Parametric Representation of Fatigue Threshold and Dislocation Free Zone

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ABSTRACT

Fatigue threshold stress intensity factor based on dislocation dynamics was formulated as a simple function of temperature and grain size. This theory corresponds well with experimental results for steel under not so high temperature, and was related to surface roughness effect on ΔK_{th}. Furthermore representation of ΔK_{th} by dislocation free zone was discussed.

KEY WORDS

Fatigue threshold; Dislocation dynamics model; Grain size effect; Temperature dependence; Surface roughness-induced crack closure; Frequency effect; Dislocation free zone

1. INTRODUCTION

Many theoretical analysis of fatigue threshold stress intensity factor, ΔK_{th} were carried out (Yokobori and Yokobori, 1981, 1982, Gerberich and Moody 1979, Mckittrick et al., 1981, Mura et al., 1984, Smith, 1981, Sadananda and Shahinian, 1977). But under fatigue threshold condition, the effect of microstructure of material is considered to be dominant. So, it is necessary to analyze ΔK_{th} taking account of this effect (Yokobori and Yokobori, 1981, 1982).

The analysis of ΔK_{th} based on dislocation emission and dynamics were carried out (Yokobori and Yokobori, 1981, 1982). From these analysis, the critical stress intensity factor of dislocation emission from the crack tip and the effect of grain size on ΔK_{th} were obtained in terms of the simple equations (Yokobori and Yokobori, 1981, 1982). But in this theory, the effect of material constants concerning temperature dependence is derived as a complex function and, therefore the characteristics of temperature dependence is not so clear.

Thus, in the present paper, at first, the effect of temperature on ΔK_{th} was investigated by assuming plastic deformation process based on dislocation dynamics is rate determining. And equation of ΔK_{th} was derived as a simple function of temperature and grain size. Furthermore, this theory was compared with experimental results.

Recently, the concept of dislocation free zone is proposed (Zhao and Li, 1985, Majumder and Burns, 1983, Lin and Thomson, 1986, Chang and Mura, 1987) but the discussion concerns only its existance and the study has not yet carried out on its chracteristics and practical application. In the present paper, dislocation free zone was also investigated with respect to various material constants. Furthermore, the representation of ΔK_{th} in terms of dislocation free zone was attempted for practical application.

2. BASIC EQUATION OF FATIGUE THRESHOLD STRESS INTENSITY FACTOR BASED ON DISLOCATION DYNAMICS

Fatigue threshold stress intensity factor, ΔK_{th} was derived by the following equation based on dislocation dynamics (Yokobori and Yokobori, 1981, 1982):

$$\frac{\Delta K_{th}-\Delta Ki^*}{\sigma_{cy}\sqrt{\varepsilon}} = \left(\frac{2}{f(\beta)}\,\frac{\tau_o^*}{\sigma_{cy}}\right)^{\frac{1+\beta}{2\beta}} \left(d\,\frac{\dot{\tau}}{\tau_o^*\upsilon_o}\,\frac{m+1}{a_1}\right)^{\frac{1+\beta}{2\beta(m+1)}} \tag{1}$$

where, $\Delta Ki^*=\mu\sqrt{b}/((1-\nu)\sqrt{\pi\xi_o})$: critical stress intensity factor of dislocation emission from the crack tip, μ: shear modulus, b: burgers vector, ν: Poisson's ratio, ξ_o: non-dimensional value of core cut off of dislocation controlled by burgers vector, σ_{cy}, β: initial yield stress and strain hardning exponent under cyclic loading, respectively. m, τ_o^*: material constant in the equation of isolated dislocation motion, that is, $v=v_o(\tau\ \tau_o^*)^m$, v_o=1 cm/s, $f(\beta)=\{(\beta+1/2)\ (\beta+3/2)\Gamma(\beta+1/2)/\Gamma(1/2)\Gamma(\beta+1)\}^{\beta/(1+\beta)}$ a_1: constant value. a_1 is approximately equal to 1.5 (Yokobori et al., 1974, 1975). ε is the length corresponding to the ε-b region near the crack tip in which the number of dislocation take some critical value N_{th} under threshold condition and expressed by the following equation (Yokobori and Yokobori, 1981, 1982):

$$\varepsilon = \frac{N_{th}}{\{k^*(m)\}^2}\,\frac{b}{\left(\dfrac{b\dot{\tau}\tau_o^{*m}}{\mu^{m+1}\upsilon_o}\right)^{2/(m+2)}} \tag{2}$$

where, $k^*(m)=1.4m^{-1.45}(m+1)/a_1$.

Since material is not so damaged by cyclic stress under threshold condition, cyclic damage is not conspicuous. Therefore, initial cyclic yield stress, σ_{cy} is assumed to be equal to monotonic yield stress, σ_y and

written by the following equation (Yokobori et al., 1977):

$$\sigma_{cy} = 2\tau_o^*\left(\frac{N^*}{\rho}\,\frac{1}{\gamma(m)}\right)^{\frac{1}{m+1}}\left(\frac{\mu b\dot{\tau}}{\upsilon_o\tau_o^{*2}}\right)^{\frac{1}{m+1}} \tag{3}$$

where, N*: the specified number of dislocations per unit area at the start of yielding, ρ: grown-in dislocation density per unit area, $\gamma(m) = 1.4\ m^{-1.45}$. Assuming $1/(m+2)\simeq 1/(m+1)\simeq 1/m$ for the value of m equal to 10~30, and substituting Eqs.(2) and (3) into Eq.(1), we get the basic equation of fatigue threshold stress intensity factor as follows:

$$\Delta K_{th} = \Delta Ki^* + M^*\left(\frac{d}{b}\right)^{\frac{1+\beta}{2m\beta}} \tag{4}$$

where, M* is expressed by Eq.(5).

$$M^* = 2\mu\sqrt{b}\left[\frac{N_{th}}{\{k^*(m)\}^2}\right]^{1/2}\left[\frac{N^*}{\rho}\,\frac{1}{\gamma(m)}\right]^{1/m}\left[\frac{m\gamma(m)\tau_o^*}{f(\beta)^m a_1\mu}\,\frac{\rho}{N^*}\right]^{(1+\beta)/2m\beta} \tag{5}$$

3. RESULT AND DISCUSSION

3-1 Temperature and Grain Size Effect on Fatigue Threshold

Eq.(4) shows grain size effect on fatigue threshold stress intensity factor, ΔK_{th} has the term of $d^{(1+\beta)/2m\beta}$ and is in good agreement with experimental results (Yokobori and Yokobori 1981, 1982). But Eq.(5) is very complex equation, and thus temperature effect is not clearly understood. Then, in this section, temperature effect on ΔK_{th}, coupled with grain size effect is derived as a more convenient function of temperature and grain size.

When the velocity of an isolated dislocation is given by a thermally activated process, material constant, m is expressed as follows (Prekel and Conrad, 1967):

$$m = H_k/4kT \tag{6}$$

where k is Boltzmann constant, H_k is kink energy, T is absolute temperature. Substituting reasonable data for steel (Hahn, 1962) (m=10 for T=293K) into Eq.(6), m is written by the following equation.

$$m = 2.93\times 10^3/T \tag{7}$$

When we take experimental values of Si-Fe (Stein and Low, 1960) as τ_o^* which are assumed to be equal to those of steel, Eq.(8) is obtained.

$$\tau_o^* = 46.67e^{-0.002959T} \tag{8}$$

For other material constants which are not so effective on temperature, we take reasonable values for steel, saying $\mu = 7.94\times10^4$ MPa, $b = 3\times10^{-10}$m, $\rho = 10^{10}/m^2$, $N^* = 10^{15}/m^2$, $\beta = 0.11$, $a_1 = 1.5$, $N_{th} = 38.94$, which is the number of dislocations when dislocation density in the ε-b region amounts to $N^* = 10^{15}/m^2$ corresponding to the dislocation density of yielding. Substituting Eqs.(7) and (8) into Eq.(5), the relation between M* in Eq.(5) and temperature is obtained as shown in Fig.1. From these results, M* is written by Eq.(9) in terms of temperature.

$$M^* = 200 \times 0.9668^T \tag{9}$$

Eq.(9) is shown by solid line in Fig.1. Substituting Eqs.(6) and (9) into Eq.(4), the equation of ΔK_{th} for steel is obtained by the following equation as a function of temperature and grain size.

$$\Delta K_{th} = \Delta Ki^* + 200 \times \left(\frac{d}{b} \times 10^{-8}\right)^{0.001722T} \tag{10}$$

or

$$\Delta K_{th} = \Delta Ki^* + C_1 \times \left(C_2 \times \frac{d}{b}\right)^{\frac{2(1+\beta)kT}{\beta H_k}} \tag{11}$$

where C_1 and C_2 is material constant ($C_1 = 200$ MPa$\sqrt{m}$, $C_2 = 1.00 \times 10^{-8}$). The relation between ΔK_{th} and T by Eq.(11) is shown in Fig.2. Experimental results of Cr-Mo-V steel obtained by Liaw (Liaw, 1984) are compared with Eq.(11) by this theory as is shown in Fig.3. Fig.3 shows this theory is in good agreement with experimental results of Cr-Mo-V steel, not only qualitatively but also quantitatively under T < 400K. The result shows fatigue threshold characteristics under not so high temperature is controlled by plastic deformation mechanism based on dislocation dynamics, and the temperature and grain size dependence are explained by dislocation dynamics model.

It is considered that fatigue threshold characteristics in this temperature range is assumed to depend on crack closure induced by crack surface roughness (Liaw, 1984), that is, so called surface roughness-induced crack closure (Liaw, 1984). Concerning crack surface roughness, some experimental results show that crack surface roughness reduces with increase of temperature (Vincent and Rémy, 1981). Therefore from these results, it may be inferred that surface roughness-induced crack closure closely connects with plastic deformation mechanism due to dislocation motion. The effect of temperature on ΔK_{th} expressed by Eq.(11) is also in

good agreement with other experimental results (Esaklul and Gerberich, 1985, Tobler,1985, Liaw et al., 1983), qualitatively.

On the other hand, it may be considered that oxide-induced crack closure is dominant under T>400K in Fig.3 (Liaw, 1984). That is, ΔK_{th} may be controlled by such mechanism under higher temperature range.

3-2 Frequency Effect on Fatigue Threshold

Eq.(4) shows that fatigue threshold stress intensity factor is not affected by applied loading rate, that is, frequency. Previously frequency effect on fatigue threshold stress intensity factor was investigated by detailed experiments for various applied stress wave form condition as shown in Table 1 (Yokobori et al., 1985). Experimental results are shown in Fig.4. These results show, although fatigue threshold stress intensity factor for slow-fast wave form is slightly larger than that for fast-slow wave form, these values are fairly equal and are not so much affected by stress rising time and decreasing time, that is, frequency. Thus experimental results are in good agreement with Eq.(4) derived from dislocation dynamics theory. This results also assure the validity of step decreasing method to estimate fatigue threshold stress intensity factor, because with decrease of each applied loading level, rising and decreasing rate of applied stress decreases, respectively.

3-3 Analysis of Dislocation Free Zone and its Application for Fatigue Threshold Stress Intensity Factor, ΔK_{th}

3-3-1: Analysis of Dislocation Free Zone. When dislocation groups emitted from the source move discretely with interactions, the region where no dislocation is located exists near the source as shown in Fig.5. This region is called dislocation free zone (Zhao and Li, 1985) and its value is defined by Eq.(12) (Zhao and Li, 1985).

$$F = \frac{x_n}{x_{n-1} - x_n} \tag{12}$$

where, x_n is the position of the nearest dislocation from the source. The concept of dislocation free zone has concerned only its existance until now, and any study has not been carried out on the characteristics and practical application (Zhao and Li, 1985, Majumder and Burns, 1983, Lin and Thomson, 1986, Chang and Mura, 1987). Therefore in this section, the characteristics of dislocation free zone is investigated and its application of fatigue threshold is discussed.

The computer simulation of the dynamic behavior of dislocation groups emitted from the source is carried out under several stress rates and material constants (Yokobori and Yokobori, 1974, 1975, 1977). In the present paper, we calculated the value of F from these results of simulation and analysed its characteristics. Detailed method of analysis is shown in Ref. (Yokobori and Yokobori, 1974, 1975, 1977). Material constants (Gorman et al., 1969, Chandhuri et al., 1962, Saka et al., 1973, Turner and Vreeland, 1970, Schadler 1964, Prekel et al., 1968, Guberman 1968) used are shown in Table 2.

For analyzing of dislocation free zone, it is necessary to consider the source activation stress of dislocation emission and the image force at crack tip. But at the first step of analysis, for clarifying qualititative chracteristics of dislocation free zone for various material constants, only back stress exerted on the dislocation source by dislocation groups emitted is considered. This is the similar as in Ref. (Yokobori and Yokobori, 1974, 1975, 1977).

Values of F obtained by the computer simulation for several stress rates and material constants are plotted as a function of dislocation number, n as is shown in Fig.6. It can be seen from Fig.6 that values of F will saturate to constant (defined as $F_{satu.}$) for $n \geq 5$, and is controlled only by m, independent of stress rate and other material constant as shown in Fig.7. The relation between $F_{satu.}$ and m is obtained as Eq.(13) and is shown by solid line in Fig.7.

$$F_{satu.} = 0.415m^{-0.628} \tag{13}$$

<u>3-3-2: The Relation Between Dislocation Free Zone and Fatigue Threshold Stress Intensity Factor, ΔK_{th}.</u> Practical application of dislocation free zone, F has not yet been clarified. But herein $F_{satu.}$ is now found to be controlled only by material constant m, independent on stress rate and other material constants. On the other hand, it is proved experimentally and theoretically that ΔK_{th} is not affected by stress rate as is shown in §3-2. Therefore, ΔK_{th} is assumed to connect with dislocation free zone. Then an attempt is carried out herein to correlate ΔK_{th} with dislocation free zone.

Value of L is taken as a representative parameter of ΔK_{th}, which is a complex function of material constants, expressed as follows:

$$L \equiv \frac{\Delta K_{th} - \Delta Ki^*}{\left(\frac{N^*}{\rho}\right)^{\frac{\beta-1}{2m\beta}} N_{th}^{\frac{1}{2}}}$$

$$= 2\mu\sqrt{b}\left\{\frac{a_1}{\gamma^2(m)(m+1)}\right\}^{\frac{1}{m}}\left\{\frac{1}{f^m(\beta)}\ \frac{m\gamma(m)\tau_o^*}{a_1\mu}\right\}^{\frac{1+\beta}{2m\beta}}\left(\frac{d}{b}\right)^{\frac{1+\beta}{2m\beta}} \tag{14}$$

If values of L is expressed by dislocation free zone, ΔK_{th} can be evaluated easily by dislocation free zone. The relation between values of L and $F_{satu.}$ for various metals was shown in Fig.8 for specified value of grain size. Fig.8 shows that good correlation was obtained between values of L and $F_{satu.}$ except for Al (fcc metal). Taking account of the effect of grain size, values of L is estimated by Eq.(15) except the case of Al:

$$L = B^{C \times Fsatu.} \tag{15}$$

where, $B = B(d/b), C = C(d/b)$. The effect of grain size, d/b on B and C in Eq.(15) are shown in Figs.9 and 10, respectively. From these results, B and C are written by the following equations, respectively.

$$B = 0.5245(d/b)^{0.2154} \tag{16}$$

and

$$C = 2.816 \times 10^3 + 7.071 \times 10^{-3}(d/b) \tag{17}$$

$$(\text{where, } 5.0 \times 10^4 \leq d/b \leq 1.5 \times 10^5)$$

Substituting Eqs.(16) and (17) into Eq.(15), value of L is written as follows:

$$L = \left\{0.5245 \times \left(\frac{d}{b}\right)^{0.2154}\right\}^{\{2.816\times10^3+7.071\times10^{-3}(\frac{d}{b})\} \times F_{satu.}} \tag{18}$$

where, $5.0 \times 10^4 \leq d/b \leq 1.5 \times 10^5$. The relation of Eq.(18) is shown by solid line in Fig.8. It can be seen from Fig.8 that Eq.(18) is in good agreement with the experimental relation between value of L and $F_{satu.}$ except for Al. So, value of L is estimated easily by saturated value of dislocation free zone, $F_{satu.}$ and grain size. Substituting the value of N* and grown-in dislocation density, ρ into Eq.(14) and using L of Eq.(18), we get ΔK_{th} as function of $F_{satu.}$, d/b, N* and ρ. N_{th} is calculated by Eq.(2) with the value of ε taken 10^{-4}m.

4. CONCLUSION

(1) Fatigue threshold stress intensity factor, ΔK_{th} based on dislocation dynamics was formulated as a simple function of temperature and grain size as follows.

$$\Delta K_{th} = \Delta Ki^* + C_1 \times \left(C_2 \times \frac{d}{b}\right)^{\frac{2(1+\beta)kT}{\beta H_k}}$$

This theory shows ΔK_{th} decreases as temperature increases, and are in good agreement with experimental result for steel under not so high temperature. This theory also corresponds well with the surface roughness effect on ΔK_{th}.

(2) From experimental result of steel, ΔK_{th} is not so much affected by stress rate. This result is in good agreement with the present theory based on dislocation dynamics, and proves the validity of step decreasing method proposed as a estimation method of ΔK_{th}.

(3) Dislocation free zone, $F(=x_n/(x_{n-1} - x_n))$ under dislocation groups

movement emitted from the source become saturated for the emitted number of dislocation n ≧ 5 and takes constant value. This saturated value, $F_{satu.}$ is controlled only by material constant, m.

(4) $F_{satu.}$ corresponds well with value of L as following equation:

$$L = \left\{0.5245 \times \left(\frac{d}{b}\right)^{0.2154}\right\}^{\{2.816\times10^{3}+7.071\times10^{-3}(\frac{d}{b})\}\times F_{satu.}}$$

where L is the representative parameter of ΔK_{th}. Using this equation, ΔK_{th} is obtained as a function of dislocation free zone and grain size.

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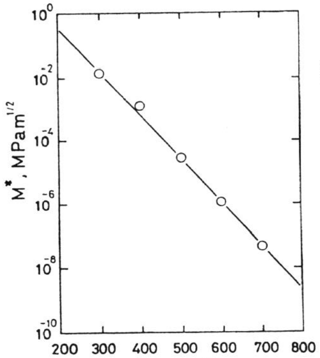

Fig. 1. Relation between M* and temperature.

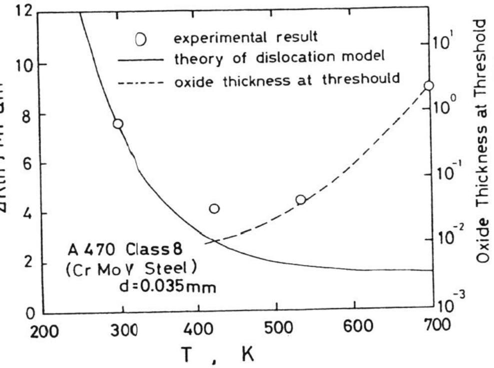

Fig. 3. Comparison between the theory based on the dislocation dynamics model and the experimental result for fatigue threshold.

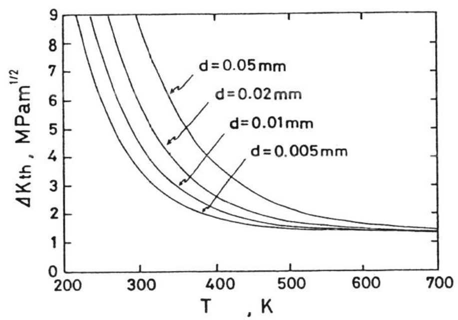

Fig. 2. The effect of temperature on fatigue threshold stress intensity factor based on dislocation dynamics model.

Table 1 Experimental condition for investigating the effect of stress rate on ΔKth

	Load	f (Hz)	P_{max} (kg) (KN)	R	T_R (sec)	T_D (sec)	T_R/T_D
I	T_D, P_{max}, P_{min}, T_R	10	700 (6.86)	0.2	0.03	0.07	0.43
			700 (6.86)		0.07	0.03	2.3
II		5	700 (6.86)	0.2	0.03	0.17	0.18
			700 (6.86)		0.17	0.03	5.7
III	T_D	2.5	700 (6.86)	0.2	0.03	0.37	0.08
	T_R		700 (6.86)		0.37	0.03	12.3

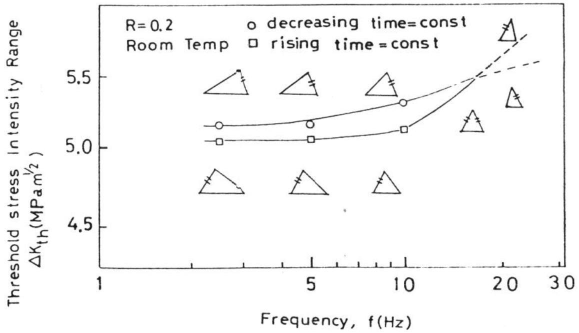

Fig. 4. Effect of stress rate on ΔK_{th}

Table 2 Material constants used for calculating.

Material	m	τ_0^*(MPa)	μ (MPa)
Al	1.0	8.33×10^{-3}	2.646×10^4
Ge	1.46	835.0	4.903×10^4
Fe screw	2.6	490.0	7.943×10^4
Fe edge	3.0	53.9	7.943×10^4
W 238-2	4.0	548.8	1.549×10^5
W	4.8	307.7	1.549×10^5
Mo	6.4	53.9	1.254×10^5
Nb	15.0	43.1	3.772×10^4

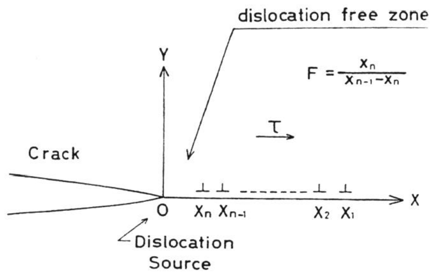

Fig. 5. Model of dislocation free zone.

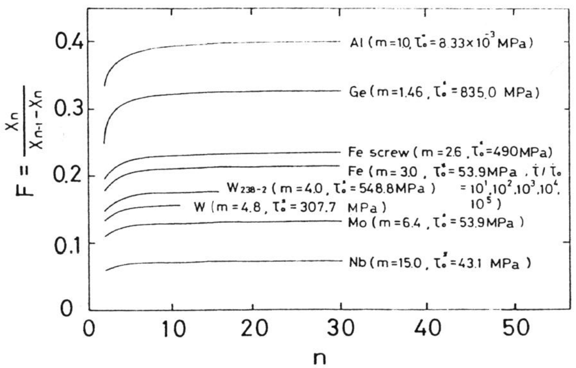

Fig. 6. Relation between dislocation free zone, $F=X_n/(X_{n-1}-X_n)$ and dislocation number, n .

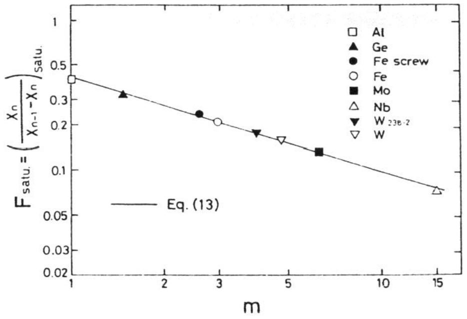

Fig. 7 Relation between $F_{satu.}$ and m.

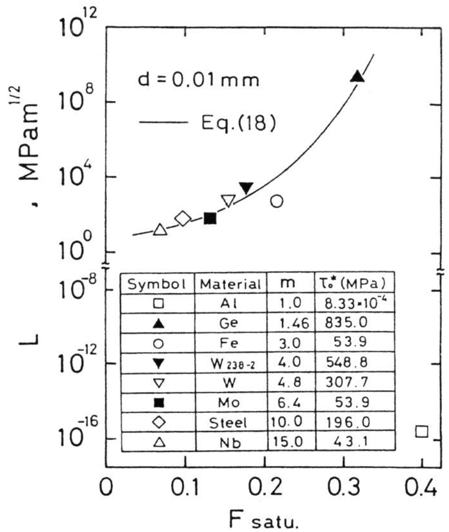

Fig. 8 Relation between L value and $F_{satu.}$.

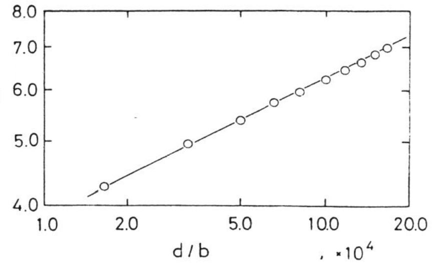

Fig. 9 Relation between B and d/b.

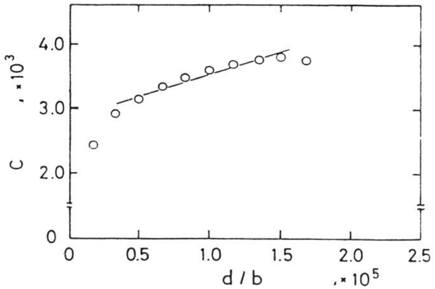

Fig. 10. Relation between C and d/b.