, inadequate for many applications. Real structures and components develop cracks which are three dimensional in geometry and which are not subjected solely to tensile opening. The Griffith criteria requires that the crack can be idealized as a two dimensional line of discontinuity and that the remote loading is tensile and normal to the crack line. It is necessary, therefore, to look beyond the Griffith criteria for many applications.

Often, anticipated cracks in components and structures form due to a stress concentration near a free surface. This phenomenon gives rise to surface cracks which cause many real world failures. In many cases, it is acceptable to assume that cracks which will form are subjected to pure tensile opening but cannot be idealized to a two dimensional line crack. In this situation, the stress intensity factor along the crack front can be used as a design criterion with acceptable safety for interior cracks. The problem, therefore, reduces to finding the distribution of stress intensity along a three dimensional crack front where the crack is subjected to pure tensile loading.

The prediction of three dimensional stress intensity factor distributions is not a straightforward process. Research in this area has been ongoing for almost ten years in earnest (some work was performed prior to the late 1970s, however, due to the limited computer resources available, the results were not accurate). FEM approaches are basically extensions of the two dimensional technology presented previously. Special singular elements have been proposed by Tracey (1973), Blackburn and Hellen (1979), Hilton (1977) and others. These singular elements are based on employment of the asymptotic displacement field in the finite element formulation directly. The element geometry is the same as the corresponding standard elements (usually either tetrahedrons or triangular prism elements as shown in Fig. 3). As analyzed elsewhere (Liebowitz and Moyer, 1987; Moyer, 1988), these approaches require the assumption of a local state of plane strain near the crack front which has not been established analytically. They must be utilized, therefore, with discretion.

As a desirable alternative to special finite elements, Henshell and Shaw (1975) and Barsoum (1976) noticed that distortion of the placement of midside nodes in higher order isoparametric elements leads to a singular strain in the element. The fifteen node prism with quarter point nodal displacement is shown in Fig. 4. While many singular fields are attainable, the square root singularity of the sharp crack is obtained when the midside nodes of a quadratic element are displaced to the quarter point on the element edge. This can be exploited in virtually any finite element code with minimal user knowledge. The major drawback with the use of

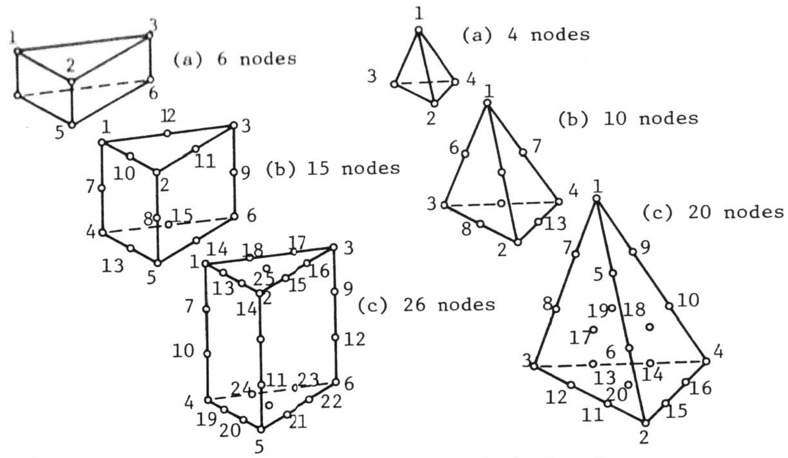


Fig. 3. Typical three dimensional tetrahedral and prism elements

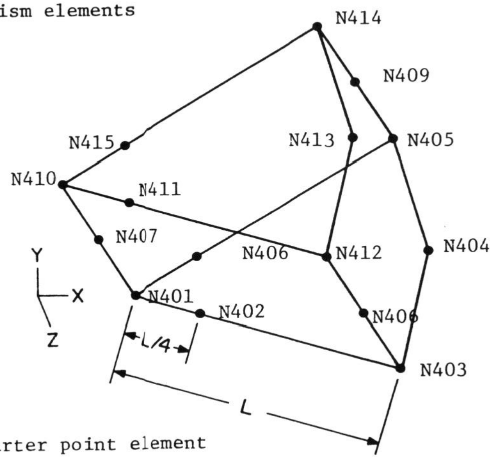


Fig. 4. Quarter point element

quarter point elements is the requirement of post processing the stress intensity factors from the FEM solutions.

Hybrid approaches to the solution of stress intensity factors in three dimensions are discussed extensively in the literature (for example, see the reviews in Liebowitz and Moyer, 1987 and Kuna, 1982). The basic concept behind hybrid elements is that neighboring elements can have different primary unknowns and can have different functional forms (e.g. one element could employ and assume stress distribution while a neighboring element employs an assumed displacement distribution). As suggested by the name, many formulations of hybrid elements can be advanced. Geometrically, hybrid elements are the same as conventional elements. Various authors have employed hybrid elements to solve three dimensional LEFM problems. Accurate and dependable solutions can be achieved using hybrid elements. The complexity of their formulation, however, makes computational implementation more difficult than with conventional elements. In addition, because uniform convergence is not guaranteed even for linear

problems, the use of hybrid elements is restricted to experienced analysts.

As stated previously, it is usually necessary to obtain stress intensity factors from FEM field solutions. After much research over the years, two approaches have emerged: the multi-term displacement field approach and the nodal force approach. The asymptotic displacement field for a stationary, three dimensional elastic crack assuming a local state of plane strain is given by

$$u_1^{\ell}=\left(\frac{1+\nu}{E}\right)\left(\frac{2r}{\pi}\right)^{\frac{1}{2}}\left\{K_I\cos\frac{\theta}{2}\left[(1-2\nu)+\sin^2\frac{\theta}{2}\right]+K_{II}\sin\frac{\theta}{2}\left[2(1-\nu)+\cos^2\frac{\theta}{2}\right]\right\}$$

$$u_2^{\ell}=\left(\frac{1+\nu}{E}\right)\left(\frac{2r}{\pi}\right)^{\frac{1}{2}}\left\{K_I\sin\frac{\theta}{2}\left[2(1-\nu)-\cos^2\frac{\theta}{2}\right]-K_{II}\cos\frac{\theta}{2}\left[(1-2\nu)-\sin^2\frac{\theta}{2}\right]\right\}$$

$$u_3^{\ell}=2\left(\frac{1+\nu}{E}\right)\left(\frac{2r}{\pi}\right)^{\frac{1}{2}}K_{III}\sin\frac{\theta}{2}\quad. \tag{4}$$

The local coordinate system is defined in Fig. 5. The multi-term displacement approach is the result of many years of numerical experimentation with various asymptotic displacement formulations. The method currently employed was formulated by Ingraffea (1980) and has many followers. The multi-term displacement formulation employs the asymptotic displacement field together with some higher order terms. Accurate predictions are obtained if a local state of plane strain exists near the point of stress intensity factor evaluation.

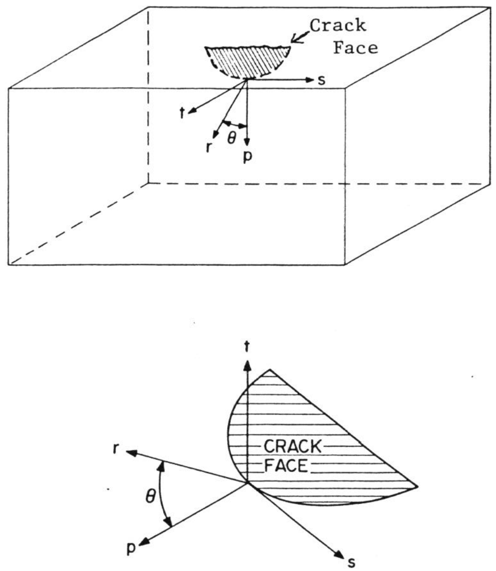


Fig. 5. Three-dimensional crack geometry

The alternative method is the nodal force approach. This approach was proposed by Raju and Newman (1977) and is based on integration of the asymptotic stress field given as

$$\sigma_{ij} = \frac{k_1(s)}{\sqrt{r}} f_{ij}(\theta) + \frac{k_2(s)}{\sqrt{r}} g_{ij}(\theta) + \frac{k_3(s)}{\sqrt{r}} h_{ij}(\theta) \quad (5)$$

where the functions f,g and h are known. It has been employed to solve a wide variety of tensile opening problems. From a theoretical viewpoint, the nodal force method is more acceptable than the multi-term displacement method. The nodal force approach utilizes the asymptotic stress field instead of the asymptotic displacement field. This approach, therefore, does not require the assumption of plane strain in the neighborhood of the crack front. Due to the assumption of local plane strain, the multi-term displacement method can yield erroneous results where crack front curvatures are large or as the crack front approaches a free surface. Unfortunately, the displacement approach is widely utilized.

An alternative approach to the understanding of LEFM phenomena involves calculation of energy release rates directly without the concern for stress intensity factors. Stress intensity factors can be predicted from these calculated energy release rates. For two dimensional problems, many path independent integrals can be related (with certain assumptions as to the direction of crack advance) to energy release rates and stress intensity factors. The famous J integral (Rice, 1968) is one example of these. The J integral is defined as

$$J = \int_\Gamma \left(U - \sigma_x \frac{\partial u}{\partial x} - \tau_{xy} \frac{\partial v}{\partial x}\right) dy + \int \left(\tau_{xy} \frac{\partial u}{\partial x} + \sigma_y \frac{\partial v}{\partial x}\right) dx \quad (6)$$

where the geometry is as defined in Fig. 6. Although J is the most widely employed path independent integral, which is related to the stress intensity factors, many alternate formulations are available. From a computational standpoint, choice of which path independent integral to employ is made based on ease of computation for the geometry and loading involved. Discussions on this approach for two dimensional problems can be found in Dexter (1987) and Liebowitz and Moyer (1987).

Extension of global energy methods to three dimensional problems is difficult, but not impossible. For pure mode I problems, two basic approaches are available: the virtual crack extension method (VCEM) (Parks, 1974) and the virtual crack closure technique (VCCT) (Newman, 1988). The VCEM works by calculating the change in stiffness produced by a small amount of crack extension into the near tip material. The VCCT works by calculating the energy required to close the crack by a small amount. Both of these methods produce an estimate of the pointwise energy release rate along a three dimensional crack front. Stress intensity factors can be obtained, therefore, assuming either plane stress or plane strain. The energy release rate calculation, however, does not require the assumption of plane stress or plane strain. Because the standard LEFM fracture criterion is based on energy release rate and not stress intensity factors, these energy methods may be quite useful for three dimensional opening mode fracture problems. The VCCT method has the computational advantage of only

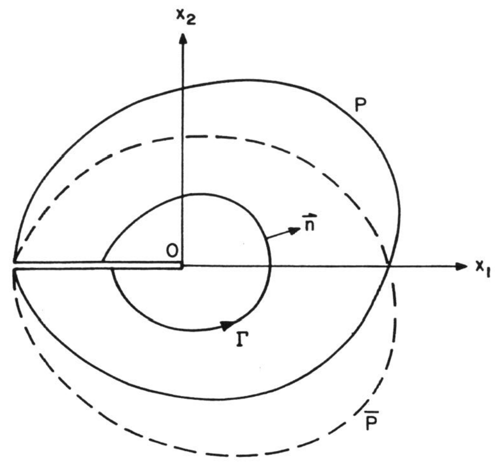


Fig. 6. Geometry for J integral calculation

requiring knowledge of the nodal forces and displacements. The computation is not based on the stress or strain fields from the analysis. The VCCT method can often predict reasonable results without the use of singularity elements.

To demonstrate the capabilities of these techniques, consider the problem of a semi-circular surface crack at the edge of a notch as shown in Fig. 7. This problem recently has been solved using the COD method, the force method and the VCCT (Newman, 1988). The results are shown in Fig. 8 using both singular and nonsingular finite element analyses. While all three methods produce good results, the VCCT results are good even without the employment of singular elements. This is the major advantage of the method.

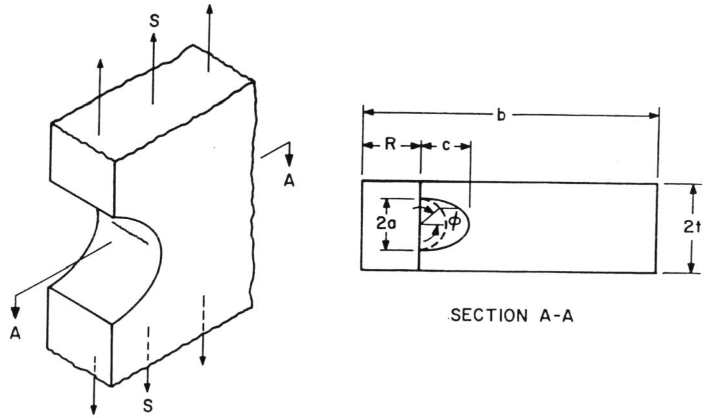


Fig. 7. Specimen configuration and loading

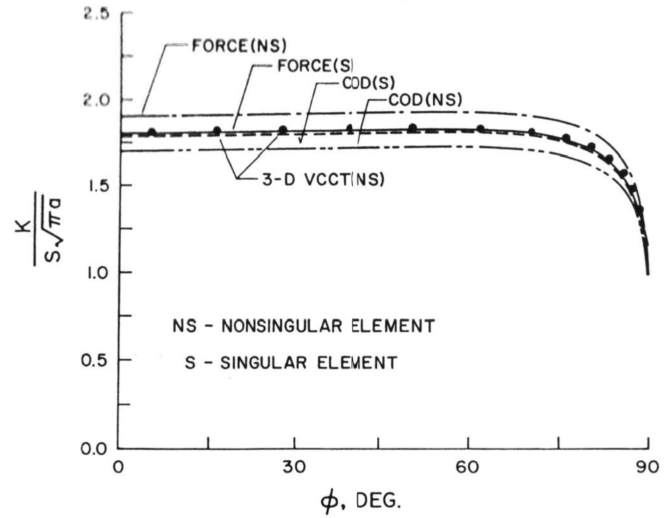


Fig. 8. Comparison of normalized K calculated from 3-D VCCT, force, and COD methods.

Global energy methods, unfortunately, are not very useful for mixed mode problems. They require knowledge of the direction of incipient crack growth. In general, this information is unavailable. Several mixed mode crack direction criteria exist which could be employed to estimate the direction of crack advance, however, the computational results are quite sensitive to advance direction on the basis of experience with two dimensional problems (Liebowitz and Moyer, 1987). Application of these methods to three dimensional mixed mode fracture problems is not expected to be very successful.

The BIEM can be employed for the solution of three dimensional crack problems in much the same way as the FEM. Much work in two dimensions demonstrates that the BIEM can produce quite accurate solutions to fracture problems and is computationally competitive with FEM. For three dimensional problems this may not be the case. BIEM methods tend to be quite expensive for three dimensional calculations, often much more expensive than the FEM. In addition, studies to date have not resolved the stress intensity distribution detail for the standard three dimensional surface crack problems used as benchmarks in three dimensions. Summary of this work to date can be found in Cruse (1987).

In addition to the study of static LEFM problems, much work has been performed for dynamic LEFM problems. In the dynamic case, two problems are important: that of a running crack and that of a static crack with elastic waves impinging. The problem of stress intensity factor calculation for static cracks in elastic materials subjected to time dependent loading is no more difficult than the corresponding static problem. The same solution methodologies are employed and the results can be calculated to the same accuracy. Computational requirements are greater, however, no new problems arise numerically.

The problem of a running crack in an elastic material is much different from the problem of a static crack. FEM solutions have had a major impact

on this area. Few analytic solutions to realistic problems are available (even in two dimensions), therefore, robust numerical approaches are essential. The first realistic solution to the problem of a running crack was presented by Anderson and King (1977). They introduced a nodal release algorithm which models the changing boundary conditions of a growing crack. The method has proved to be very robust and easy to implement -- even in commercial finite element codes -- and is widely employed. This algorithm has allowed many researchers to study running crack problems for a wide variety of geometries and loadings. Many examples are available in the literature; a good review is given by Williams and Knauss (1985).

For two dimensional running crack problems, the decision to employ either conventional elements or singular elements is not totally established. Many authors have produced excellent results to difficult problems employing only conventional elements (e.g. Williams and Knauss, 1985). Other authors employ singular elements with equal accuracy and claim computational superiority (e.g. Liebowitz and Moyer, 1987). Many examples exist in the literature, however, where one method or the other produce marginal results to seemingly simple problems. These examples demonstrate that the modeling of running cracks require careful study of solution convergence and stability for each new problem. This aspect is probably more important than the choice of element type.

An interesting example of dynamic crack propagation simulation involves the problems of interacting cracks. Consider the problem of two cracks in a sheet which are opened by wedge loads (Swenson and Ingraffea, 1987). The geometry is shown in Fig. 9 where the arrows indicate the wedge loading. The cracks are slightly misaligned to provide initial asymmetry. Figure 10 shows the cracks at three stages of the analysis. Initially, the cracks repel each other and, as propagation continues, they attract. At the final stage, the two cracks intersect. Figure 11 shows the stress intensity factor histories as a function of crack length. The positive mode II component is evident during the avoidance stage and the negative mode II is evident during attraction.

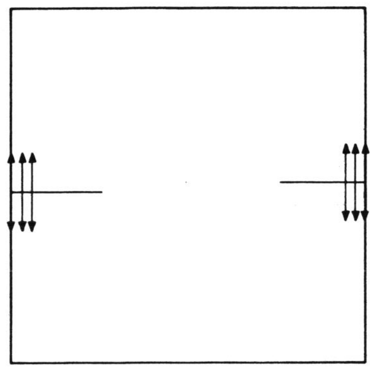
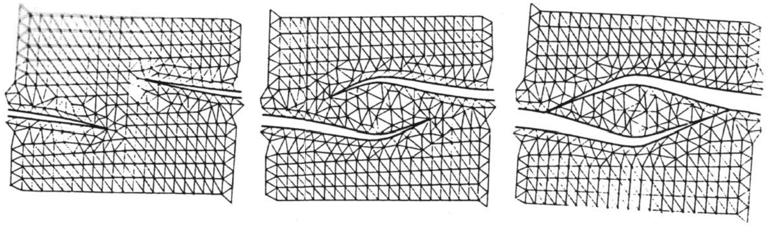
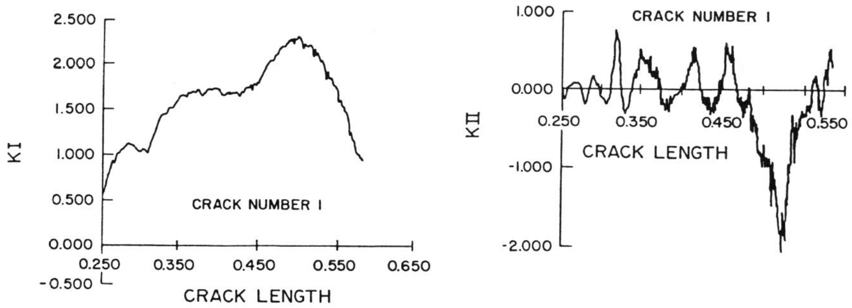


Fig. 9. Geometry of problem



(a) Crack avoidance (b) Attraction (c) Final separation

Fig. 10. Displacement plots during crack propagation (width of detail 0.25 inches)



(a) Mode I (b) Mode II

Fig. 11. Calculate stress intensity histories

An alternative approach to modeling running cracks involves the use of a moving element. Atluri and others have employed this concept in their moving singular element studies (Atluri and Kathisesan, 1980). They use a hybrid element which produces reasonable results. The main problem with the hybrid approach is the complexity of formulation and the difficulty in implementation. In addition, hybrid elements tend to behave unpredictably in convergence studies, making their routine employment problematic. Few authors have pursued this approach. Moving crack tip elements can be formulated with standard elements. This has been done for the case of ductile crack growth, but not for dynamic fracture. The computational

requirements for moving elements, however, appear to be on the same order as for standard approaches with nodal release (Moyer and Liebowitz, 1984).

To date, little work has been done on three dimensional dynamic problems. The few studies in the literature employ numerical modeling too coarse for the obtaining of accurate results. This is mainly due to the prohibitive computational costs for three dimensional dynamic fracture calculations and the lack of experimental data for comparison.

For problems of two dimensional LEFM, the FEM and BIEM technology available today is sufficient for the solution of most problems. The convergence nature and the computational requirements are well established, and routine employment is feasible and, in fact, in place. For three dimensional problems, however, much more research is required before routine analysis can be performed. Specifically, the problem of the intersection of a crack front with a free surface is still an open question. In addition, the optimal methodology for calculation of mixed mode fracture parameters (assuming the desired parameter is definable) is still open. Research in this area is needed and discussed in depth in a later section.

PROBLEMS OF DUCTILE FRACTURE

Numerical methods have probably provided more understanding of the field of ductile fracture mechanics than of any other discipline in the field of fracture. Due to the extreme mathematical complexity of the problems, analytic methods are able to provide only qualitative insight for very idealized situations. Often, the physical processes of interest are lost due to simplifications required for the attainment of an analytic approximation. Numerical methods, therefore, are required for fundamental understanding of the basic physical problem.

For the purposes of this discussion, ductile fracture will be divided into four subtopics: elastic-plastic problems without subcritical crack growth, elastic-plastic problems with subcritical crack growth, rate dependent plasticity (or creep) problems without subcritical crack growth, and rate dependent plasticity problems with subcritical crack growth.

The problem of elastic-plastic (rate independent) fracture without prior, subcritical crack growth is the easiest problem to study. An extension of the energy release rate concept to nonlinear elastic materials appears to provide a suitable fracture criterion. Although computation times may be long, two dimensional geometries can be analyzed quite accurately with current FEM technology. This approach can be extended to three dimensional mode I problems through the use of domain integrals. The energy release rate for a stationary crack in a monotonically responding medium can be written as (Shih et al., 1988)

$$\bar{G} = \int_V \Big[\sigma_{ij}\frac{\partial u_i}{\partial x_k}\frac{\partial q_k}{\partial x_j} - (W+L)\frac{\partial q_k}{\partial x_k} + \rho\Big(\frac{\partial^2 u_i}{\partial t^2}\frac{\partial u_i}{\partial x_k} - \frac{\partial u_i}{\partial t}\frac{\partial^2 u_i}{\partial x_k \partial t}\Big)q_k + \Big(\sigma_{ij}\frac{\partial^2 u_i}{\partial x_j \partial x_k} - \frac{\partial W}{\partial x_k}\Big)q_k\Big]dV \qquad (7)$$

where W is the strain energy density, L is the Lagrangian of the system, u_i is the displacement vector and q_k is a weight function. This formulation holds for static and dynamic responses. The major limitation is the required assumption of monotonic field response and a stationary crack front. This parameter is useful for characterizing fracture which is not preceded by ductile tearing or any non-monotonic load history. Unfortunately, many real fracture problems do not satisfy these criteria.

An important problem for the prediction of the early part of fracture life is the study of cracks subjected to mixed mode loading. The problem of ductile crack solutions for mixed mode problems has been addressed numerically by many authors in the past (e.g. Shih, 1974 and Moyer and Haegele, 1988). No mixed mode ductile fracture criteria, however, have been established, therefore, these solutions can only provide insight into field distributions. While this is an important contribution, fundamental work toward a general fracture criteria is needed.

A major problem in extending the current knowledge of ductile fracture is the lack of understanding of the local crack tip (or crack front in three dimensions) fields in the elastic-plastic regime. For two dimensional problems with highly restrictive constitutive relations, an asymptotic analysis has suggested fields in the form

$$\sigma_{ij} = \left(\frac{J}{\alpha\varepsilon_y\sigma_y I_n r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}(\theta) \qquad u_i = \left(\frac{J}{\alpha\varepsilon_y\sigma_y I_n}\right)^{\frac{n}{n+1}} r^{\frac{1}{n+1}}\tilde{u}_i(\theta) \tag{8}$$

$$\varepsilon_{ij} = \left(\frac{J}{\alpha\varepsilon_y\sigma_y I_n r}\right)^{\frac{n}{n+1}}\tilde{\varepsilon}_{ij}(\theta) .$$

The required assumptions for the derivation of these equations, however, are met only in a negligibly small region near a crack tip. Many numerical studies over the past 15 years have demonstrated that these fields fail to characterize the material response in physically realistic dimensions. A recent study in three dimensions could not demonstrate a region in which this solution dominates even on the plane strain plane (Parks and Wang, 1988).

To demonstrate the considerable complexity, a recent asymptotic study was undertaken numerically. The near crack tip region was studied by applying an elastic stress field (K dominant) to a region far from the plastic zone. Various mixed mode loading ratios were studied. The resulting stress, strain and energy fields were predicted for a power law hardening material. Small strain theory was employed. This problem employed the same assumptions and boundary conditions as in the asymptotic analyses. The major difference is that the elastic response is not ignored. Figure 12 is a log-log plot of the effective stress ahead of the crack tip for various mixed mode loading ratios. The mode I solution clearly delineates the transition from the elastic to the elastic-plastic regime. In the elastic-plastic region, the degree of singularity changes abruptly, transitions for a distance, and then continually increases toward the asymptotic. For no reasonable dimension, however, is the asymptotic solution demonstrated. For higher ratios of mixed mode loading, the transition decreases and approaches the asymptotic for pure mode II. The

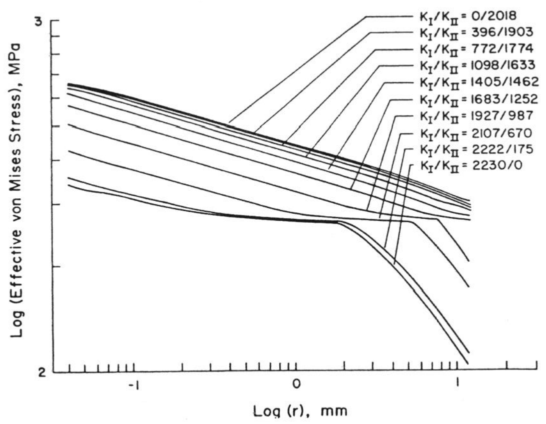


Fig. 12. Full logarithmic representation of effective von Mises stresses along line θ=0° for all of mixed-mode cases considered

assumption of geometrically infinitesimal deformations, however, is violated for mixed mode loading as significant rotation is observed. As is demonstrated in McMeeking (1987), it is critical to assess the influence of finite deformations before an understanding of the near tip stress fields can be obtained.

Most ductile fracture problems exhibit significant stable crack growth prior to final instability. The simplest problem of a mode I crack growing in a plane strain material yields a significant mathematical problem. Analytic attempts at asymptotic solutions produce dubious results at best. This is easily understood as the assumption required to attain the solution restricts its possible validity to a region so small that the assumption of a homogeneous, isotropic continuum must be suspect. For the static crack problem, the HRR field is often restricted to a zone which is of the order of the physical process zone (Dean and Hutchinson, 1980). Because asymptotic solutions for growing cracks are restricted to regions (on the average) of between 1/5 and 1/10 of the corresponding HRR region, the solutions are not very useful.

Fortunately, the problem of stable mode I crack growth has been extensively studied numerically over the past ten years. The nodal release algorithm is most frequently employed for the modeling of crack growth. Early investigations yielded qualitative insight into the effect of crack growth and local yielding on field variables relatively far from the crack (e.g. Dean and Hutchinson, 1980, and Hoff et al., 1986). These quantities were often "matched" with asymptotic solutions to construct a "full near field" solution. As large-scale computational facilities have increased, however, more sophisticated numerical studies have demonstrated that the asymptotic fields are not observable at finite distances from the crack. Instead, a continual transition is observed from the crack tip to the elastic field (assuming free surface effects do not complicate the phenomena). Fracture criteria based on assumed asymptotic dependencies, therefore, are not expected to be successful. The interested reader is referred to Dodds and Read (1988) and to Moyer and Kunze (1988).

In addition to nodal release approaches, several authors have proposed moving crack tip meshes with the changing boundary conditions modeled directly. This approach has been studied using both hybrid elements and conventional elements (Atluri and Kathisesan, 1980 and Moyer and Liebowitz, 1985). Comparison among algorithms using conventional elements has demonstrated that both the nodal release and the moving crack elements provide good, convergent solutions to problems of ductile crack growth. Unfortunately, the added complication of formulation of the moving element approach yields no significant computational advantages. Similar comparisons for hybrid elements are not currently available.

Crack growth criteria for the stable tearing regime can be divided into two categories: those based on global integral approaches and those based on local crack tip parameters. It has been suggested in the literature that the early stage of ductile fracture is characterized by a constant dJ/da rate and that subsequent growth is characterized by a constant crack opening angle (Hoff et al., 1986). Many other criteria have been proposed; however, they have been insufficiently compared with experiment to asses their utility. Stable crack growth criteria currently employed for the prediction of ductile tearing are summarized in Table 2.

Table 2. Summary of ductile crack growth theories

Criteria	Type	Comments
Tearing Modulus	Global	Contrary to experiment for most materials When applicable, range is extremely limited
Crack-tip Opening Displacement	Local	Definition is arbitrary Difficult to measure and verify
Crack-tip Opening Angle	Local	Difficult to measure and verify Definition is arbitrary
Strain Energy Density Theory	Local	Untested in application Material parameters difficult to determine
Plastic Energy Approach	Global	Parameters are specimen dependent
Stochastic Approaches	Usually local	Parameters are non-physical Difficult to verify for application

To evaluate crack growth criteria, numerical simulation of experimental histories are required. Such simulations, however, must employ the raw experimental data as the only input without the bias of the fracture parameter being studied. Few such simulations have been performed to date. A recent study, however, followed these criteria for a common steel material (Moyer and Kunze, 1988). The results indicate that the dJ/da rate is not constant over any reasonable proportion of crack growth in a standard test specimen. The J integral versus the crack extension is plotted in Fig. 13, and the crack tip opening angle is plotted against crack extension in Fig. 14. For large amounts of crack growth, the crack opening angle appears to be approaching a constant value. A more stable parameter appears to be the slope of the crack tip opening displacement versus crack extension curve (Fig. 15). For large amounts of crack growth, the curve becomes extremely linear. Even in the short growth range, the linearity is reasonable. Unfortunately, any crack opening parameters are quite sensitive to the choice of definition. It is impossible to obtain an unambiguous and specimen independent definition. Employment of these criteria, therefore, is limited to a modeling role and should only be employed until a valid, field variable based, local crack extension criterion is established.

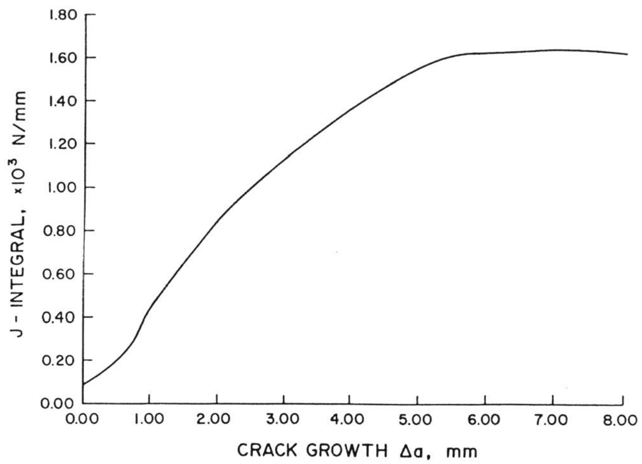


Fig. 13. J-integral vs crack growth

Similar trends can be seen in the literature regarding rate dependent fracture problems. A wide class of applications involves materials which are rate sensitive. For extremely simple constitutive assumptions, FEM studies have been successful at obtaining accurate solutions. The most widely studied rate dependent problem is that of secondary creep (both in the transient and steady state conditions). Static studies have been performed historically; however, as in ductile fracture, most problems of interest involve stable crack growth. Stable crack growth in a creeping solid has only recently gained attention in the literature. This is primarily due to the extreme run times required to study problems of interest. The available studies, however, indicate the same trends as for rate independent materials (e.g. that the asymptotic predictions hold primarily for stationary cracks, and that growing crack asymptotics are not observed at finite distances; see Hawk and Bassani, 1986 and Moyer and Liebowitz, 1987).

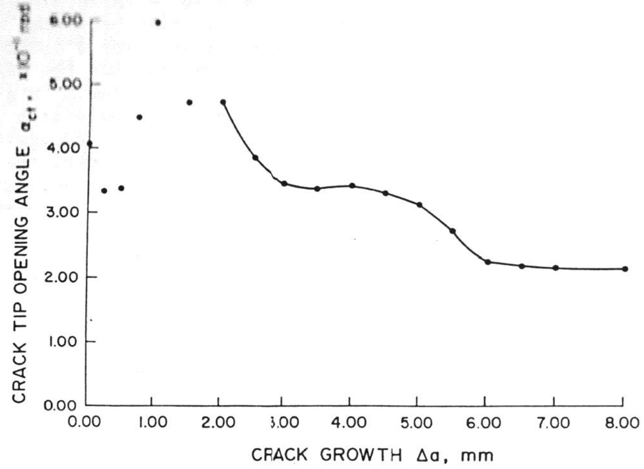


Fig. 14. Crack tip opening angle vs crack growth

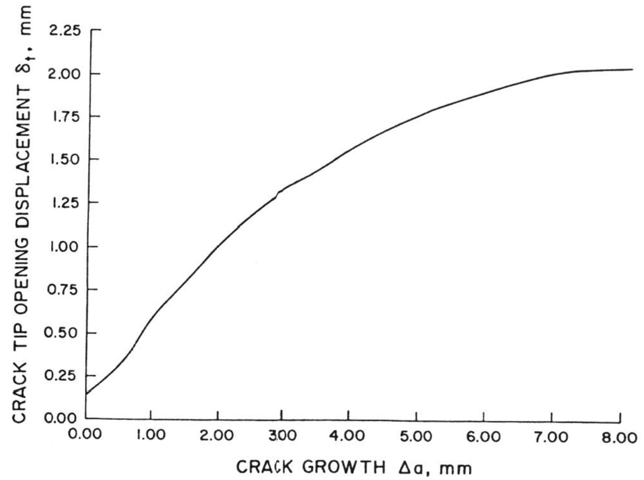


Fig. 15. Crack tip opening displacement vs crack growth

An important aspect of ductile fracture which is starting to receive attention by researchers is the effect of finite deformations on the local crack tip fields. The existence of significant yielding indicates that strain and displacement fields cannot adequately be assumed to be infinitesimal. For both rate independent materials and rate dependent materials, recent research has revealed that finite deformations significantly effect the local stress and strain response of the material (McMeeking, 1987; Moyer and Liebowitz, 1987). Even for a static crack, significant effects can be observed (McMeeking, 1987). Because fracture

criteria for stable crack growth will probably involve local field quantities, studying the effects of finite deformation will be imperative. This is even more important for criteria based on local crack opening characteristics. While geometric nonlinearities often increase computational costs, if the local crack region is modeled with a very fine mesh, the large displacement problem may be cheaper to solve as the field may not be singular (as in the infinitesimal theory problems).

An interesting comparison between the asymptotic two dimensional, plane strain fields in the transient, power-law creep problem recently has been studied (Moyer and Liebowitz, 1987). In this problem, the short time solutions were examined while crack opening was still small (so that the asymptotic fields would remain dominant). Figure 16 shows the von Mises stress ahead of the crack tip. The problem was solved numerically using both small and finite strain theory. The problem parameters were chosen so that the transition zone would be negligible and an abrupt transition from the elastic to the creep singular zones would occur. The agreement between the small strain finite element solution and the asymptotic solutions demonstrate this effect. The large deformation solution shows significant differences even for this problem in which deformations are quite small. The elements very near the crack tip experience large enough strains that the entire singular field is influenced. This preliminary study clearly indicates that, as in the elastic-plastic case, finite strain analysis is required to understand ductile and creep fracture problems.

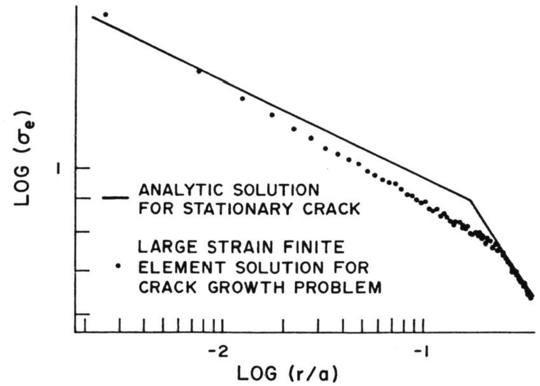


Fig. 16. Log-log plot of asymptotic effective stress ahead of the crack tip after crack growth (P-1500 lb)

Three dimensional ductile fracture studies for stationary cracks are starting to appear in the literature. As mentioned previously, the domain integral approaches are being explored for fracture characterization in the absence of stationary crack growth. The advantage of the domain integral approach is that local solution state need not be accurately represented or predicted. Unfortunately, the local state is the important issue. Current literature is starting to address this problem, however, this work is in its infancy and no conclusions are validated to date.

Numerical solution of ductile crack growth problems provides the opportunity for full field simulation of a complicated physical phenomenon. Laboratory experiments, while extremely important, cannot provide the full field data required to establish a fracture theory to

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predict stable crack growth. Numerical simulation holds great potential for this use. As discussed subsequently, the need for development of local stable crack growth criteria and their testing by numerical experiment (or computer simulation) is one of the major requirements of current fracture research. FEM solutions should provide the tool to address this research.

SUMMARY OF THE STATE OF KNOWLEDGE NEAR CONTINUUM CRACKS

To fully understand the fracture of solids from a continuum scale, several pieces of knowledge are required: the field variables must be known near the fracture point (stress, strain, displacement, strain rate, etc.), the thermodynamic laws governing the fracture processes (e.g. stable tearing, ductile rupture, brittle rupture) must be known and the coupling of these must be understood. The theoretical models of the constitutive and fracture behavior of materials must be postulated, understood and verified for real materials. Numerical methods can aid in this process by obtaining solutions to mathematical problems which cannot be solved in closed form; however, numerical solutions cannot provide any information as to the accuracy or validity of postulated theoretical models.

In two dimensions, the asymptotic nature of the field variables is known for elastic materials, as was discussed previously (Eqs. 4 and 5). These are accurate near the crack tip, and the body geometry influence is described by the stress intensity factor. If the crack is near enough to a geometrical boundary, however, the above relations do not adequately describe the stress state (see Fig. 5). Because this rarely occurs in applications, it does not represent an important problem. For two dimensional, plane strain fracture, knowledge of the stress intensity state is sufficient to predict the onset of brittle fracture. The failure curve (in stress intensity space) is geometry independent and accurate predictions can be made.

For elastic-plastic materials which do not exhibit stable crack growth prior to ductile instability, the local field parameter response is given in Eq. (8) for a power law hardening material. For other simple constitutive relations, similar expressions can be derived analytically. These solutions are two dimensional (either plane stress or plane strain). The loading must be monotonic and no large strain effects can be present. When all these criteria are met, the asymptotic solutions are valid but only in a very small region near the crack tip. In many applications, they are not valid beyond estimates of the process zone size. The utility of these relations, therefore, is extremely restricted. Under these very restrictive assumptions, the J integral represents the energy release rate for mode I failure and can be employed as a fracture criterion. In most applications, however, this is a poor approximation.

For stationary ductile cracks, numerical solutions exist which describe the local stress, strain and displacement fields. Small strain solutions have been used to study the transition zone between the highly restrictive ductile asymptotic zone and the elastic asymptotic zone. Typical mode I yield zones are shown in Fig. 17. This work has been done for both mode I and mixed mode problems in two dimensions (e.g. Shih, 1974; Moyer and Kunze, 1988; Moyer and Haegele, 1988). The transition zone is the region which probably characterizes the local state most accurately and is the region fracture criteria must address. To date, however, the only local fracture criteria for two dimensional ductile tearing are based on crack opening profiles or strain energy density arguments (e.g. Moyer and

Haegele, 1988; Sih, 1985; K. Hellen, 1984). Crack opening criteria are difficult to employ due the fact that these quantities are difficult to measure or precisely predict in an unambiguous manner. More research in this area must be performed to accurately compare numerical and experimental predictions under controlled growth conditions.

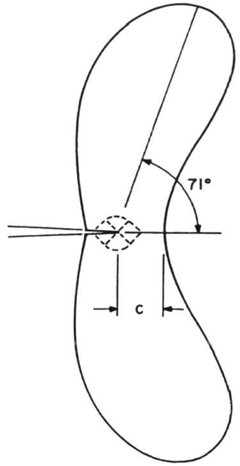

Fig. 17. Shape of the ideal-plastic small-scale yield zone in plane strain, mode I.

Strain energy density approaches for the prediction of ductile crack growth have been proposed for many years. While a large body of literature has grown in this area, little in the way of experimental verification exists. The primary reasons are that the material properties in the formulation are difficult to obtain and the approach is highly sensitive to constitutive and large strain assumptions (Sih, 1985). Recent work seems to indicate that ductile tearing is an essentially three dimensional process and that two dimensional approaches (such as crack opening profiles) may not be able to predict ductile fracture. Strain energy density approaches are attractive because they address the problem of element fracture without unnecessary assumptions concerning global response. Much more work in comparing predictions with experiments is required to establish or nullify the approach.

A third local approach to predicting fracture employs continuum damage theory. A constitutive model of a postulated damage parameter is proposed as a function of local stress, strain, strain rate and deformation. A typical damage evolution equation can be given in the form

$$\dot{D} = f(\underset{\sim}{\sigma}, \underset{\sim}{\varepsilon}, \dot{\underset{\sim}{\varepsilon}}, D) \tag{9}$$

where D is the damage parameter. The equation is integrated over the loading history and the damage law predicts the material failure dynamically. This approach is being widely researched in Europe and appears promising (Chaboche, 1987; Benallal et al., 1987). Unfortunately, continuum damage is largely ignored by American fracture researchers. The most common approach to addressing ductile fracture in the United States is through the use of global integrals. The J integral was the first to be employed for fracture resistance characterization in the ductile region. Unfortunately, the predictions are far from observed material response unless the loading is very near the brittle range. Crack growth and ductile rupture cannot be predicted accurately. Alternative integrals have been proposed which address the restrictions of the J integral to some degree. Unfortunately, none of these address the material failure or load history problem. In addition, the global integral approaches require unrealistic proportionality assumptions which are not valid for ductile tearing. In spite of the lack of success, much research is focused in this area. It is interesting that research on global integral approaches to address ductile rupture has been largely abandoned except in the U.S.

The problem of fracture at elevated temperatures is very similar to the problem of ductile tearing. Many similarities between viscoplastic and elasto-plastic fracture can be observed in fracture data. For extremely restrictive constitutive models, static crack asymptotic solutions analogous to equation (8) can be established. For example, for steady state power-law creep, the stress and strain rate fields are given by (Reidel and Rice, 1980)

$$\sigma_{ij} = \left[\frac{C(t)}{BI_n r}\right]^{1/n+1} \tilde{\sigma}_{ij}(\theta) \qquad (10)$$

$$\dot{\varepsilon}_{ij} = \left[\frac{C(t)}{BI_n r}\right]^{n/n+1} \dot{\tilde{\varepsilon}}_{ij}(\theta)$$

where

$$C(t) = \lim_{\Gamma \to 0} \int_\Gamma \left(\frac{n}{n+1}\right) \sigma_{ij}\dot{\varepsilon}_{ij}dx_2 - \sigma_{ij}n_j \frac{\partial \dot{u}_i}{\partial x_1} ds \; . \qquad (11)$$

Unfortunately, creep fracture always exhibits stable crack growth prior to catastrophic failure. These solutions, therefore, have little utility. Early work in creep fracture attempted to employ global integrals to the problem of predicting creep crack growth. As with the J approach in ductile failure, global integrals were not successful. Little additional work has appeared in the literature with the exception of some recent continuum damage predictions (e.g. Walker and Wilson, 1984; Chaboche, 1987). This work is promising but not yet mature. To date, strain energy approaches have not been applied to viscoplastic problems.

For three dimensional brittle geometries, the deformation and stress state near the crack front has the same asymptotic form as for the two dimensional case. The stress intensity factors vary as a function of position along the crack front. This solution holds except near the intersection of the crack front with a free surface. The dimension of this "boundary layer" region and the nature of the fields in this region have received much attention with little conclusion. Work on this question is ongoing and recent studies -- both analytically (Folias, 1988) and numerically (Cruse, 1987) -- are beginning to provide qualitative insight. Resolution of this problem is still pending.

Prediction of brittle failure in three dimensions is accomplished using pointwise energy release rate predictions. This approach works well for many practical problems provided the crack extension is planar (as is the case of pure mode I, II or III or for combined modes I and III). For noncoplanar fracture problems, however, the issue remains open. Further research is needed in the area of three dimensional mixed mode brittle fracture.

For ductile problems, as indicated previously, the prediction of fracture may be inherently three dimensional for all problems. This could explain the lack of success of the current approaches in the literature. The deformation and stress state near a three dimensional ductile crack front is just now being studied. Due to the difficult nature of three dimensional nonlinear problems, little analytical work exists to address this problem. Because large, high speed computers are becoming more available, three dimensional ductile fracture problems are being addressed by many researchers. Preliminary results (e.g. Parks and Wang, 1988; Moyer et al., 1986) indicate that two dimensional asymptotics do not dominate the near crack region in three dimensions. As suspected, large strain effects appear important and must be modeled. The problem appears very complicated and much additional research is needed. Fortunately much work is ongoing.

Dynamic fracture phenomena (where inertia effects are important) constitute an important problem in many applications. The two dimensional brittle fracture problem is well understood for the dynamic case. The dynamic stress intensity factor represents a valid fracture criterion in this regime. The asymptotic deformation and stress fields have the same form as in the static case except that the stress intensity factor becomes time dependent. For example, a mode I crack running with speed v has the asymptotic stress field given by (Williams and Knauss, 1985)

$$\sigma_{\alpha\beta} = \frac{K(t,v)}{\sqrt{2\pi r}} f_{\alpha\beta}(\theta,v)+O(1) \qquad \alpha,\beta = 1,2 \text{ as } r\to 0 \qquad (12)$$

where r and θ are the polar coordinates centered at the crack tip. Material fracture resistance is well understood for brittle dynamic fracture. In three dimensions, crack front energy release rate is employed analogous to the static problem. Limited experimental verification indicates that this approach is promising.

Dynamic ductile fracture has received some study; however, little progress has been made in the prediction of fracture phenomena. No asymptotics exist for this problem and little is known about the nature of the deformation response. Numerical studies have simulated experiments reasonably well, although no fracture prediction methodology has resulted. The fields near the crack have not been studied adequately. Some three dimensional work exists; however, this work has focused on global, qualitative comparisons. A recent study focused on determination of the region where the HRR field given by (Parks and Wang, 1988)

$$\sigma_{ij}(r,0) \to \sigma_0 \cdot [J/(\alpha\varepsilon_0\sigma_0 I_n r)]^{\frac{1}{n+1}} \cdot \tilde{\sigma}_{ij}(0,n) \equiv \sigma_{ij}^{HRR}$$

$$\varepsilon_{ij}(r,0) \to \alpha\varepsilon_0 \cdot [J/(\alpha\varepsilon_0\sigma_0 I_n r)]^{\frac{n}{n+1}} \cdot \tilde{\varepsilon}_{ij}(0,n) \equiv \varepsilon_{ij}^{HRR} \; . \qquad (13)$$

This study demonstrated little dominance when the plastic zone was not negligible relative to the crack dimensions. The field of three dimensional, dynamic ductile fracture is a new frontier.

In general, asymptotic relationships, where applicable, characterize fracture fields and provide the necessary fundamental understanding. When their applicability breaks down, however, little understanding exists. As an example, the problem of two or more interacting cracks demonstrates this. As long as the crack distance separation is large enough, the asymptotic solution is sufficient and the stress intensity factors relate the interaction information (Sih, 1978). If they are too close, however, the fields are unknown. Little work in this area (either experimental, numerical or analytical) has been performed. In light of recent interest in micromechanical modeling of fracture processes (e.g. void nucleation, void coalescence, crack-void interaction) this is an important issue which warrants further research. At the moment, no uniform method for determining the extent or validity of asymptotic solutions exists. The asymptotic behavior of the near-tip stress strain and energy fields for all fracture regimes are summarized in Table 3.

Table 3. Near crack field singularities

Problem type	Stress	Strain	Energy	Comments
2-D Elastic	$1/\sqrt{r}$	$1/\sqrt{r}$	$1/r$	Exact asymptotic solution
3-D Elastic	$1/\sqrt{r}$	$1/\sqrt{r}$	$1/r$	Restricted to interior domain
2-D Nonlinear Elastic	$r^{-1/n+1}$	$r^{-n/n+1}$	$1/r$	Applicable region very small - limited plastic size and constitutive behavior - limited to monotonic loading
2-D Creep	$r^{-1/n+1}$	$r^{-n/n+1}$	$1/r$	Applicable region very small - limited creep size and constitutive behavior - limited to monotonic loading
3-D Nonlinear	???	???	???	Unsolved to date elastic analytically - numerical solutions forthcoming
2-D Plastic with Local Unloading	???	???	???	Unsolved to date analytically - numerical solutions forthcoming
3-D Plastic with Local Unloading	???	???	???	Unsolved to date analytically - numerical solutions forthcoming

APPLICATION OF FRACTURE MECHANICS

Fracture mechanics has found a wide variety of applications over the years. The design of aircraft structures, aerospace structures, submarines, ships, land vehicles and civil structures employ fracture mechanics based rules in the design code standards. An excellent review of this topic can be found in Hellen (1987). For emerging disciplines, the characterization of fracture resistance (or fracture toughness) is a demanding and challenging extension of the technology.

From a fracture mechanics standpoint, the field of composite materials has many open questions with regard to theoretical issues. Numerically, however, linear elastic composite problems can be solved quite accurately using FEM or BIEM, albeit, often at great computational expense. The field of theoretical basis for composites needs to be greatly expanded before the numerical needs can be established. For the time being, the problems of interface cracks, free surface cracks and laminate interface modeling will occupy much numerical research effort. These problems are soluble with current technology; however, much effort will be required.

An important problem area where fracture mechanics finds application is in the prediction of fatigue crack growth rates and fatigue life. A full discussion of the problem of fatigue is beyond the scope of this paper. It is important to note, however, that while fracture mechanics approaches to fatigue crack propagation have been used for twenty-five years, only problems involving relatively long cracks in long-life (high-cycle fatigue) materials can be adequately addressed with these approaches. Work is ongoing and promising in this area, especially approaches based on local energy criteria. Unfortunately, the level of effort on this problem is not commensurate with its importance. More effort and funding are required in this important area as the majority of real world failures and component retirements are due to fatigue.

An area in which fracture mechanics technology is currently being tested for applicability is the simulation of metal cutting. For the prediction of tool wear, safe cutting speeds, optimal cutting depths, etc., it is desirable to have a predictive technology to determine the stress and strain fields arising from the process. While work in this area is complicated due to dynamic, thermal and strain rate effects, promising research is underway. Examples of this work can be found in Strenkowski and Carroll (1985).

Fracture mechanics has been successfully applied to the failure characterization of ceramic materials for many years. An excellent review of the fracture of ceramic materials can be found in Bradt et al. (1974). Ceramics often exhibit viscoelastic and anisotropic material behavior complicating their analysis; however, because they tend to be brittle, energy release rate predictions tend to characterize fracture quite well.

An area of active research in mechanics and material science is in the processing and performance characterization of thin film coatings. Coating base materials can provide increased strength and tribological properties for engineering components. The coated material often has superior performance capabilities to the base material alone. Fracture characterization of thin film coatings is receiving attention by researchers. Several approaches to fracture characterization have been proposed and are under study. Promising results indicate that methodology to measure fracture toughness and cohesive properties should be available in the next few years.

An application of thin film technology which is currently receiving much attention is the deposition of superconducting thin films. Mechanical characterization of superconducting thin films is complicated by the temperature sensitivity and anisotropy of these materials. Technology developed for characterizing room temperature thin films should be extensible to liquid nitrogen environments. Orthotropy in the coatings' mechanical properties, however, is an issue requiring further research.

Recent research on thin film technology has led to the development of nanostructured thin films. Typically these coatings are constructed of two materials alternately layered. The total coating may consist of as many as 500 layer pairs with layer thickness on the order of 10 angstroms. Different layer thicknesses and volume ratios produce varied material properties. The nanostructuring of these coatings often yield mechanical properties superior to those of a single material of equivalent coating thickness. Research is ongoing to determine optimal layer spacing and proportions for fracture resistance, tribological properties and strength. Current research indicates that single coating film techniques for fracture toughness determination may be applicable (with appropriate modifications) for the characterization of nanostructured thin films (Moyer et al., 1988).

Application of fracture mechanics principles and extension of the theory to emerging material technologies is one of the most challenging tasks for the field. Due to the complexity of the arising applications, computational approaches increasingly will be employed. Vigorous research is needed in these areas. Preliminary indications are that fracture mechanics will provide a framework for the characterization of many of these emerging applications.

FRACTURE MECHANICS RESEARCH NEEDS AND THE ROLE OF NUMERICAL ANALYSIS

Research needs in fracture mechanics are quite varied and still pose formidable tasks for engineers and scientists. The failure of isotropic, brittle materials is well understood today. Existence fracture theories for mode I failure are well established in two and three dimensions. Existing FEM and BIEM technology can routinely solve LEFM problems with the exception of the problems of interface crack and the intersection of a crack with a free surface. These problems require further study, although current research seems to be converging on an understanding of these phenomena.

LEFM problems involving mixed mode fracture have been less precisely theorized. Sufficient theoretical framework exists; however, it has not been tested adequately. Interest in mixed mode fracture of metals historically has been given less importance than warranted by the existence of mixed mode failures. This imbalance needs to be addressed if LEFM is to become a truly mature technology. Current numerical methods for two-dimensional mixed mode LEFM problems are adequate. Three dimensional problems can be solved; however, methods for extraction of relative fracture parameters (e.g. stress intensity factors or energy release rates) are not quite mature. When the appropriate parameters are theoretically established, existing numerical techniques can be evaluated. (In fact, such evaluations are ongoing as interest in three-dimensional composite

problems increases). Completing the development of numerical techniques for LEFM problems should be easily accomplished in the next several years.

The problems involving material nonlinearities constitute the greater challenge for both numerical analysts and fracture theorists. The need for a richer fracture theory which can predict the onset and propagation of a stable crack is by far the greatest research need. Such a theory, by the very local nature of the deformation and the complication of simultaneous loading and unloading of the material, will require local fracture theory in the neighborhood of the crack. From a purely continuum viewpoint, such a criterion must address the local energetics of the crack propagation and incorporate the energy loss due to crack growth, energy loss due to plastic dissipation and energy input from applied loading in a self consistent manner. Several researchers are addressing this issue; unfortunately, the majority of the community is pursuing other, unlikely, avenues.

From a computational perspective, the major needs in support of ductile fracture research are algorithms which can more quickly solve nonlinear problems, better time integration schemes for rate dependent problems and better methods for extracting local crack front field quantities. Nonlinearities occurring due to material behavior and finite deformations must be addressed at the outset in a consistent manner. Specifically, crack growth modeling algorithms must be studied to determine mesh sensitivity and convergence with respect to near crack front field quantities. To date, these studies are lacking. Of primary interest will be the convergence of stress, strain (or deformation gradient) and energy fields as these are the quantities which must make up a viable local fracture criteria.

An area which has received some recent interest is the problem of short cracks. While much testing has been performed, little in the way of fundamental understanding has emerged. The propagation characteristics of short cracks and its influence on future instability would appear to be a critical question. Microstructure effects are more important for short crack problems and should be included in the theory. More research on developing predictive methodology and a theoretical framework is needed.

The focus of this paper has been on deterministic fracture mechanics. Indeed, where applicable, deterministic methodology for the prediction of fracture is desirable as uncertainty is eliminated. For applications involving random loading or imprecise knowledge of the loading, geometry or material response, deterministic fracture methodology is not applicable. For these applications, stochastic methods are required. Stochastic methods for the prediction of fatigue and fracture have been employed for more than twenty years. These methods can often provide an estimate of safety and life prediction for problems which are otherwise not addressable. Because stochastic approaches employ random variables to represent physical parameters, extreme care must be exercised to validate the model and the data before application. Validation methodology and assessment of accuracy of these techniques require considerable research before routine application can be safely accomplished.

In the short term, the FEM must provide the solution methodology and improved algorithms desired, as it is the most firmly established and robust methodology for nonlinear problems. The open questions discussed above must be attacked from a purely numerical (mathematical) approach to establish accurate results to the posed problems. These solutions, therefore, will establish the validity (or lack thereof) of the assumptions built into the theoretical formulation. A much more concentrated, broader effort than is currently ongoing is required.

Concurrent with FEM research, the BIEM method should be studied. This must be viewed as a long-range investment as the method is not ideally suited to nonlinear problems. However, because one dimension of discretization is removed (i.e., FEM for three dimensional problems requires a three dimensional mesh while BIEM requires a two dimensional mesh), the approach is extremely attractive. Also, because for linear problems the BIEM not only produces accurate displacements but accurate gradients as well, robust BIEM for nonlinear problems may improve one of the major problems with FEM solutions: namely the low resolution of gradient quantities.

An important issue is the application of fracture mechanics to real problems. It is necessary to take methodology from the laboratory and the computer and employ it in applications to obtain a full assessment. This aspect of integration of fracture mechanics into the engineering design and analysis world lags significantly behind research in fracture mechanics. A strong cooperative, interdisciplinary effort in this area is needed.

A discipline which may assist in the integration of fracture mechanics to engineering design practice is the field of artificial intelligence and expert systems. It may be possible to create design systems with an integrated expert knowledge base and a hierarchical, rule based decision process. While application of these techniques to fracture mechanics has yet to be explored, the success of knowledge based systems in other disciplines indicates that application to the design process with fracture mechanics information should be attempted.

Another issue which needs concentrated effort is development of high-risk, long-term issues which may or may not directly influence current applications. Unanswered questions in the real nature of local deformation response when material nonlinearities and geometric nonlinearities are present, where crack tip blunting exists, where thermal gradients are present, etc., remain as obstacles to furthering the field. Because these problems cannot be addressed analytically with the current state of mathematics, numerical approaches must be employed. More refined modeling, more robust algorithms and significant computational resources will be required. Going into such investigations, the payoffs will be totally unknown. For this reason (as well as many practical considerations), research in this area is almost nonexistent. Future breakthroughs, however, will only come if the local deformation state can be fully understood. Research on these issues, therefore, is essential.

The National Agency for Finite Element Methods and Standards (NAFEMS) has recognized that Finite Element Methods are a vital part of the design and safety verification of many major engineering structures and in a variety of industries. They form an integral part of the design/redesign cycle. In the U.K., the National Engineering Laboratory has brought together leading users of finite element methods to form the National Agency for Finite Element Methods and Standards, which has as its objective the aim of producing tests which can easily result in performance figures which will then be useful for potential users of commercial finite element codes. The benchmarks themselves have tried to aim for known or likely weaknesses in commercial systems.

The NAFEMS Fracture Mechanics Working Group under the chairmanship of Dr.

Trevor K. Hellen is working on benchmark tests applicable to the fracture of structures. The forthcoming results should be important for the users of the various codes being utilized today in fracture mechanics. Such studies should be encouraged.

While much progress has been made in the solution of the complicated problems in fracture mechanics over the years, it should be obvious to the reader that fracture mechanics is far from a mature discipline. The issues summarized above (in addition to many other topics beyond the scope of this paper) warrant a significant research effort far beyond the current level. Major advances can only be made through a large, international, cooperative effort which is well supported by both government and industry. Fracture mechanics historically has enjoyed a great degree of international cooperation. This is to be congratulated and encouraged.

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