# Significance of Crack Closure due to Residual Stretch and Stresses

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#### ABSTRACT

Fatigue crack closure characteristics due to residual stretch and stresses are analyzed. A real stress intensity factor is defined by using a superposition technique in linear elastic fracture mechanics. The analysis gives the theoretical definition for the effective stress intensity and shows a guide how to detect associated crack opening points. It is shown that this approach can be applicable for the prediction of fatigue crack growth rates in residual stress field.

#### KEYWORDS

Fatigue; fracture mechanics; crack closure; residual stretch; residual stresses.

# INTRODUCTION

Due to fatigue crack closure, only a fraction of applied stress intensity factor range,  $\Delta K$ , plays an important role in actual crack growth (Elber, 1971). This fraction is termed the effective stress intensity factor,  $\Delta K_{\rm eff}$ . The validity of the crack closure concept has been confirmed by experiments and analyses. However, the following important points remain still unclear.

- (1) Definition of  $\Delta K_{\rm eff}$ : For plasticity induced crack closure as originally found by Elber, a range of the "effective"  $\Delta K$  is clear. However, when partial crack surface contact occurs with an original crack tip open at minimum stress intensity factor,  $K_{\rm min}$ , the conventional definition for the opening stress intensity factor,  $K_{\rm op}$  (the tip opening point), can not be applied.
- (2) Accuracy of unloading compliance method: In unloading compliance technique, as a nonlinear portion of the *P-V* curve approaches to the opening point asymptotically, the detected point might have some errors. Of course, the amount would depends on the closure mechanisms. So, the accuracy should be cleared up for many cases.

As the crack grows by fatigue, the asperities cause the crack closure which yields a contact (compressive) stress behind the crack tip. By using a

superposition technique, it is shown that the real stress intensity factor at the crack tip can easily obtained from the contact stress. From this, more theoretical definition will be given to  $\Delta K_{
m eff}$  in this paper. The validity is confirmed by the analysis of fatigue crack growth under various crack closure mechanisms, such as residual plastic stretch (Elber, 1971), fracture surface roughness (Minakawa and McEvily, 1981), or fretting oxide debris (Stewart, 1980). The results are compared with experiments (Nakamura and Kobayashi, 1988).

Further approach to fatigue crack growth rate prediction in residual stress field includes a method incorporating partial crack surface contact. The validity of the method is confirmed by comparing the prediction with experiments.

## ANALYTICAL PROCEDURE

 $T_{\text{O}}$  assess the effect of asperities on fatigue crack closure, a fatigue crack growth simulation was conducted by using a modified Dugdale model developed by Newman (Newman, 1981). The model can leave elastic-plastic elements in the wake of the advancing crack tip as shown in Fig. 1. The simulation includes a loading and an unloading processes with advancing a crack tip (the growth increment per cycle was 0.05 of the maximum plastic zone size,  $\omega_{\rm max}$ ) at each loading cycle under the condition of  $K_{\rm max}$  = constant and R=0 until the steady state of  $K_{\rm op}$  and  $K_{\rm cl}$  is reached. After that, several types of asperities were attached on the crack faces as shown in Fig. 2. These are

case (a): the amount of residual stretch  $\delta_R$ , being constant along the crack faces (plasticity induced crack closure alone, no

case (b): with asperities remote from the crack tip (near the initial notch tip in this case).

case (c): with asperities near the crack tip.

The details of the analytical procedure are shown elsewhere (Nakamura and Kobayashi, 1988).

The validity of the unloading compliance technique is discussed by analyzing the relation between the applied K and the crack mouse opening displacement COD. The detected opening point is denoted by  $K_{\rm op}$ .

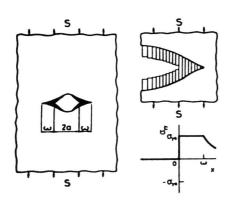


Fig. 1 Modefied Dugdale model.

As discussed in the previous study,  $K_{\mathrm{Op}}$  is superior than  $K_{\mathrm{C1}}$  as a parameter describing the crack growth rate, de/dN (Nakamura and Kobayashi, 1988). Thus, the discussions are made only for  $I_{\text{OD}}$  in this study.

# REAL STRESS INTENSITY FACTOR IN RESIDUAL STRESS FIELD

when a initial residual stress distribution (the stress is  $\sigma_R^{\,i}(x)$ ) is given for an uncracked body, the associated stress intensity factor, KR, for a cracked body is calculated by applying a negative  $\sigma_R^i(x)$   $(-\sigma_R^i(x))$  on the erack surfaces. By introducing a superposition technique, it is given as

$$K_R = \int_a^a \sigma_R^i(x) \ m(a,x) dx \tag{1}$$

$$= \int_{-\sigma_R}^{\alpha} (x) M(a,x) dx$$
 (2)

where a is the crack length, m(a,x) and l(a,x) are weight functions for the residual stresses of the uncracked body and for the traction acting on the crack surfaces, respectively. Of course, these functions are in a relation

m(a,x) = -M(a,x).The real stress intensity factor range,  $M_r$ , and stress ratio,  $R_r$ , are given

displacement COD(x), associated with the traction  $-\sigma_R^i$  is given as

$$V_R(a,x) = (1/E) \int_{x}^{a} K_R(a) m(a,x) da$$
 (6)

under the plane stress condition, where E is the Young's modulus.

When a crack grows from a compressive stress field, Vp(a,x) becomes negative shown by doted lines in Fig. 3(a) which is physically unacceptable. Actually, a crack surface contact occurs in this case. Thus, a contact stress,  $\sigma_{co}$ , is applied (Fig. 3(c)) to prevent such crack surface

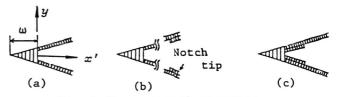


Fig. 2. Three cases of asperities.

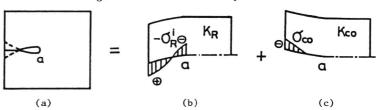


Fig. 3. Superposition technique considering crack surface contact.

overlapping (Todoroki et al., 1988b). A stress intensity factor,  $K_{CO}$  ( $\ge 0$ ), associated with  $\sigma_{CO}$  is given as

$$K_{\text{CO}} = \int_{a}^{a} \sigma_{\text{CO}}(x) M(a, x) dx. \tag{7}$$

By using Eqs. (1) and (7), the real stress intensity factor,  $K_r$ ,  $\Delta K_r$ and  $R_r$ can be defined as

$$K_{r} = K + K_{R} + K_{CO} \tag{8}$$

$$\Delta K_r = (K_{\text{max}} + K_R + K_{\text{CO}}(\text{max})) - (K_{\text{min}} + K_R + K_{\text{CO}}(\text{min}))$$

$$R_r = (K_{\min} + K_R + K_{CO}(\min)) / (K_{\max} + K_R + K_{CO}(\max))$$
 (10)

#### CRACK CLOSURE DUE TO ASPERITIES

In plasticity induced crack closure, the residual plastic stretch yields residual stresses behind the crack tip and hence  $K_R$ . So, by Eq.(9),  $\Delta K_{
m eff}$ 

$$K_{\text{eff}} = K_{\text{max}} - (K_{\text{min}} + K_{\text{CO}}(\text{min})) \tag{11}$$

unknown during fatigue crack growth, it is difficult to calculate  $K_r$ . So, it is convenient to introduce a new parameter, an applied stress intensity  $(K_{app})$ , defined as

$$K_{\rm app} = K + K_{\rm co}. \tag{13}$$

$$K_{\text{app}} = K + K_{\text{co}}.$$
By using  $K_{\text{app}}$ ,  $K_{\text{op}}$  is defined as
$$K_{\text{op}} = K_{\text{app}} \qquad (\text{at } K = K_{\text{min}}).$$
(13)

The relation between  $K_{\rm app}$  and K for the case (a) is shown in Fig. 4. Until the crack tip opens,  $K_{\rm app}$  remains constant ( $K_{\rm app}/K_{\rm max}=0.51$ ). As the crack tip is closed during  $K_{\rm min} \leq K \leq K_{\rm op}$ , the crack tip singularity vanishes. Thus,  $K_{\rm r}$  in Eq.(8) is zero and the following equation can stands.

$$K_r = K_R + (K + K_{CO})$$

$$= K_R + K_{app} = K_R + K_{op} = 0 (for K_{min} \le K < K_{op}). (15)$$
From these results,  $K_{op}$  for case (a) is equal with

- (i) the negative value of stress intensity factor due to asperities,  $-K_R$ ,
- (ii)  $K_{\rm app}$  during  $K_{\rm min} \le K < K_{\rm op}$ , or (iii) K when the crack tip opens (or when  $K_{\rm CO}$  becomes zero).

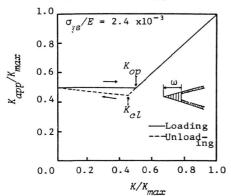


Fig. 4.  $K_{app}$ -K relation for case (a).

Of course, the last definition agrees with the conventional one (Elber, 1971). From Eq.(10), it is shown that the real stress ratio is given as

The relation between  $K_{
m app}$  and K for the case (b) is typically shown in Fig. ), where the total asperity thickness,  $\delta_R$ ', near the notch tip along  $x'/\omega_{max}$ • 1.2-2.4 is twice of the original residual plastic stretch,  $\delta_R$ . For this (ABC, the first surface contact occurs near the notch tip with the original crack tip opens (partial crack surface contact). During the partial crack surface contact, the relation between K and  $K_{\rm app}$  becomes nonlinear. However, asperities have no influence on  $K_{\rm op}$ . It is shown that the crack tip actually opens at  $K_{\text{OD}}$  defined by Eq.(14).

Further increase in thickness of asperities brings the condition where the **Crack** tip is still open at  $K=K_{\min}$ , which brings increase in  $K_{\mathrm{op}}$ . Thus, \*\*sperities remote from the crack tip have a influence on  $K_{\mathrm{OD}}$ , only when the partial crack surface contact occurs. Among the above three definitions [(1)-(iii)] for  $K_{OD}$ , only the second one (Eq.(14)) is applicable in this

The relation between  $K_{\mathrm{app}}$  and K for the case (c) shows the same trend as the (a), because no partial crack surface contact occurs.

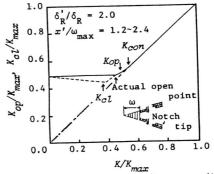


Fig. 5.  $K_{app}$ -K relation for case (b).

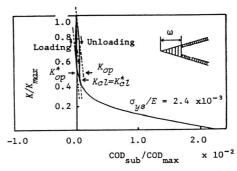


Fig. 6.  $K-COD_{sub}$  relation for case (a).

## DETECTION OF CRACK CLOSURE

The relation between K and the subtracted COD, COD<sub>sub</sub>, for the case (a) is shown in Fig. 6, where  $COD_{sub}$  is the nonlinear component of the crack mouse displacement COD,  $COD_{max}$  is COD at  $K = K_{max}$  and dashed lines are the tangents of the unloading curves at  $K = K_{max}$ . The intersections between the hysteresis loop and the dashed lines give the detection of  $K_{op}^* = 0.51 K_{max}$  (from loading portion of the curve) and it was found that it agrees with both the actual tip opening point and  $K_{op}$  by Eq.(14).

The relations between K and  $C\!O_{\mathrm{Sub}}$  for the cases (a) to (c) are typically shown in Fig. 7. For the case (b), the first crack surface contact occurs near the notch tip with the crack tip open which causes a sudden compliance change. Thicker the asperities attached, higher the detected value of  $K_{\mathrm{op}}^{\ *}$ . As stated earlier, however, the asperities have no (or a little depend on the asperity thickness) influence on actual  $K_{\mathrm{op}}$ . The detected value of  $K_{\mathrm{op}}^{\ *}$  (point B) gives the overestimated value of  $K_{\mathrm{op}}$ .

For the case (c), thicker the asperities attached, increasing the actual values of  $K_{\rm Op}$  (point B). Thus, asperities near the crack tip have a significant influence on  $K_{\rm Op}$ . However, it becomes more difficult to detect  $K_{\rm Op}$  precisely because the compliance change near  $K=K_{\rm Op}$  becomes smaller. Thus, the detected value of  $K_{\rm Op}^{\phantom{Op}}$  (point C) tend to give the underestimated value of  $K_{\rm Op}$  and  $K_{\rm Cl}$ . This phenomena always happens under relatively high  $K_{\rm Op}/K_{\rm max}$  (under high stress ratio or near the threshold where the oxide and/or fracture surface roughness induced crack closure prevail).

Experimentally, the validity of the crack closure concept has been confirmed by comparing da/dN versus  $\Delta K_{\rm eff}$  curve with closure-free da/dN versus  $\Delta K$  curve. Sometimes, it is difficult to deduce the candidate for the closure-free curve. Although the Dugdale model adapted here has inherent limitations in terms of its application, the results obtained here may give some suggestion regarding this point.

The da/dN versus  $\Delta K$  curves at R=0.1 for a high strength steel, AISI 4340, tempered at 773 K tested in air and in synthetic seawater are shown in Fig. 8 (Kobayashi and Nakamura, 1987). No environmental effect or no interaction effect between stress corrosion cracking (SCC) and fatigue crack growth is observed. On the other hand, da/dN versus  $\Delta K_{\rm eff}$  curve in synthetic seawater tends to deviate to upwards as  $\Delta K$  decreases. The threshold stress intensity for SCC,  $K_{\rm ISCC}$ , is 69.6 MPa/m for this material and is considerably higher compared with the level of  $\Delta K$  tested. Accordingly, this acceleration has been interpreted as the effect of stress induced corrosion dissolution at

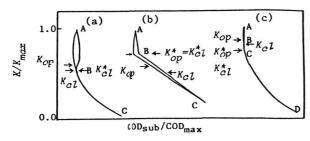


Fig. 7.  $K-COD_{sub}$  relation for cases (a) to (c).

the crack tip (Endo *et al.*, 1983). However, thick corrosion products were observed only near the initial notch tip on the fracture surface. As suggested from Fig. 7(b), the detected value of  $K_{op}$  in this case, which is believed to have yielded the above mentioned results.

# PREDICTION IN RESIDUAL STRESS FIELD

Results of Eqs. (8)-(10) can be directly applicable to the prediction of fatigue crack growth rates in residual stress field due to welding or plastic work. Unlike stresses due to asperities, such initial residual stresses without the contribution of asperities may be measured which enable us to treat such stresses as one of applied stresses. For the same  $\Delta K_r$  and  $R_r$ ,  $K_{\rm Op}$  due to asperities (residual plastic stretch, fracture surface roughness, etc.) becomes the same. Thus, da/dN is estimated from the da/dN-  $\Delta K$  curve for the base metal at  $\Delta K = \Delta K_r$  and  $R = R_r$ .

Conventionally,  $\Delta K_r$  and  $R_r$  have been calculated from Eqs. (4) and (5) for given initial residual stresses. Under a compressive residual stress field with tensile residual stresses only near the crack tip, however, the partial crack surface contact as shown in Fig. 3 can occur which is the same as the case (b) in Fig. 2. Thus, in life prediction, Eqs. (9) and (10) must be used. A typical example is shown in Fig. 9 (Todoroki, et al., 1988b). Without considering the effect of partial crack surface contact (doted curve), the prediction gives the overestimation for experiments (open symbols). If that effect is considered (solid curve), the prediction shows a good agreement with experiments.

## CONCLUSION

Fatigue crack closure characteristics due to residual stretch and stresses

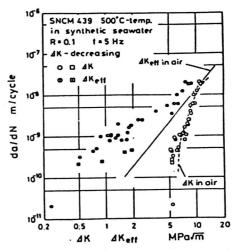


Fig. 8.  $da/dN-\Delta K$  and  $da/dN-\Delta K_{\rm eff}$  relations at R=0.1 in synthetic seawater (AISI 4340 tempered at 773K).

are analyzed. A real stress intensity factor is defined by using a superposition technique in linear elastic fracture mechanics. The analysis gives the theoretical definition for the effective stress intensity and shows a guide how to detect associated crack opening points. It is shown that this approach can be applicable for the prediction of fatigue crack growth rates in residual stress field.

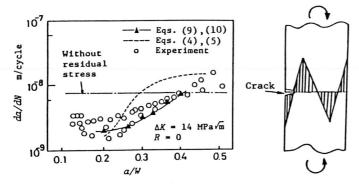


Fig. 9. Comparison between predictions and experiments of da/dN in residual stress field induced by plastic bending.

#### REFERENCES

Elber, W.(1971). The Significance of Fatigue Crack Closure. ASTM STP, 486, 230-242.

Endo, K., Komai, K and Shikida, T. (1983). Crack Growth by Stress-Assisted Dissolution and Threshold Characteristics in Corrosion Fatigue of a Steel. ASTM STP, 801, 81-95.

Kobayashi, H. and Nakamura, H. (1988). Role of Closure Mechanisms on Fatigue Crack Growth in Steels under Severe Service Conditions. In: Advanced Materials for Severe Service Applications (K. Iida and A. J. McEvily Ed.), Elsevier Applied Science, pp. 325-342.

Kobayashi, H., Ogawa, T., Nakamura, H. and Nakazawa, H. (1984). Oxide Induced Fatigue Crack Closure and Near-Threshold Characteristics in A508-3 Steel. In: Proc. 6th ICF Intl. Conf. on Fract. 4, Pergamon Press, pp. 2481-2488.

Minakawa, K and McEvily, A. J. (1981). On Crack Closure in the Near-Threshold Region. Scripta Metallurgica, 15, 633-636.

Newman, J. C. (1981). A Crack-Closure Model for Predicting Fatigue Crack Growth under Aircraft Spectrum Loading. ASTM STP, 748, 53-84.

Stewart, A. T. (1980). The Influence of Environment and Stress Ratio on Fatigue Crack Growth at Near Threshold Stress Intensities in Low-Alloy

Steels. Engineering Fracture Mechanics, 13, 463-478.
Todoroki, A. and Kobayashi, H. (1988a). Prediction of Fatigue Crack Growth Rate in Welding Residual Stress Fields. Proc. of Intl. Conf. on Residual

Stresses 2, Nancy, France, to be appeared.

Todoroki, A., Kobayashi, H. and Nakamura, H. (1988b). Effect of Partial Crack Surface Contact on Fatigue Crack Growth in Residual Stress Fields. *Proc. of Intl. Conf. on Residual Stresses* 2, Nancy, France, to be appeared.