Near Threshold Fatigue Curve Based on the Dislocation Shielding Model of Fracture*

S.-J. CHANG* and T. MURA**
*Engineering Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA
**Department of Civil Engineering, Northwestern University, Evanston, IL 60201, USA

ABSTRACT

Dislocation pileup along an inclined direction from a crack tip has been observed in metals during tensile deformation in an electron microscope. Dislocations are emitted from the crack tip, move through a dislocation free zone and pile up in the plastic zone. The problem of inclined pileup of screw dislocations at a crack tip with a dislocation free zone has been solved by applying the Wiener-Hopf method. The result can be used to calculate the crack tip blunting and the local stress intensity K for the applied stress intensity K_3. According to the fatique theory of Laird or Neumann, the crack propagation per cycle has the order of magnitude of the blunting. The blunting versus K_3 is then plotted. It has the shape of the near threshold fatigue curve. To plot this curve, the local stress intensity K has been assumed to be the critical value K_g for the dislocation emission at the crack tip.

INTRODUCTION

Recent electron microscope studies of dislocation behavior at the crack tip of various metals have shown a dislocation-free zone (DFZ) often located between the crack and the plastic zone.[1-3] As the crack moved, the crack tip generated, primarily, edge dislocations on planes that were inclined to the crack plane. Although the slip geometry with two inclined pileups of dislocations symmetric with respect to the crack was occasionally observed, the crack often emitted dislocations on only one plane which was inclined to the crack plane.

---

*Research sponsored by the Division of Materials Sciences, U.S. Department of Energy under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc. and by ARO DAAG 29-85-K-0134, NSF-MRL DMR-8216972, Northwestern University.

The distribution of dislocations in the plastic zone coplanar to the crack was first treated by Bilby, Cottrell, and Swinden[4] (BCS). The BCS theory was recently extended to include the DFZ as part of the crack tip equilibrium configuration.[5-8] The presence of a DFZ was attributed to difficulty in generating dislocations at the crack tip, and this difficulty was expressed in terms of a critical stress intensity factor K_g required for dislocation generation.[5-8] The constant K_g can be derived from the model of Rice and Thomson[9] in terms of the core radius of a dislocation. General discussions on the shielding effect were given by Thomson[10] and Weertman.[11] Effects of pileup were also studied by Majumdar and Burns[12] and Li.[13] A review was written by Thomson.[14] The present calculation can evaluate the stress intensity factor at the crack tip in the presence of a dislocation pileup. The result is a function of the length e of the DFZ and the length ℓ of the plastic zone.

In the present work, we have treated the distribution of screw dislocations on a plane which is inclined to the crack. The problem is formulated as an integral equation which represents an equilibrium condition between the dislocations and the crack similar to the BCS model. This dislocation pileup equation is solved by applying the extended Wiener-Hopf technique. The extended Wiener-Hopf technique is required because the boundary condition in the DFZ is different from that in the pileup region. A detailed description of the method of solution has been published elsewhere.[16] The problem of inclined pileup of screw dislocations without a DFZ was solved recently by using the usual version of the Wiener-Hopf technique.[15] Although the mathematics involved is tedious, the method yields simple analytic expressions for the BCS condition and the local stress intensity factor. It is interesting to observe in this paper that a rearrangement of the numerical results leads to the near-threshold fatigue curve. A discussion on fatigue threshold was given by Mura and Weertman.[17]

PILEUP INTEGRAL EQUATION

As shown in the previous paper,[16] the stress distribution at z ($z = x + iy$) due to screw dislocations located at z_k ($k = 1, 2, \ldots$) is

$$\sigma = \sigma_y + i\sigma_x = \frac{K_3}{\sqrt{2\pi z}} - \frac{\mu b}{4\pi}\left[\frac{1}{2z} + \frac{1}{z + |z|}\right] + \sum_k \frac{\mu b_k}{4\pi\sqrt{z}}\left[\frac{1}{\sqrt{z} - \sqrt{z_k}} - \frac{1}{\sqrt{z} + \sqrt{z_k}}\right] \qquad (1)$$

where the summation excludes the self image term due to the dislocation at z and K_3 is the applied stress intensity factor of mode 3 deformation. The force on the dislocation at z along the inclined angle $\Psi = a\pi$ is the real part of $\sigma e^{i\Psi}$. If this force is balanced by the friction stress σ_f, then the integral equation of equilibrium can be derived by replacing the summation in Eq. (1) with an integration, that is,

$$\frac{K_3}{\sqrt{2\pi r}}\cos\frac{\Psi}{2} - \sigma_f = \frac{\mu b}{4\pi\sqrt{r}}\int_e^{\ell} f(r')\left[\frac{1}{\sqrt{r'} - \sqrt{r}} + \mathrm{Re}\,\frac{1}{\sqrt{r} + \sqrt{r'}\,e^{i\Psi}}\right]dr' \qquad (2)$$

where $f(r)$ is the distribution function, e denotes the length of DFZ,

and ℓ denotes the length of the plastic zone. The self image term, which is the second term in Eq. (1), is neglected because it is a differential of second order.

After applying the Mellin transform to the nondimensional form of Eq. (2), we obtain the extended Wiener-Hopf equation in the complex plane s in the form

$$t_e^s\, G_+(s) + T(s) + H_-(s) = B(s)\, K(s) \qquad (3)$$

where $t_e = \sqrt{e/\ell}$ and T, G_+, and H_- are the Mellin transforms of the applied stress, the DFZ stress, and the stress outside of the pileup region, respectively. Also, $B(s)$ is the transform of $rf(r)$ and $K(s)$ is the transform of the dislocation interaction force. Equation (3) is in a form suitable for solution by using the extended Wiener-Hopf technique.

SOLUTION OF THE PILEUP INTEGRAL EQUATION

In this paper, we shall not describe the method of solution, but only illustrate and discuss the results obtained. Numerical results were also obtained by using the finite difference method directly applied to the integral equation. The results from the two methods are in good agreement.

The DFZ condition which is derived by imposing a finite magnitude of stress at the end of the pileup region is obtained in a simple expression,

$$2\sigma_f(2\pi\ell)^{1/2}/(K_3\pi) = g(e/\ell,a)\sin(a\pi)(1+a)^{(1+a)/2}(1-a)^{(1-a)/2}/(a\pi) \qquad (4)$$

where $g(e/\ell,a)$ is a function which becomes unity when the DFZ is not present.

For $e/\ell = 0$, Eq. (4) is plotted in Fig. 1. We shall use the BCS condition to denote the DFZ condition at $e/\ell = 0$. For $e/\ell \neq 0$, the function $g(e/\ell,a)$ is plotted in Fig. 2 as $g(e/\ell,a) = \sqrt{F_{\ell g}}$. It is shown numerically that $g(e/\ell,a)$ is essentially independent of the angle Ψ and is smaller than one for all e/ℓ. Equation (4) provides a relationship between the applied force K_3 and the pileup length ℓ for a range of values of e/ℓ at any angle of inclination Ψ.

The total number of dislocations N has been obtained as

$$\sigma_f bN/G = h(e/\ell,a)\sin(a\pi)/(a\pi)\,, \qquad (5)$$

where $G = K_3{}^2/2\mu$ and $h(e/\ell,a)$ is plotted in Fig. 2 as $h(e/\ell,a) = F_{\ell g}\cdot F_{n\ell}$. The function $h(e/\ell,a)$ is equal to one for $e/\ell = 0$ and this function is essentially independent of the angle of inclination Ψ. For $e/\ell = 0$, Eq. (5) is also plotted in Fig. 1.

A combination of Figs. 1 and 2 will provide the numerical values of ℓ/G, ℓ/N, and N/G for a given value of e/ℓ at any angle of inclination Ψ.

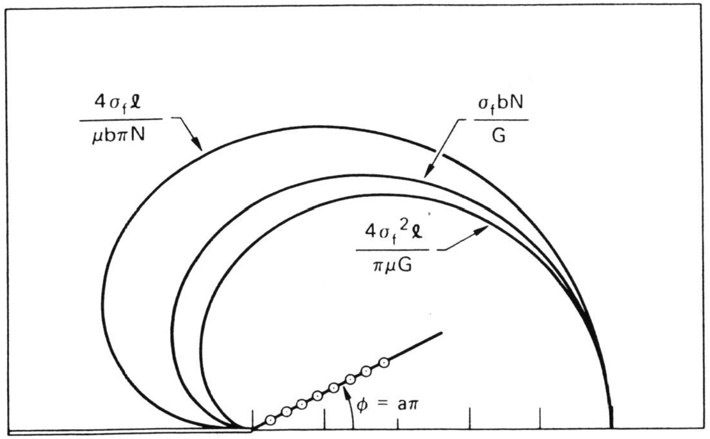

Fig. 1. BCS condition as expressed in terms of the pileup length ℓ, the applied stress G, and the total number of dislocations N in polar coordinates.

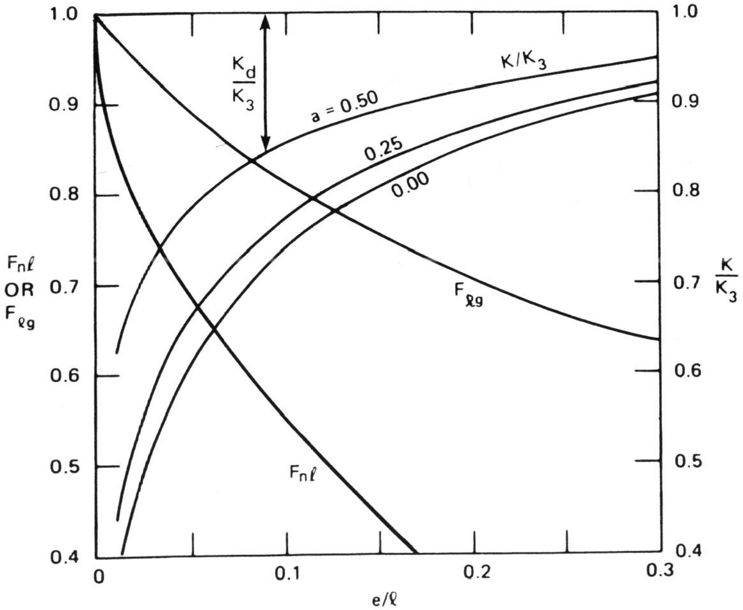

Fig. 2. Stress intensity factor K and reduction factors $F_n\ell$ and $F\ell_g$ for ℓ/N and ℓ/G, respectively, in the BCS condition as functions of the ratio e/ℓ.

The local stress intensity factor K at the crack tip is defined as the difference between the applied stress intensity factor K_3 and the factor K_d due to the presence of the shielding dislocations. The numerical values of K/K_3 are also plotted in Fig. 2. The value of K/K_3 depends strongly on the angle of inclination Ψ. For increasing Ψ, it is observed that dislocations located at the same distance from the crack tip contribute less to the shielding of the crack tip, thus, more dislocations (or smaller values of e/ℓ) are required to reach the same value of K/K_3.

INCLINED EMISSION OF DISLOCATIONS VERSUS BRITTLE EXTENSION

During direct experimental observation of the inclined pileup of dislocations, fast emission of dislocations along an inclined angle occurs first. The emission then stops and an equilibrium configuration of the inclined pileup is observed. At this moment the driving force for the emission of dislocations must have been reduced to the critical value K_g owing to the shielding effect of the emitted dislocations. For a measured value of e/ℓ, Fig. 2 will provide the critical value of K. After the emission theory is applied, this critical K must be equal to its theoretical value K_g defined by the core radius. The value of K_g is then compared with the brittle extension intensity K_c to determine whether the crack will emit dislocations or propagate in a brittle mode, as discussed in previous papers.[6,7]

As mentioned in previous papers the dislocation emission along an inclined angle is controlled by the critical emission constant K_g. For the local stress intensity K smaller than K_g, dislocations will not be emitted from the crack tip and, therefore, blunting will not be created.

FATIGUE CRACK PROPAGATION CURVE NEAR THRESHOLD

Numerical values plotted in Fig. 2 show that the local stress intensity factor K and the total number of dislocations N for the inclined pileup are functions of the size of the dislocation-free zone e/ℓ. As e/ℓ decreases, the local K tends to zero and the total number of dislocations N reaches that as required from the BCS case.[15] We replot Eq. (4) and a combination of Eqs. (4) and (5) with e/ℓ as a single parameter. The resulting curve of $b \cdot N$ versus K_3/K is shown in Fig. 3. In this plot, we eliminated ℓ by the solution for $K/\sqrt{\ell}$ and assumed $K = K_g$ where K_g is a material constant. This curve has the shape of the frequently observed near-threshold fatigue curve. It has the property that for large K_3 $b \cdot N$ tends to $K_3{}^2$ and K_3 decreases to a threshold value K_g as its lower limit. The two ends of the curve correspond to $e/\ell \rightarrow 0$ and $e/\ell \rightarrow 1$ if the curve is viewed from its parameter e/ℓ.

By applying the fatigue theory of Laird or Neumann, the numerical result shown in Fig. 3 is a near-threshold curve of the following problem. Cyclic load is applied to a solid body containing a semi-infinite crack. The applied stress intensity K_3 is increased to a finite value and then reduced to zero repeatedly. During the loading phase N dislocations are piled up

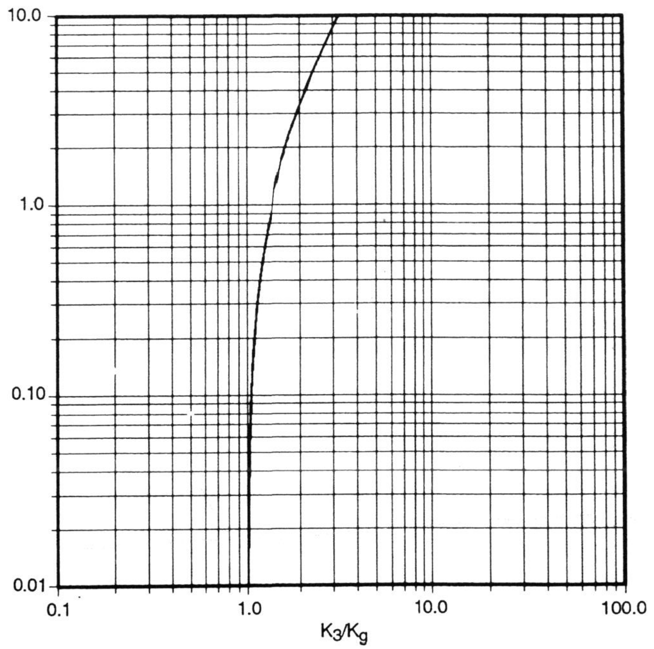

Fig. 3. The inclined pileup solution for b·N versus K_3/K_g where K_g is the local stress intensity factor equal to the dislocation emission intensity. Applying the fatigue theory of Laird or Neumann, the value b·N is equivalent to fatigue crack propagation per cycle dc/dN and K_3/K_g is equivalent to ΔK. $dc/dN \sim K_3^2$ for large ΔK and ΔK threshold tends to K_g.

along the direction 45 degrees from the crack plane and create a COD of b·N. According to the theory of Laird or Neumann, the crack will advance along the direction of the crack plane with an order of magnitude equal to b·N. A new crack tip is created. The applied K_3 is then reduced to zero. The inclined dislocation pileup is left behind the new crack tip and possibly results in some residual stress. The next loading phase will generate new inclined pileup of dislocations along slip plane, from the newly created crack tip. The crack tip then will advance another b·N with new crack tip. The unloading is then followed. The process repeats. Therefore, the plot shown in Fig. 3 represents fatigue crack propagation per cycle versus the applied K_3 with the threshold value K_g as its lower limit.

DISCUSSIONS

Perhaps we need to emphasize that in contrast to the BCS theory the finite threshold value $K_3 = K_g$ derived in this problem is a consequence of the finite dislocation emission resistance K_g and the existence of a dislocation-free zone in the fracture model. It is interesting to observe that as the applied K_3 increases the crack propagation per cycle dc/dN tends to K_3^2. Various ways to extend the K_3^2 to a higher exponents term K_3^n have been proposed in the literature such as treating the crack tip region as wedge crack.[18,19]

The near-threshold curve is derived based on a mode 3 solution with inclined dislocation pileup solution. The more realistic solution is the edge dislocation pileup problem. An even better problem probably is the double pileup. As the crack advances in each loading and unloading phase, residual stress is created due to the remaining dislocations in the earlier pileup. The subsequent pileup does not consider the influence of the residual stress.

REFERENCES

[1]Ohr, S. M. and J. Narayan (1980). *Philos. Mag.* A41, 81.
[2]Kobayashi, S. and S. M. Ohr (1980). *Philos. Mag.* A42, 763.
[3]Ohr, S. M., J. A. Horton, and S.-J. Chang (1983). Defects, Fracture and Fatigue, Ed. by G. C. Sih and J. W. Provan (Martinus Nijhoff Publishers, Boston, p. 3.
[4]Bilby, B. A., A. H. Cottrell and K. H. Swinden (1963). *Proc. R. Soc. London* A272, 304.
[5]Chang, S.-J. and S. M. Ohr (1981). Dislocation Modelling of Physical Systems, ed. by M. F. Ashby, R. Bullough, C. S. Hartley, and J. P. Hirth (Pergamon, New York), p. 23.
[6]Change, S.-J. and S. M. Ohr, (1981). *J. Appl. Phys.* 52, 7174.
[7]Ohr, S. M. and S.-J. Chang (1982). *J. Appl. Phys.* 53, 5645.
[8]Chang, S.-J. and S. M. Ohr (1983). *Int. J. Fracture* 23, R3.
[9]Rice, J. R. and R. Thomson (1974). *Philos. Mag.* 29, 73.
[10]Thomson, R. and J. E. Sinclair (1982). *Acta Metall.* 30, 1325.
[11]Weertan, J., I.-H. Lin, and R. Thomson (1983j). *Acta Metall.* 31, 473.
[12]Majumdar, B. S. and S. J. Burns (1983). *Int. J. Fracture* 21, 229.
[13]Li, J. C. M. (1984). Dislocations in Solids, Ed. by X. Markenscoff (ASME, AMD-Vol. 63) p. 35.
[14]Thomson, R. (1986). *Solid State Physics*, Ed. H. Ehrenreich and D. Turnbull, Vol. 39, pp. 1-129.
[15]Chang, S.-J. and S. M. Ohr (1984). *J. Appl. Phys.* 55, 3505.
[16]Chang, S.-J. and T. Mura (1987). *Int. J. Engng. Sci.* 25, 561.
[17]Mura, T. and J. Weertman (1983). *Fatigue Crack Growth Threshold Concepts*, Ed. by D. L. Davison and S. Suresh (AIME Conf. Proceedings), p. 531.
[18]Weertman, J. (1981). *Mechanics of Fatigue*, Ed. T. Mura (ASEE, AMD, Vol. 47), p. 11.
[19]Ohr, S. M., S.-J. Chang and R. Thomson (1985). *J. Appl. Phys.*, 57, 1839.