Near-tip Fields During Creep Crack Growth by Grain Boundary Cavitation

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ABSTRACT

A synopsis of recently obtained results (Li, Needleman and Shih, 1988b) on near tip fields during transient creep crack growth under plane strain and small scale creep conditions is presented. In the analyses, full account is taken of the finite geometry changes accompanying crack tip blunting and the material is characterized as an elastic-power law creeping solid with an additional contribution to the creep rate arising from a given density of cavitating grain boundary facets. When the crack growth rate is faster than the growth rate of the creep zone, HR singular fields (Hui and Riedel, 1981) dominate over the crack tip region. For a crack that grows more slowly than the creep zone, HRR type fields (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) dominate over the crack tip region. Regardless of which of the two singular fields dominates for the growing crack, finite strain effects are significant over a size scale of the order of the crack opening displacement at crack growth initiation.

KEYWORDS

creep crack growth; grain boundary cavitation; small scale creep; finite deformation.

INTRODUCTION

There is considerable interest in elucidating near tip behavior during transient crack growth, (Riedel, 1987, Wilkinson and Vitek, 1982, Bassani, 1981, Wu *et al.*, 1986, Hui and Banthia, 1984, Hawk and Bassani, 1986 and Bassani *et al.*, 1988). One impetus for this stems from the search for characterizing parameters such as K_I, C^* and J. The characterizing parameter formulation for analyzing crack growth presumes that the damage (or at least most of the damage) is confined to a rather local process zone and that the fields in the surrounding annular region are well approximated by certain singular fields. Presuming that the mechanism of crack growth does not change and that the magnitude of the singular fields can be related to the crack geometry, remote loads and material parameters, the characterizing parameter approach provides a framework for predicting fracture behavior in structures from measurements on test specimens. Furthermore, when this approach is applicable, the single characterizing parameter controls the evolution of damage and hence the rate of crack growth. A different approach to the analysis of creep crack growth involves the solution of full boundary value problems based on a constitutive relation that has one or more internal variables that characterize damage evolution. This methodology has been used by various investigators, for example (Hayhurst *et al.*, 1984, Chaboche, 1988 and Tvergaard, 1986).

Here, a synopsis of results on near tip fields, during transient creep crack growth under plane strain and small scale creep conditions is presented (Li *et al.*, 1988b). The analyses use the micromechanically based constitutive framework of (Tvergaard, 1984ab). A more complete description of the results and of the analysis procedure is given in (Li *et al.*, 1988b).

BOUNDARY VALUE PROBLEM FORMULATION

A small scale creep boundary value problem is analyzed in (Li *et al.*, 1988b). Attention is confined to quasi-static deformations and body forces are presumed absent. Traction free conditions are imposed on the crack surface, and tractions corresponding to the mode I plane strain elastic stress fields are imposed along a circular arc remote from the crack tip at time $t = 0$ and held constant thereafter.

The material is characterized as an elastic-power law creeping material, with an additional contribution to the rate of creep deformation arising from a given density of cavitating grain boundary facets as depicted in Fig. 1. A full finite deformation phenomenological constitutive description of this process has been developed (Tvergaard, 1984ab), extending previous work (Hutchinson, 1983 and Rice, 1981).

The rate of deformation tensor, $\mathbf{d}$, is written as the sum of an elastic part, $\mathbf{d}^e$, and a creep part, $\mathbf{d}^c$, so that $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^c$, where

$$\mathbf{d}^e = (1+\nu)\hat{\boldsymbol{\sigma}}/E - \nu(\hat{\boldsymbol{\sigma}} : \mathbf{I})\mathbf{I}/E \tag{1}$$

and

$$\mathbf{d}^c = \dot{\epsilon}_0\left(\frac{\bar{\sigma}}{\sigma_0}\right)^n \left[\frac{3}{2}\frac{\boldsymbol{\sigma}'}{\bar{\sigma}} + \rho\left\{\frac{3}{2}\frac{n-1}{n+1}\frac{\boldsymbol{\sigma}'}{\bar{\sigma}}\left(\frac{s-\sigma_n}{\bar{\sigma}}\right)^2 + \frac{2}{n+1}\left(\frac{s-\sigma_n}{\bar{\sigma}}\right)\bar{\mathbf{n}}\otimes\bar{\mathbf{n}}\right\}\right] \tag{2}$$

where $\hat{\boldsymbol{\sigma}}$ is the Jaumann rate of Cauchy stress, $\mathbf{I}$ is the identity tensor, E is Young's modulus and ν is Poisson's ratio. The notation $\mathbf{A} : \mathbf{B}$ denotes the dyadic product; i.e. $\mathbf{A} : \mathbf{B} = A^{ij}B_{ji}$. In (2), n is the creep exponent, $\mathbf{x}\otimes\mathbf{y}$ denotes the tensor product having components $x^i y^j$ and $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \sigma_m\mathbf{I}$, $\sigma_m = (1/3)(\boldsymbol{\sigma} : \mathbf{I})$, $\bar{\sigma}^2 = (3/2)\boldsymbol{\sigma}' : \boldsymbol{\sigma}'$. Here, $s = \bar{\mathbf{n}}\cdot\boldsymbol{\sigma}\cdot\bar{\mathbf{n}}$, representing the macroscopic normal stress on cavitating facets with normal $\bar{\mathbf{n}}$ in the current configuration. The parameters σ_n and ρ are internal variables; σ_n is the average normal stress in the vicinity of the voids and ρ is a measure of the density of the cavitating facets, which, in the calculations is prescribed to have the constant value 0.20.

Evolution equations are specified for $\bar{\mathbf{n}}$ and for σ_n. The evolution of $\bar{\mathbf{n}}$ is obtained from the geometrical relation $\bar{\mathbf{n}} = \mathbf{n}\cdot\mathbf{F}^{-1}/|\mathbf{n}\cdot\mathbf{F}^{-1}|$, where $|\quad|$ denotes the norm of a vector and $\mathbf{n}$ is the normal to the cavitating grain facet in the reference configuration. The evolution equation for σ_n comes from the description of grain boundary void growth. The voids on the grain boundary facet grow by the combined processes of grain boundary diffusion and plastic dislocation creep in the adjoining grains. The physical situation modelled is shown in Fig. 1 and specific expressions for the evolution equations for the internal variables are as given by (Tvergaard, 1984ab) and can also be found in (Li *et al.*, 1988b). Failure is taken to occur when the cavities on the grain boundary facets coalesce to form open microcracks, i.e. when the cavity radius equals the cavity half spacing, (Tvergaard, 1984ab).

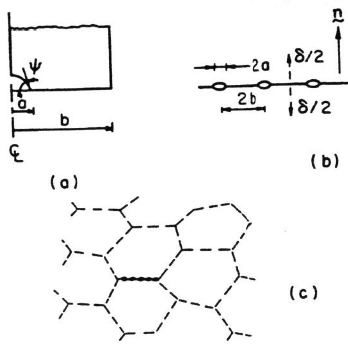

Fig. 1. (a) Spherical-caps shape of a single cavity. (b) Equally spaced cavities on a grain boundary. (c) An isolated, cavitated grain boundary facet in a polycrystalline material.

To explore the characterization of stress and deformation fields for growing cracks and the dependence of these fields on crack growth rate, values of the stress intensity factor K_I were chosen (Li *et al.*, 1988b) to produce two very different responses—a fast growing crack and a slowly growing crack. For the fast growing crack case, the stress intensity factor K_I is $(K_I/\sigma_0)^2/\Delta_0 = 1250$, where $\Delta_0/2$ is the initial radius of the semi-circular notch representing the crack tip. In the other case, $(K_I/\sigma_0)^2/\Delta_0 = 312.5$ is specified, which leads to a much more slowly growing crack and, hence, to a larger creep zone.

NUMERICAL RESULTS

The radius of the cavitating grain boundary facets, the initial void half spacing and the initial void radius are specified by $R_0 = 0.05\Delta_0$, $b_0 = 0.1R_0$, $a_0 = 0.1b_0 = 0.01R_0$. It is assumed that the cavities are crack-like initially so that the initial void volume is taken to be zero. The material properties are specified by $\sigma_0/E = 0.002$, Poisson's ratio $\nu = 0.3$ and $n = 5$. The material parameter $\dot{\epsilon}_0$ serves to set a characteristic time scale, $1/\dot{\epsilon}_0$. Although the focus is on the characterization of near-tip fields for growing cracks, we begin by briefly considering the near-tip stress and deformation fields for a stationary crack.

Near Tip Fields prior to Crack Growth

Based on the constitutive equation (2) with $\sigma_n = 0$, (Hutchinson, 1983) has shown that the HRR (Hutchinson, 1968 and Rice and Rosengren, 1968) type asymptotic near-tip stresses and strain rates for a stationary crack have the form,

$$\sigma_{ij} = \sigma_0\left[\frac{C(t)}{\dot{\epsilon}_0\sigma_0 I(n,\rho)r}\right]^{1/(n+1)}\tilde{\sigma}_{ij}(\theta,n,\rho)$$
$$\dot{\epsilon}^c_{ij} = \dot{\epsilon}_0\left[\frac{C(t)}{\dot{\epsilon}_0\sigma_0 I(n,\rho)r}\right]^{n/(n+1)}\tilde{\dot{\epsilon}}_{ij}(\theta,n,\rho) \tag{3}$$

where r and θ are the polar coordinates centered at the crack tip. The constant I and the θ-variations of the suitably normalized functions $\tilde{\sigma}_{ij}$ and $\dot{\tilde{\epsilon}}_{ij}$ depend on the creep exponent n and the density of the cavitating facets ρ. With ρ constant, the amplitude parameter $C(t)$ is

$$C(t) = \int_{\bar{s}} [W\bar{n}_1 - \bar{\nu} \cdot \sigma \cdot \dot{\mathbf{u}}_{,1}]\, d\bar{s} \tag{4}$$

which can be shown to be path independent for all contours $\bar{s}$ around the crack tip in the region where the strain rate is dominated by the creep strain rate. In (4), $\bar{\nu}$ is the normal to the path $d\bar{s}$ in the current configuration, and $W = [n/(n+1)]\sigma : \mathbf{d}$ for a power law creeping solid.

The numerical results, (Li *et al.*, 1988ab), show that before crack growth and after well developed creep deformation, the angular variation of the numerically computed stress and strain rate fields, away from the region where finite strain effects dominate, are well approximated by HRR type field (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) angular variations. Finite deformation effects are significant within a zone that has dimensions of the order of the crack tip opening displacement.

Near Tip Fields for a Fast Growing Crack

A comparison between numerically computed near tip field quantities and the HR asymptotic fields (Hui and Riedel, 1981) for growing cracks is shown in Fig. 2. For a crack growing at a rate $\dot{\ell}(t)$, when $n > 3$, the asymptotic stress and strain fields take the form (Hui and Riedel, 1981),

$$\begin{aligned}\sigma_{ij} &= A_n\sigma_0 \left[\frac{\dot{\ell}(t)}{\dot{\epsilon}_0(E/\sigma_0)r}\right]^{1/(n-1)} \tilde{\sigma}_{ij}(\theta, n)\\ \epsilon_{ij} &= A_n\frac{\sigma_0}{E} \left[\frac{\dot{\ell}(t)}{\dot{\epsilon}_0(E/\sigma_0)r}\right]^{1/(n-1)} \tilde{\epsilon}_{ij}(\theta, n)\end{aligned} \tag{5}$$

where A_n is a numerical factor depending on n.

In (5) $\dot{\ell}(t)$, the crack growth rate, is undetermined by the asymptotic analysis and, in fact, is unspecified by usual continuum analyses. It is determined by the micromechanics of the failure process and the HR field (Hui and Riedel, 1981) values in Fig. 2 were obtained by substituting the numerically computed crack growth rate into (5). In these calculations, as well as in the slow growing crack calculations, crack growth occurred straight ahead along the initial crack line, (Li *et al.*, 1988b). In Fig. 2, the Cartesian components of Cauchy stress, σ_{11} and σ_{22}, are normalized by the reference stress σ_0 and the distance r from the current crack tip is normalized by the crack opening at the time that crack growth initiates, Δ_I. The solid line is obtained from the HR (Hui and Riedel, 1981) singularity, (5), and the points are obtained from the finite element solution. In Fig. 2c the effective strain $\bar{\epsilon}$ is calculated from $\bar{\epsilon} = (2/3)(\epsilon_1^2 + \epsilon_2^2 - \epsilon_1\epsilon_2)^{(1/2)}$, $\epsilon_1 = \log\lambda_1$ and $\epsilon_2 = \log\lambda_2$, where λ_i are the principal stretches.

Figure 2 shows very good agreement between computed field quantities and the HR (Hui and Riedel, 1981) asymptotic values over the range $2 \leq r/\Delta_I \leq 10$. Since the current crack tip opening is less than Δ_I, the range of dominance of the HR field (Hui and Riedel, 1981) is quite large when expressed relative to the current crack tip opening. Near the notch tip, the lowered value of σ_{11} and the increased effective strain, $\bar{\epsilon}$, are both consequences of the finite strains accompanying blunting. However, it should be noted that the numerical results do not

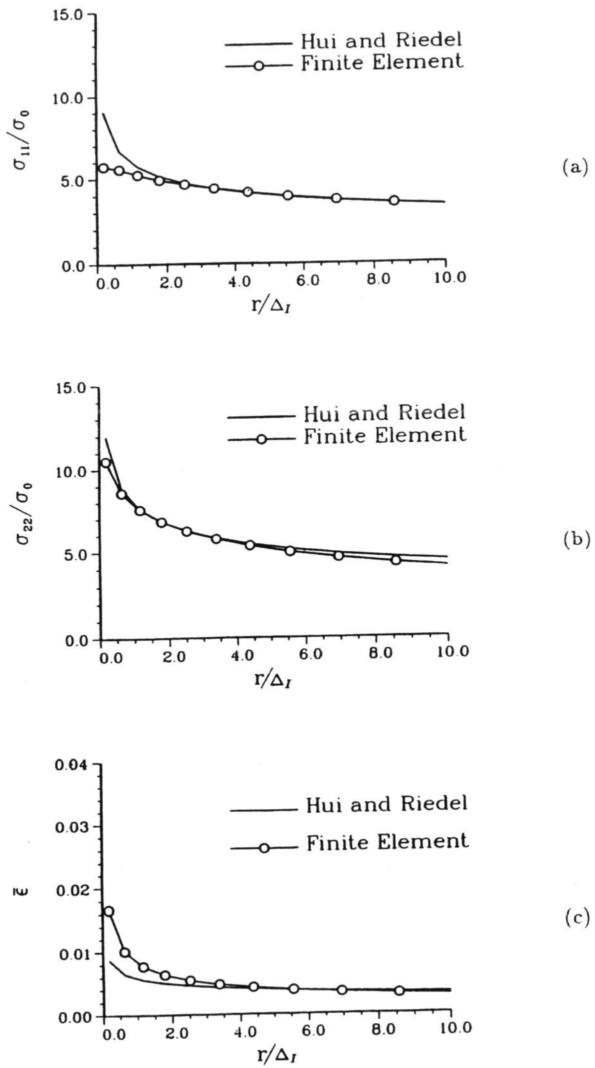

Fig. 2. Crack tip stress and strain distributions in the row of elements closest to the initial crack line at $t/t_I = 2.20$, for the case with $(K_I/\sigma_0)^2/\Delta_0 = 1250$, (a) σ_{11}/σ_0, (b) σ_{22}/σ_0, (c) $\bar{\epsilon}$.

accurately resolve the distribution of field quantities very near the crack tip since, for example, the traction free condition on the notch tip requires $\sigma_{11} = 0$ and the transition in Fig. 2a from $\sigma_{11}/\sigma_0 \approx 5$ to 0 occurs over one element.

Near Tip Fields for a Slowly Growing Crack

Figure 3 shows a comparison of computed field quantities with the HRR type field (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) solution, (3), for the slowly growing crack. Here, the Cartesian components of Cauchy stress, σ_{11} and σ_{22}, are normalized by the reference stress σ_0 and the distance r from the current crack tip is normalized by the crack opening at the time that crack growth initiates, Δ_I. In this figure, the solid line is obtained from the HRR (Hutchinson, 1968 and Rice and Rosengren, 1968) type singularity, (3), and the points are obtained from the finite element solution. This slowly growing crack is well contained

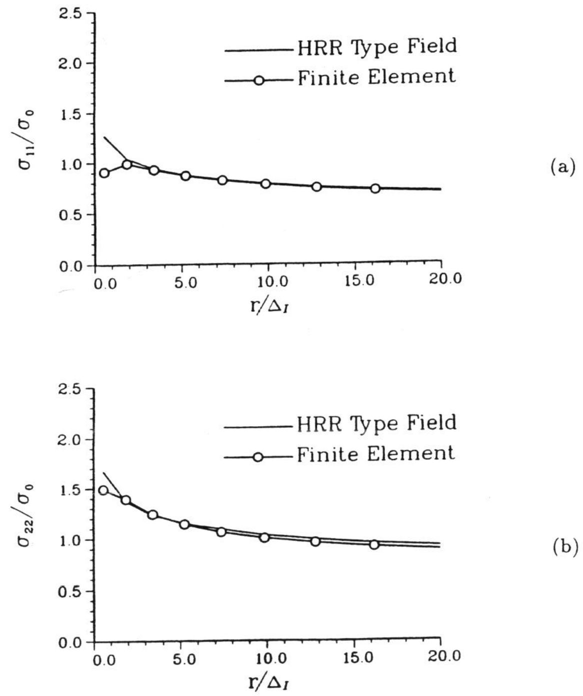

Fig. 3. Crack tip stress distributions in the row of elements closest to the initial crack line at $t/t_I = 128$, for the case with $(K_I/\sigma_0)^2/\Delta_0 = 312.5$, (a) σ_{11}/σ_0, (b) σ_{22}/σ_0.

within the creep zone. The HRR type field (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) values are obtained taking the value of $C(t)$ appearing in (3) equal to its computed, path independent value within the creep zone. It can be seen from these figures that the computed values of stresses are in excellent agreement with the HRR type field (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) values over the range of about 3–20 times the crack opening at the time of crack growth initiation, Δ_I. The comparison for the effective strain rate is similar to that for the stationary crack, (Li *et al.*, 1988b). Within the context of small strain theory for a mathematically sharp crack, the existence of HR (Hui and Riedel, 1981) singular fields at the crack tip does not depend on crack speed, but the finite strain region at the crack tip suppresses the HR (Hui and Riedel, 1981) fields in the calculation for the slowly growing crack, (Li *et al.*, 1988b).

CONCLUDING REMARKS

The calculations in (Li *et al.*, 1988ab) show that prior to crack growth, the stress and strain rate fields are well approximated by HRR type (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) singular fields, (3), in a region that is roughly a fifth of the creep zone. For a growing crack, at a high K_I value, so that the rate of crack growth is faster than the rate of growth of the creep zone, HR (Hui and Riedel, 1981) singular fields, (5), dominate over a zone that is more than ten times the crack tip opening at the initiation of crack growth (denoted by Δ_I), see Fig. 2. The magnitude of the fields scales with the crack growth rate $\dot{\ell}$. At a lower K_I value, where the rate of crack growth is slower than the rate of growth of the creep zone, HRR type fields (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) dominate over a zone which is about twenty times Δ_I, see Fig. 3. For durations up to ten times the initiation time t_I, the HRR type fields (Hutchinson, 1968, Rice and Rosengren, 1968 and Hutchinson, 1983) scale with $C(t)$. Regardless of which of the two singular fields dominates for the growing crack, finite strain effects were found to be significant over a size scale of the order of the crack opening displacement at crack growth initiation, (Li *et al.*, 1988b).

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