Some Studies on Fatigue Damage as a Criterion for Dynamic Strength Optimization

M. K. PADMANABHAN*, S. SURYANARAYAN** and M. S. S. PRABHU***

**Senior Engineer, Aeronautical Development Agency, Bangalore–37, India*

***Professor, Aeronautical Engineering Department, I.I.T. Bombay–76, India*

****Head, Structural Analysis, ISRO Satellite Centre, Bangalore–17, India*

ABSTRACT

Fatigue Damage as a concept for dynamic strength optimization is presented in this paper. Various, typical cases of load spectra are considered and the effect of the combined participation of a number of vibrating modes of the structure are studied. The study demonstrates that the concept is very appropriate and useful, particularly for situations involving low stresses for long durations.

KEYWORDS

Fatigue Damage, Dynamic strength optimization, Modal participation.

INTRODUCTION

Optimisation of structures under dynamic loads is a subject of great interest and significance to Aerospace researchers. Aerospace structures experience, besides static loads, spectra of dynamic loads for different durations. They are usually designed to meet the requirements of minimum weight and specified operating life. The design of these structures for dynamic loading conditions poses a variety of optimisation problems in which the objectives and constraints can be formulated in terms of requirements on mass, stress, frequencies, displacements and other parameters which are a measure of the dynamic strength and performance of the structure.

Pierson (1972) and McIntosh (1974) have provided a survey of optimal structural design problems enunciating the work and the methodology adopted by various researchers. A review of the optimality criteria approaches to structural optimisation is given by Berke and Venkayya (1974). Venkayya (1978) has also provided a review and recommendation on the problem of structural optimisation. It is seen that the majority of the work is in maximising the fundamental frequency or minimising the dynamic displacements or stresses. The authors of the present paper have studied the problem of design of beams under dynamic loads, taking into consideration the participation of a number of modes of the structure in the overall response and have presented optimal beam configurations (Padmanabhan et.al.,1987a,b,1988). While these problems were considered from the stress point of view, it is important to examine fatigue damage as a concept in the optimisation study as fatigue is a measure of dynamic strength for a class of dynamic loading conditions. The present study is a step in this direction which considers various typical cases of load spectra and examines the effect of the combined participation of a number of vibration modes of the structure in the characterisation of its dynamic response.

Configuration of the structure and the parameters for optimisation

As a typical structural component, the beam has been chosen for a detailed study to bring out the concepts and illustrate the feasibility of optimisation for dynamic loads, keeping the computational efforts tractable. A beam with uniform width and variable depth has been chosen as the configuration for the study. The beam geometry and coordinate system are shown in Fig.1. For the case of cantilever beam, eleven design variable nodes (beam depths) and for the simply supported beam (due to symmetry) six variable nodes are used.

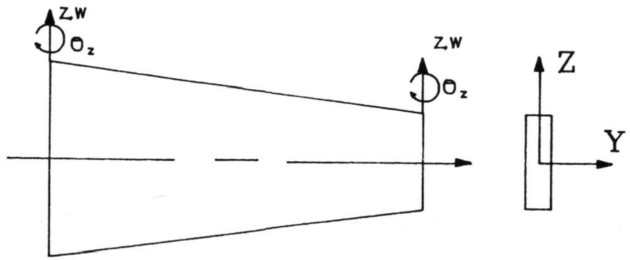

Fig. 1. Coordinate System and Beam Element

Nature of excitation

The specification of the excitation to which the beam is subjected to, requires the description of the spatial distribution, spectral contents and its temporal characteristics.Here for the case of cantilever beam (CN) two types of spatial distributions, Tip Excitation (TE) and Base Excitation (BE) are considered. For the simply supported beam (SS) Central Excitation (CE) and Uniformly Distributed Excitation (UDE) are considered.

Spectral Contents

The spectral contents of the excitation decide the extent of participation of each mode of vibration in the overall dynamic response of the structure. In the present study four typical spectra are considered. The first three spectra correspond to the individual response of each of the first three

modes (the amplitude levels of these spectra are scaled up by suitable factors such that the stress are above the endurance value so as to produce damage). The fourth spectrum is chosen to represent a case wherein the uniform beam exposed for a specific duration experiences fatigue damage composed of three equal components of damage due to each of the three modes at the reference section.

Temporal Characteristics

For the analysis in this study, the excitation spectrum is assumed to be obtained by a combination of a sequence of narrow band excitations over the frequency range as achieved in a sine sweep test. Different types of sine .plsweep tests can be formulated to achieve a specified load spectrum. In the present analysis it is assumed that a spectrum is achieved in a single sweep with a constant sweep rate in the linear scale.

Constraints

Mass and stiffness constraints are imposed such that (i) the mass of the optimum beam is kept the same as that of the reference uniform cross-section beam and (ii) the dynamic stiffness of the first three frequencies of the optimised beam are always equal to or greater than those of the reference beam.

METHODS AND PROCEDURE

The schematic of the optimisation procedure is shown in Fig. 2. The procedure is initiated by choosing a reference starting beam configuration, whose geometry is specified by its basic dimensions and boundary conditions.

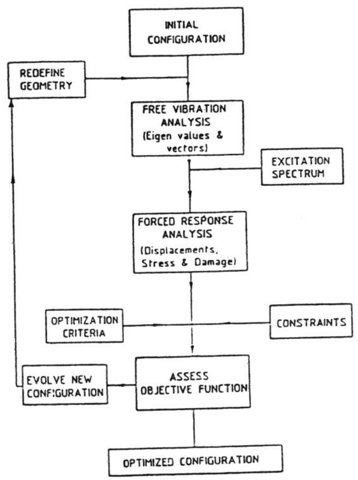

Fig. 2. Optimization Procedure

Free vibration Analysis

The next step involves finding the free vibration characteristics of the beam. The element stiffness and mass matrices are generated and assembled to obtain the system matrices. From the stiffness and mass matrices the eigenvalues and eigenvectors are obtained.

Forced Response and Evaluation of the Objective Function

For the forced response analysis the equations of motion formed in terms of normal modes are solved to obtain modal amplitudes and dynamic displacements along the beam span. The dynamic stresses are then obtained by employing strain-displacement and stress-strain relationships.

Optimisation Criteria

The criterion for optimisation is the fatigue damage. For spectra exciting the individual modes, (M1, M2 or M3 defined earlier), the damage distribution due to the individual modes is considered for the optimisation. When multi-mode participation is considered in the total response, the damages produced by each of the modes is summed up to obtain cumulative damage. This sum cumulative damage, (SCD), either due to single mode excitation or sequential excitation of each of the modes is also a function of the spanwise location in the beam and the maximum value of this SCD is minimised in the optimisation process.

Evaluation of Cumulative Damage

Deterministic excitations enveloping the number of modes (three, in the present study) are considered. The excitation is assumed to be such that it sequentially sweeps over the frequency range of interest. The duration of excitation of each mode is assumed to be same as that obtained in a sine sweep test with a constant sweep rate in the linear frequency scale.

To evaluate damage, the widely accepted design practice of using the cumulative damage criteria based on Miner's hypothesis which is valid for the vibration environments whose descriptive parameters remain constant and are applied sequentially has been adopted.

Analysis Procedure

The analysis cycle begins with calculating stresses at each of the nodal points in the beam for any given excitation for each of the modes. Then with the known rate of linear sweep over the frequency f_1 the damage D^i at the i-th location of the beam can be obtained. Repeating it for all the locations of the beam, the damage distribution D_1 varying over the span of the beam for the first mode is obtained. Similarly, damage distribution for second and third modes are obtained as D_2 and D_3. Since it is assumed that the excitation loads in each of the modes is sequentially applied (as frequencies are swept over the range) the cumulative damage at any location i of the beam can be obtained as

$$D^{(i)} = D_1^{(i)} + D_2^{(i)} + D_3^{(i)}$$

This cumulative damage distribution over the span varies and a measure of uniformity of damage over the span ($\bar{D}$) is then calculated. The maximum value of damage anywhere in the span (maximum of D's), the average damage D or the deviation from average damage $D_{av.}$ could be used as the objective function for optimisation.

Optimisation Process

The objective function (cumulative damage) is F ($\vec{d}$) where ($\vec{d}$) is the vector of the design variables. We can set {F ($\vec{d}$)} = Max (D) or = $D_{av.}$ or = $\bar{D}$. The constraints on mass and stiffness can now be imposed as

$$\text{Min} \quad \{F(\vec{d})\} = D_{av.}(\vec{d}) = \bar{D}(\vec{d}) = \text{Max } D(i) \qquad i = 1,2....n$$

$$\text{with } M(\;) = \text{constant} = M^{(r)} = \text{mass of the reference uniform beam}$$

$$f_1^{opt.} > f_1^{(r)}$$

$$f_2^{opt.} > f_2^{(r)}$$

$$f_3^{opt.} > f_3^{(r)}$$

Optimization Procedure

Once the objective function and constraints are known, the problem now is one of minimization of the objective function, satisfying the constraints. The constrained minimisation problem is converted into a sequence of unconstrained minimisation problems by merging the constraints into the objective function thereby forming a new objective function which is then minimised. This conversion into the unconstrained minimisation problem is done by resorting to indirect method of exterior penalty function. The resulting unconstrained minimisation problem is solved in the present study using Sutti's method (1975), due to certain specific advantages. In this method the approach to the generation of the conjugate directions is similar to the others but once a significant progress is made using a few of the basis direction, a new conjugate direction is generated. This ensures that the directions along which the functional reduction is insignificant are not lost during the process of building a set of conjugate directions. This results in good conditioning of the basis and smooth progression towards minimum.

DISCUSSION OF RESULTS

Damage Optimised Profiles of Cantilever Beams for Single Mode Excitation

For the case of the single mode excitation spectra it may be noted that the damage at any location is a function entirely of a single stress component, namely, the stress amplitude at that location corresponding to the excitation mode. Consequently the location of maximum damage is same as the location of maximum stress amplitude. Therefore, a beam configuration optimised from a stress amplitude angle will also be the optimum configuration for minimum damage.

Tip Excitation

It could be seen (Fig.3) that optimised configurations result in a much lower value of maximum damage than that of the reference beam for M1 and M3 excitation spectra. For M1 spectrum the maximum damage per test cycle is reduced to 0.12×10^{-4} from the value of 0.15×10^{-2} for the reference beam. Alternatively, we can say that the life of M1 optimised configuration is 8.2×10^{4} test cycles as against 6.4×10^{2} for the reference beam. Similarly, for the M3 spectrum the damage is reduced from 0.75 to 0.16×10^{-1} and the life is increased from 1.3 to 60 test cycles by the optimisation process.

For the M2 spectrum, the benefit of optimisation appears to be marginal(a reduction in the maximum damage from 0.58×10^{-1} to 0.55×10^{-1}). The reason for this has been traced to the stress distribution as obtained by the 10 element FEM analysis.Due to the limited number of elements used, the zero stress value at the stress modes of the corresponding vibration modes sometimes get reflected in the stress profile and sometimes not depending on whether there is an element node close to the stress node of the vibration mode or not. Consequently, the average stress value and the stress deviation suffer some distortion during the optimisation process as the stress node of the vibrating mode crosses the element node. Another factor that contributes to this distortion is the error stress value at the tip of the beam, particularly when the beam cross section is small.

Base Excitation

For the case of base excitation (whose results are not reproduced here) the trends are basically the same and the optimisation process was found to be quite successful in achieving the objective of lowering the damage values. The results are shown below:

Spectrum	Maximum damage/test cycle & (expected life) Uniform	Optimum
M1	0.15×10^{-2} (6.6×10^{2})	0.74×10^{-3} (13.5×10^{2})
M2	0.58×10^{-1} (1.7×10^{1})	0.73×10^{-3} (18.86×10^{2})
M3	$0.44 \times 10 (2.3 \times 10)$	0.74×10^{-3} (29.4×10^{2})

Damage Optimised Configurations for Modal Combination Spectra

In contrast to the case of single mode excitation spectra, for the case of modal combination excitation, the damage at any location of the beam is a combined function of each of the individual modal stress amplitudes and their frequencies. As the damage produced by a lower stress amplitude applied for a large number of cycles can be higher than that produced by a relatively higher stress amplitude applied for much lesser number of cycles, the point of maximum stress and maximum damage are different.

For the BE cantilever beam MC-ECD spectra, the resultant optimised beam configuration for all the cases (not reproduced here) has also resulted in

a large reduction of the maximum damage.It was also seen that the process of making the stress uniform is normally more easily achieved by the optimiser. Making the damage uniform is found to be a more difficult objective to realise as the process of damage calculation involves the use of an inverse power relationship as given by the S-N curve. A small variation of the stress, especially around the endurance value can appear as large variation of damage in the linear scale. Taking this into account,it could be stated that the optimised configurations have a good uniformity of damage distribution over the span, in addition to lesser damage magnitude.

Damage optimised profiles of simply supported beams

The optimised configuration and the damage distribution for the simply supported beams subjected to central and uniformly distributed excitations for M1, M3 and MC-ECD spectra, also result in a substantial reduction and a reasonably uniform damage distribution over the span.

CONCLUSIONS

Fatigue damage was considered as a criterion for dynamic strength optimisation.Beam configurations optimised for damage and the corresponding damage distributions were compared with those of reference uniform beam for different cases of spectral and spatial distributions of excitation for cantilever and simply supported beams. The study demonstrates that fatigue damage as a criterion for dynamic strength optimisation is very appropriate for situations involving low stresses for long durations. For all the spatial distributions considered for single mode spectra, both for cantilever and simply supported beams, that significant reduction in maximum damage is obtained.

For modal combination spectra, damage is a function of both individual modal stress amplitudes and frequencies in contrast to the single mode spectra where it is a function of stress alone. In all the cases considered one could arrive at optimum configurations with considerably reduced damage values. Also, a good uniformity of damage distribution over the span is obtained if one keeps in mind the inverse power relation between stress and damage as given by the S-N curve.Optimisation for damage involving redistribution of material has to take into account both the stiffness as well as inertia distribution. Frequency constraints limit the optimisation process.

Selection of optimisation criterion for damage minimisation is very important.One could start with configurations of high stress (infeasible even from stress considerations), but reach optimised configurations feasible even from damage considerations. In other words, it is necessary to incorporate a certain degree of flexibility in the optimisation criteria so that the optimisation process itself could lead from infeasible designs to feasible designs.

REFERENCES

Berke, L and Venkayya V.B.(1974) Review of Optimality Criteria Approaches to Structural Optimization. ASME Winter Annual Meeting, New York.

Mc Intosh Jr, S.C.(1974) Structural Optimization via Optimal Control Techniques:A Review. Structural Optimization Symp., AMD-7, ASME,New York.
Padmanabhan, M.K. Suryanarayan, S. Prabhu, M.S.S. (1987) Design for Maximum Dynamic Stress of Beams undergoing Multi-modal response. National conference on Mechines and Mechanisms, Bombay, India.
Padmanabhan, M.K. Suryanarayan, S. Prabhu, M.S.S. The effect of load distribution on the optimum profiles of tapered beams designed for Dynamic Load Spectra. 32nd Congress of Indian Society of Theoretical and Applied Mechanics, Bombay, India .
Padmanabhan,M.K. Suryanarayan,S. Prabhu, M.S.S.(1988). The effect of end conditions on the design for dynamic stress of beams. Technical Sessions of the 39th AGM of Aero Society of India .
Pierson, B.L.(1972) A Survery of Optimal Structural Design Under Dynamic Constraints. Int. J. Num. Methods. Engg., 4, 491-499.
Sutti, C.(1975) A New method for Unconstrained Minimisation without derivatives. Towards Global Optimization, North Holland, Amsterdam .
Venkayya,V.B.(1978) Structural Optimization: A Review and Recommendations. Int. J. Num. Math. Enff.13(2).

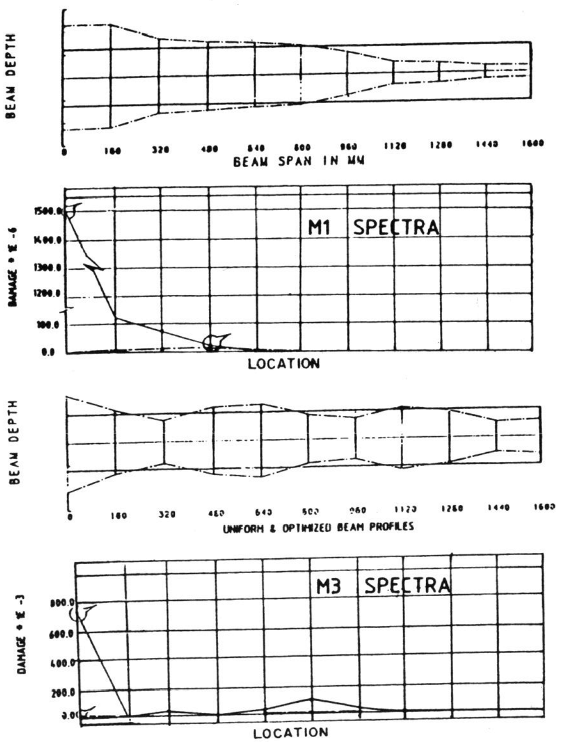

Fig. 3. Optimized Profiles and Spanwise Variation of the Peak Stress: TE,Cantilever,M1 & M3 Spectra