The Accumulated Internal Energy in the Fatigue Strength Region

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ABSTRACT

In this paper an energy model of fatigue process is proposed and its experimental verification presented. The model was based on the assumption that some critical amount of internal energy stored under fatigue cycling is responsible for failure. The energy stored in a specimen was detemined in an indirect way as the difference between the work done over the specimen and the heat emitted. The energy dissipated per cycle (equal to the work done on the specimen) was measured using a modified method of dynamic hysteresis loop and the heat liberated was determined by electrically modelling internal heat sources in the specimen. The experimental values were aproximated with a statistically tested constant function. It was shown that the hypothesis could be accepted at the significance level 0.01. The tests were carried out in the fatigue strength region on round specimens made of Ck 35 carbon steel and loaded with uniaxial stress.

KEYWORDS

Fatigue; energy theories of failure; internal energy measurement.

INTRODUCTION

A considerable number of fatigue failure theories are those of the energy type, e.g. (Miner, 1945, Hanstock, 1947, Enomoto, 1955, Feltner and Morrow, 1961, Martin, 1961, Ivanova, 1963, Troshchenko, 1966, Gurevitch and Gaevoy, 1966, Chang et al., 1968, Ostergen, 1976, Leis, 1977). They all assume that failure is due to reaching a critical value of energy E=C (a constant) accumulated in unit volume of material. This class of theories adopts "various" energies as responsible for failure. They may be for example the total energy dissipated within a material or a part of it or the difference between the dissipated energy and that liberated as heat. A review of energy criteria can be readily found elsewere, e.g. in (Morrow, 1965, Troshchenko, 1971, Klesnil and Lukas, 1980, Puskar and Golovin, 1985), and will not repeated in this paper.

The energy theory adopted for the present paper assumes the internal energy stored within a material during its service life as being a decisive factor controlling failure. This energy according to the first law of thermodynamics can be defined as the difference between the work done over the system and the liberated heat

$$E_a = E_w - E_q \tag{1}$$

where: E_a - total internal energy accumulated in material,
E_w - work done over system,
E_q - heat energy emitted from material

The following assumptions were made for the purpose of the proposed fatigue process model:

1. Energy E_f needed to cause fatigue failure (in general, different from E_a) referred to unit volume of material is a material constant, i.e.

$$\frac{E_f}{V} = C \text{ (const)} \tag{2}$$

where: E_f - energy required to cause fatigue failure
V - volume of reference

2. Distribution of energy within material volume is uniform, i.e.

$$e = \lim_{\Delta V \to 0} \frac{\Delta E}{\Delta V} = 0 \tag{3}$$

where: e - damage energy density

3. Ambient temperature does not change.

The proposed model may be written as

$$\frac{E_f}{V} = \int_0^{N_j} \left[W(N_j) - Q_i(N_j) \right] \cdot f_{II}(N_j - N)dN = C \tag{4}$$

where the intergrand is assumed regular, time expressed in cycles and dN taken suitably small.
The quantities in (4) read as follows:

W - energy dissipated within unit volume of material per cycle (equal to the hysteresis loop area in the $\sigma - \varepsilon$ coordinate system)
Q - heat energy emitted by unit volume of material per cycle
N - current number of cycles
N_j - specimen life
f_{II} - rheological function that accounts formerly for any reversible processes occurring in a specimen under cyclic loading. In a particular case the function can become unity.

In general, both W and Q are functions of not only time N but also of material properties and load vector which is not accounted for by the symbols used for the sake of clarity. In the particular case when $f_{II} = 1$, can be written in a simpler form

$$\frac{E_f}{V} = \sum_{i=1}^{i=j} (W_i - Q_i) N_i = C \tag{5}$$

A detailed description of theory (4) and interpretation of the so-called fading memory function $f_{II}(N_j - N)$ may be found in (Blotny and Kaleta, 1978, 1981). The present investigation was aimed at experimental verification of the theory.

MEASUREMENT OF UNIT ENERGY W AND UNIT HEAT Q

The energy accumulated in a specimen was determined according to (1) in an indirect way. To find the dissipated energy W the dynamic hysteresis loop method was adopted. Measurement of the energy W was carried out by measuring the area of hysteresis loop in the $\sigma - \varepsilon$ coordinate system, since by definition

$$W = \oint_{(\varepsilon)} \sigma \, d\varepsilon = \int_{(t)} \sigma(t) \frac{d\varepsilon(t)}{dt} dt \tag{6}$$

Since the strain $\varepsilon(t)$ and stress $\sigma(t)$ signals were periodic, their expansion into Fourier series permitted a simple algorithm for determining the energy W to be constructed:

$$W = \pi \cdot \sum_{i=1}^{n} i(a_i d_i - b_i c_i) \tag{7}$$

where: a_i, b_i - respectively sine and cosine terms of Fourier series for strain signals
c_i, d_i - respectively sine and cosine terms of Fourier series for stress signals

In the present investigation it was sufficient to retain the first three terms of the two series to obtain the desired accuracy. The above methods enabled software filtration against interferences and analytic compensation of the phase shift angle between measurement channels. As a result the adopted technique proved to be a sensitive method for measuring the hysteresis loop area and thus for measuring W at the fatigue limit level (Blotny and Kaleta, 1986, Pisarenko et al.,1971).

The heat energy Q was determined by a method of electric modelling of internal heat sources as described by the authors in detail in (Blotny and Kaleta 1986a, b). The following assumptions underlie the method:
- under steady-state conditions each particular value of internal heat source power in a specimen is characterized by a single distribution of temperature along the specimen length (and reversely) , (Fig. 1).
- the temperature distribution is independent of the nature of internal heat sources, i.e. flow of current and internal friction yield the same temperature distribution.

With these assumption in mind it is possible to use current heated specimens Each specimen is heated with current of a fixed power providing that conditions of heat transfer do not change with time. Each power p_i yields one temperature distribution being a kind of pattern (Fig. 2). Having collected a large number of such patterns it is possible to determine the heat power Q produced in fatigue loaded specimens (this is done by arranging the pattern temperature profiles and the experimental ones from fatigue tests in pairs in which the two profiles match each other):

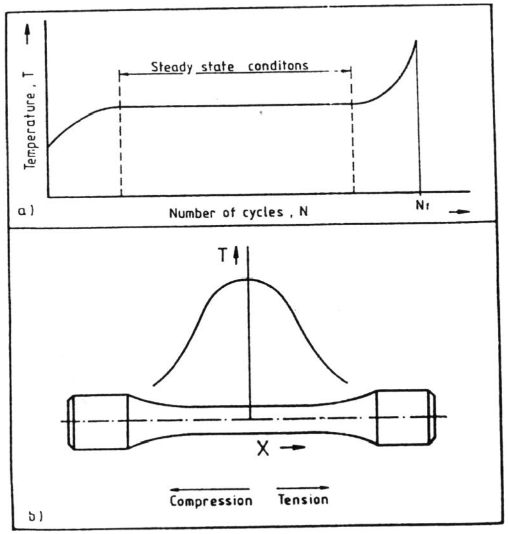

Fig. 1. a) Variation of temperature at a fixed point of the gauge length with the number of cycles.
b) Temperature distribution T(x) along the specimen length obtained in a fatigue test.

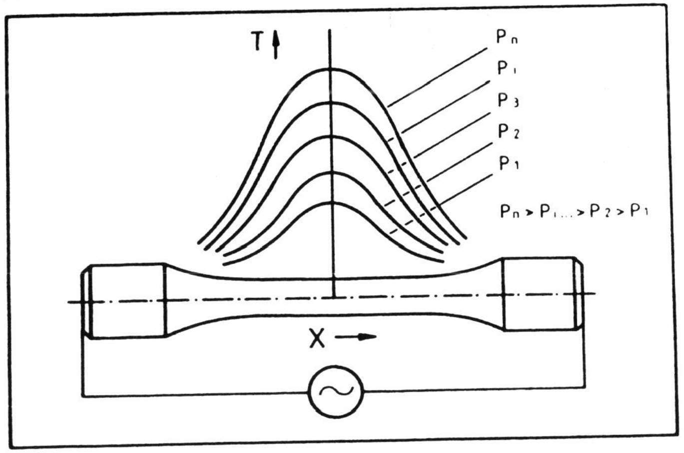

Fig. 2. Patterns of temperature distribution obtained by electrically heating a specimen and their corresponding values of current power p_i.

MEASURING-PROCESSING SET-UP

Two different measurement set-ups were used, one-for conducting measurements under fatigue loading and the second for modelling internal sources of heat. In order to be capable of measuring the unit energy W in real time it was necessary to have the opportunity of frequent sampling of the stress $\sigma(t)$ and strain $\varepsilon(t)$ signals. The measurement set-up configuration used in the test is shown in Fig. 3. In addition to what is typically measured the temperature of the middle portion of the specimen was recorded with a thermocouple.

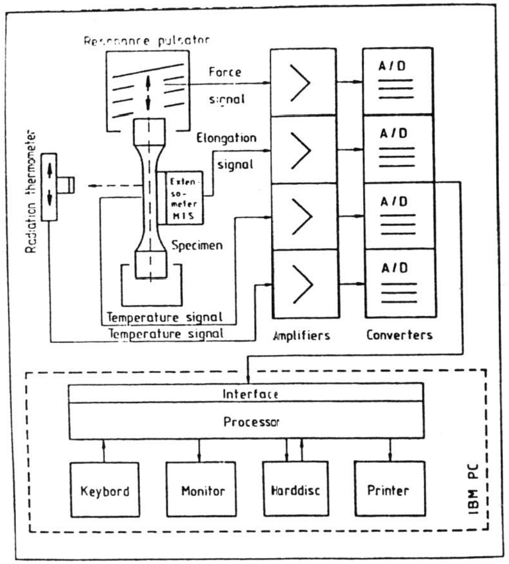

Fig. 3. Block diagram of measuring-processing set-up.

An additional radiation thermometer was used to record discrete values of the temperature distribution along the specimen gauge length. Discretization of the analogue signals and further steps of signal processing were performed with A/D converters connected with IBM PC. In order to obtain a collection of pattern temperature distributions the experimental set-up shown in Fig. 4 was designed. The investigations were carried out on specimens made of carbon steel Ck 35. Properties of the material are presented in Table 1.

ANALYSIS OF RESULTS AND FINAL REMARKS

The energy value E_f/V corresponding to failure as a function of lifetime N_i is shown in Fig. 5. Below they are analysed with only the amount of accumulated energy taken into account and without any emphasis put on specific conditions of loading. In addition shown is in Fig. 6. accumulation of the energy in one of the specimens. Having in mind the obvious advantages of the simplest models, the authors used a constant function $E_f/V=f(N_i)=$ $=$ constans $=25409$ $\mathrm{MJ/m^{-3}}$. The hypothesis was verified by using the Student

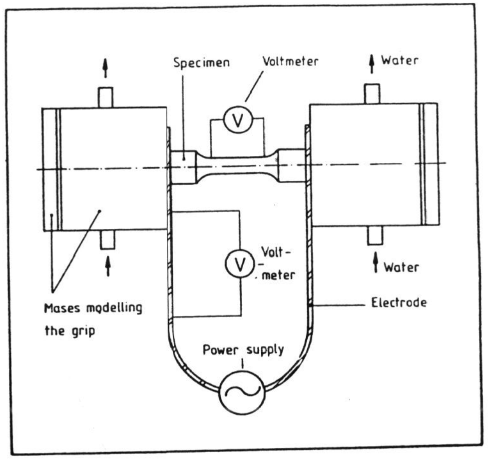

Fig. 4. Schematic illustration of experimental set-up for modelling the internal heat sources.

Table 1. Properties of Ck 35 steel

Ultimate tensile strength (MPa)	Yield strength (MPa)	Young's modulus (GPa)	Elongation (%)	Fatigue limit (MPa)
560	365	205	35	220

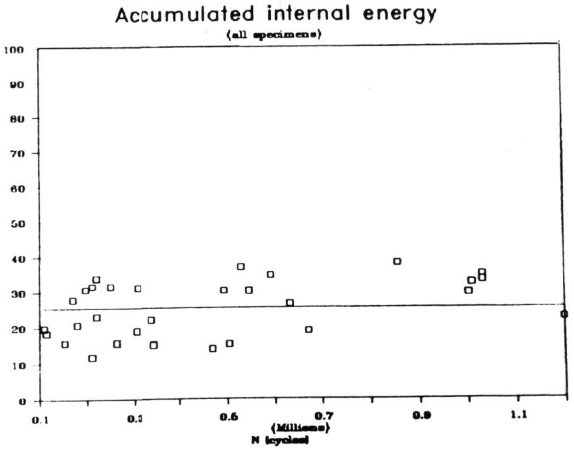

Fig. 5. Energy value E_f/V corresponding to failure as a function of lifetime N_j.

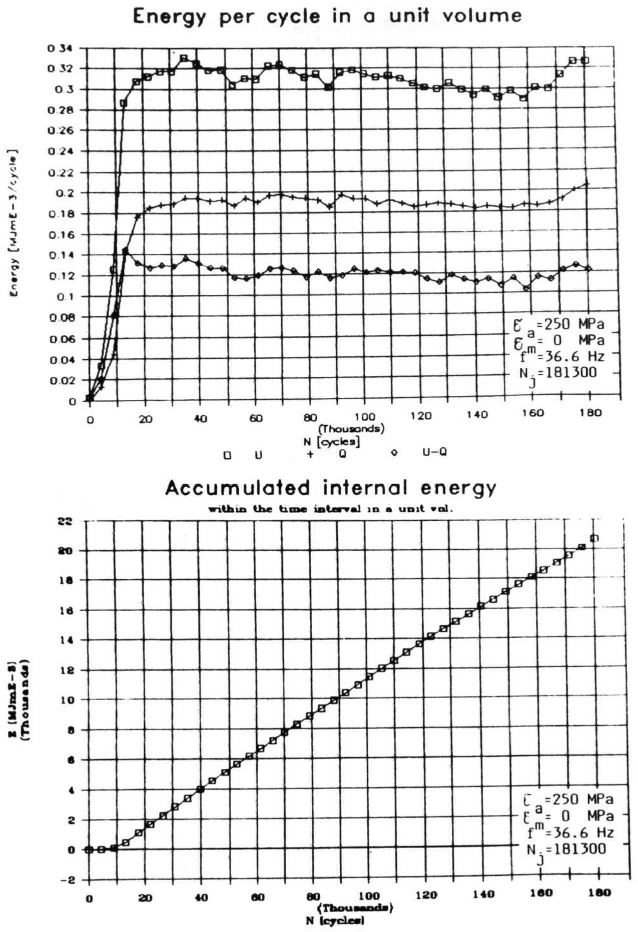

Fig.6.a) Energies U=f(N), Q=f(N) and U-Q=f(N), respectively in one of the specimens
b) Accumulation of the energy in one of the specimens as a function of time N.

test. The hypothesis was found valid at the significance level =0.01. Adoption of the hypothesis leads via relationship (4) to a statement that the fading memory function $f_{II}(N_j-N)$ is identically equal to unity and this in turn allows rheological effects to be safely neglected. Then model (4) can be assumed in a simpler form (5). The lifetimes N_j determined from this model differ from the actual values from 4% to 85%. Extension of the tests to the fatigue limit region can make it necessary to introduce a fading memory function different from unity, e.g. in the form proposed in (Blotny and Kaleta, 1981, 1988). It is to be noted that an indirect way of determining the internal energy according to formula (1) affects the accuracy of the method. It was shown in (Blotny and Kaleta,1988) that at atress equal to fatigue limit $\sigma_a = \sigma_f$ the errors in W and Q are 4.7% and 3%, respectively. Taking the most unfavourable circumstances, the error of the difference U-Q might reach 19.6%.

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