Several Models to Predict the Low Cyclic Corrosion Fatigue Life

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ABSTRACT

Two methods have been developed to estimate the failure probability of spherical tanks. They have been compared with Monte Carlo simulation and Hasofer and Lind's reliability index method.

KEYWORDS

Failure probability; COD.

INTRODUCTION

Crack opening displacement design criterion, $\delta_c \geq 3.5$ ea, of the JWES-2805, is considered as a probabilistic failure mode. Here δ_c is the critical crack opening displacement, a is the surface crack fepth, e, is the total strain, $e=e_1+e_2+e_3$, e_1 is tensional membrane strain and bending strain, e_2 is the residual strain, e_3 is the concentrated strain. The fatigue crack size, a, is assumed to obey Paris's crack growth law

$$da/dn = C\ (\Delta K)^m \qquad (1)$$

The stress intensity factor range of a surface crack of depth, a, is expressed as

$$\Delta K = K_t \Delta\sigma \sqrt{\pi a} \qquad (2)$$

where K_t is the stress concentration factor due to angular distortion, K_w, as well as due to misfit, K_h. Substitution of ΔK from eq.(2) into eq.(1) and integration of eq.(1) lead to

$$a = \left[a_o^{1-m/2} - (m/2-1)\; K_t^m \,(\Delta\sigma)^m \pi^{m/2} N \right]^{2/2-m} \qquad (3)$$

where, a_o, is the initial defect size.

In what follows, the Paris exponent, m, is considered as deterministic. Other variables i.e, the initial defect size, a_o, the Paris constant, C, the stress concentration factor, K_t, the stress range, $\Delta\sigma$, and the tensional membrane stress and the bending stress, σ_1, are considered as random variables. (see table-1)

Table-1

Random Variables	Probability Density Function	Distribution Paremeters	
a_o (mm)	$\lambda e^{\lambda x} / (e^{-\lambda x_1} - e^{-\lambda x_u})$	$\lambda = 0.65$,	$x_1 = 0$, $x_u = 34$
σ_1 ($^{Kg}/mm^2$)	Normal	$\mu = 14.5$,	$\sigma = 2$
$\Delta\sigma$ ($^{Kg}/mm^2$)	Normal	$\mu = 8.8$,	$\sigma = 2$
C	Lognormal	$\mu = -10.81$,	$\sigma = 0.0648$
K_h	Weibull	$\alpha = 1.16$,	$\beta = 0.134$
K_w	Weibull	$\alpha = 2.98$,	$\beta = 0.475$
δ_c	Weibull	$\alpha = 1.91$,	$\beta = 0.138$

If all the random variables, a_o, C, K_t(due to K_w and K_h), and are considered then the failure probability can be calculated from $\delta_c \geq 3.5$ ea as a multidimensional integral.

Some approximations are considered in the following two methods to estimate the failure probability.

APPROXIMATE METHODS FOR FAILURE PROBABILITY CALCULATION

METHOD-1

In this method m, the Paris's Law exponent, is considered to be deterministic and is taken equal to 4 for safety. The product "$C\ K_t^4\ (\Delta\sigma)^4 \pi^2\ N\ a_o$" is considered to be one single random variable equal to x. Then

$$a = a_o / (1-x) \qquad (4)$$

The first five statistical moments of a can be determined from the first n terms of infinite series of the following expressions

$$a^2 = a_o^2 / (1-2x+x^2) \qquad (5)$$

$$a^3 = a_o^3 / (1-3x+3x^2-x^3) \qquad (6)$$

$$a^4 = a_o^4 / (1-4x+6x^2-4x^3+x^4) \qquad (7)$$

The moments of the random variable, the crack opening displacement, can then be determined from $\delta = 3.5$ ea. Using these first five moments of δ, the best approximation to the probability distribution of δ is selected from the following five distribution functions. They are: the normal distribution, the lognormal distribution, the Weibull distribution, the exponential distribution and the gamma distribution. Some graphical methods is used for this purpose.

The distribution function of the critical crack opening displacement, δ_c, is determined from experimental data. The failure probability is obtained by

$$P_f = \text{Prob}\ (\delta \geqslant \delta_c) \qquad (8)$$

METHOD-2

In the first method seven variables, namely $K_w, K_h, \sigma_1, \Delta\sigma, C, \delta_c, a_o$ have been considered to calculate the failure probability. A paremeter sensitivity index (PSI) analysis (Shinozuka, 1976) is carried out by Monte Carlo simulation technique to select random variables. The results of the PSI analysis is presented in table-2. The PSI for the random variables, K_w, K_h, and C are less compared to these of other random variables. Therefore K_w, K_h and C are considered to be deterministic in this method. The analysis of failure probability remains the same as for method -1.

Table-2

Variables	PSI						
	0	5	10	15	20	25	30 Years
K_w	0.53	0.58	0.61	0.63	0.67	0.67	0.74
K_h	0.14	0.15	0.17	0.17	0.18	0.19	0.19
1	2.02	2.02	2.02	2.02	2.02	2.02	2.02
	0	1.16	2.41	3.53	5.12	6.63	8.24
C	0	0.02	0.04	0.06	0.08	0.11	0.13
c	3.19	3.19	3.19	3.21	3.21	3.22	3.22
a_o	2.59	2.65	2.72	2.77	2.86	2.94	3.02

EXAMPLE PROBLEM, RESULTS AND DISCUSSION

Method-1 and method-2 are employed to determine the failure probability of a spherical tank. Relevant data for the problem are, Young's modulus, $E=2.1\times10^4\ Kg/mm^2$, thickness, t=34mm, initial crack depth=0 mm, final crack depth=30mm, and number of loading cycles per year=252. The different probability density functions and their associated parameters for the various random variables are shown in the table-1. The probability of failure is determined as a function of T, the number of years of service. The failure probability results are shown in figure-1.

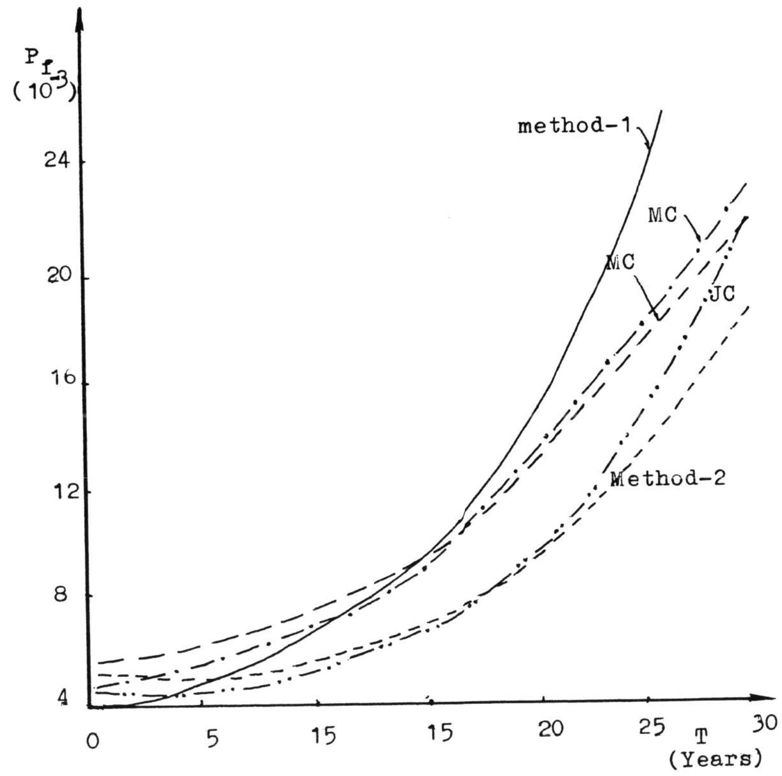

Fig-1

These data are compared with results obtained from a) Hasofer and Lind's reliability index method, JC method, (Hasofer and Lind', 1974), b) Monte Carlo simulation and c) improved Monte Carlo simulation using dual random variables as variance reduction technique.

From the P_f results it can be shown that at a given probability of failure, the ratio of life obtained from Monte Carlo method to method-1 is 7:1. and ratio of life obtained from Monte Carlo method to method-2 is 2:1 and Monte Carlo method and JC method life ratio is 10:1.

It can be concluded that the second method, as proposed in this paper, provides a more reliable method for estimating the failure probability.

REFERENCES

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