Fracture Control Via DRM–Algorithm

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ABSTRACT

A method for automatic fracture control is proposed. The method is based on representative parameters measured continuously or at times which are employed as inputs to the DRM-Algorithm. The algorithm automatically checks the consistency of the input data on the basis of the known past history, forms the predictor-controller regression model and yields, as outputs, predicted values of the same parameters or of certain "safety" function corresponding to a particular engineering problem. On the basis of DRM-Algorithm the automatic safety control systems can be built to monitor and control the mechanical stress or fatigue situation.

KEYWORDS

Fracture control; stress control; safety control.

INTRODUCTION

"An inspection of five Boeing 747 jetliners operated by Japan Air Lines (JAL) showed about 80 cracks on each of them, but non big enough to cause ruptures, the transport ministry said yesterday. ... Earlier reports have said cracks have been found in two British Airways and two Pan American 747s, but all are considered airworthy..." (Reuter, Gazette, February 19, 1986, Montreal).

"... The U.S. Federal Aviation Administration has urged inspection of all 747s with 10,000 or more landings because of reported cracking of the frames it says could cause failure of the aircraft in flight." (Reuter, Gazette, February 13, 1986, Montreal).

Let y(t) be a "sensor" function (e.g., a variable proportional to a damage factor in the case of brittle or fatigue fracture) which can be measured (continuously or at times) and which indicates dangerous situation in the sense that there exists some critical value y* such that if y(t) $\geq$ y*, then fracture of the system occurs in a short time. In simple cases y(t) may represent stress in material or displacements with y* representing, e.g., plasticity zone. In complex cases y(t) may be some composite parameter affecting and implying fracture when y(t) $\geq$ y*.

The evolution of y(t) in complex cases is usually unknown. Much research has been devoted to the study of y(t) through its representation as a stochastic process /1-4/ with its characteristics assumed a priori or determined on the basis of some known statistics or by experimentation. Whatever the results, this approach implies that, say, a plane may crash with some positive probability.

The present approach via DRM algorithm is deterministic in its nature and, if y(t) is really representative of the problem, then under certain conditions the method provides a means to reconstruct the evolution of y(t) and, thus, to prevent approaching a dangerous zone.

THE REPRESENTABILITY QUESTION

The answer on this question about y(t) depends on the process under consideration and on the knowledge of the natural laws governing the process. Even if such knowledge is scarce, some essential results about the behavior of the process can be obtained by applying the DRM system itself. For example, the DRM algorithm can distinguish between linear and nonlinear processes (see test example in /14/) and/or establish that some particular parameter is not representative.

In application to the crack growth process under cyclic/sustained loads on the aircraft wings and in the gas turbine engine the crack length can be chosen as the representative parameter y(t), see /1-5/. The crack growth is described by the equation

$$dy/dt = f(y, \sigma, \gamma, \mu) \quad (1)$$

where σ is the stress, γ is the geometry parameter and μ is the material parameter. The function f is, in general, unknown; up to certain precision it can be determined by the DRM algorithm or simply bypassed by a DRM system, if y(t) is measured directly.

Now, a DRM safety control system can be built on the basis of stress measurements (which is easy) with subsequent identification of f in (1), or on the basis of the crack propagation gauge (not to be confused with the gauge for measuring cracks) with the DRM program employed to check the consistency of the crack length measurements and to make crack growth predictions. Another interesting idea /5/ is to build the special on-board crack simulation machine with stress measurements as inputs to induce and simulate crack propagation in the machine used as input into a DRM predictor system. All three systems can be built so as to

account for the "retardation phenomenon", see /4/. In fact, if the peaks of stress are measured or modelled, then the DRM predictor takes them into account automatically.

THE PREDICTOR DRM MODEL

Suppose that the parameter y(t) is measured at times

$$y_n = y(t_n), \quad t_n = t_0 + n\Delta t, \quad n = 0, 1, \dots \quad (2)$$

and consider the sequence of equations:

$$e_n = -y_{n+r} + a_1 y_n + a_2 y_{n+1} + \dots + a_r y_{n+r-1}, \quad a_i = \text{const}, \ n = 0, 1, \dots \quad (3)$$

where $2 \leq r \leq N/4$, and N is the number of observations taken as a basis for identification. If for some r it happens that $e_n = 0$ for all n = 0, 1, ..., then the evolution of y_n is a linear process. Usually it is not the case. Consider the following linear programming problems indexed by s:

$$\min p_s = a^s_{r+1} \quad (4)$$

under the conditions:

$$a^s_{r+1} + e_n \geq 0, \quad n = s, s+1, \dots, s+N-r \quad (5)$$

$$a^s_{r+1} - e_n \geq 0, \quad n = s, s+1, \dots, s+N-r \quad (6)$$

$$a^s_{r+1} \geq 0 \quad (7)$$

with e_n defined by (3) and let s run through the sequence of natural numbers s = 0, 1, ... until the last available observation. For each s this problem has a solution $a_s = (a^s_1, \dots, a^s_r, a^s_{r+1})$ whatever r and $\{y_n\}$.

Let $p_0 > 0$ be the precision of observations y_n. Those s for which $a^s_{r+1} \leq p_0$ indicate time intervals over which the evolution of y_n can be represented by the model (3) with the accuracy p_0. Increasing r and N allows to improve the precision a^s_{r+1} and to adapt to the unknown physical realities given in the sequence of observations y_n. Once the model has been obtained, the evolution of y(t) can be forecast so as not to approach the fracture zone.

THE CONTROLLER DRM MODEL

Consider two sequences of observed data

$$y_0, y_1, \ldots, y_N, \ldots \quad (8)$$

$$u_0, u_1, \ldots, u_N, \ldots \quad (9)$$

which are in the input-output relation and suppose that the input u_i is applied after the output y_i has been measured, the latter being just an enumeration agreement. The two sequences describe a control system or a subsystem of unknown structure.

Consider the sequence of equations:

$$e_n = -y_{n+r} + a_1 y_n + \ldots + a_r y_{n+r-1} + b_1 u_n + b_2 u_{n+1} + \ldots + b_r u_{n+r-1}; \quad (10)$$
$$a_i, b_i = \text{const},\ n = 0, 1, \ldots$$

If the observed data (8), (9) fit to an equation of the type (10), then parameters r, a_i, b_i can be identified and the equation so obtained can serve as a controller for the process represented by (8), (9).

THE PRECISION-VALIDATION TRADE-OFF

To construct a plausible model, one should leave aside a subset of observations and use them to check a model already identified. To accomplish this task, the orders much less than N/3 should be considered. Let M be the number of observations set aside for validation of a model determined by earlier observations. Then for confident prediction the following requirements should be met, see /14/:

$$a^s_{r+1} \leq p_0 \qquad \text{for all} \quad s = 0, 1, \ldots \quad (11)$$

$$N - 3r \geq M > 0 \quad (12)$$

Take an integer $r \leq (N-M)/3$ and solve the linear programming problems (4)-(7). If (11) is satisfied, then a DRM predictor is found, otherwise, weaken, if possible, the requirements (11), (12) and repeat the solution for increased r. For more details one is referred to the papers /6/-/14/.

The system has numerous applications, mainly, in mechanical and aeronautical engineering (to prevent cracks and ruptures in aircrafts and spacecrafts and to ensure the reliability of engines) and in civil engineering (reliability of bridges).

REFERENCES

1. Effect of Load Spectrum Variables on Fatigue Crack Initiation and Propagation, ASTM STP 714, 1979.
2. Methods and Models for Predicting Fatigue Crack Growth under Random Loading, ASTM STP 748, 1981.
3. R. Arone, Estimation of life time and reliability of components with fatigue cracks. Proc. Fatigue Prevention and Design Conf., Amsterdam, 1986.
4. R. Arone, Fatigue crack growth under random overloads superimposed on constant amplitude cyclic loading. Engineering Fracture Mechanics, to appear.
5. R. Arone, Personal communication (Letter of Feb. 21, 1986, Technion, Haifa, Israel).
6. E.A. Galperin, An algorithm for long-term prognosis of deterministic time series. Proc. 1st Int. Conf. Math. Modelling, Vol. 1, 1977, pp. 437-446.
7. E.A. Galperin, Linear programming solution of the discrete stationary linear predictor problem. Proc. Int. Conf. Cyber. Soc., USA, 1979,pp. 707-711.
8. E.A. Galperin and R. Labonté, Two algorithms for solution of the discrete stationary linear predictor problem. I,-Proc. 3rd Int. Symp. Large Engrg. Systems, St. John's, Canada, 1980, pp. 133-138, II,-Proc. JACC, Vol. 2, TP6G, USA, 1980.
9. E.A. Galperin, Regression models for prediction and control. Proc. 8th IFAC World Congress, Vol. 6, Kyoto, Japan, 1981, pp. 119-124.
10. E.A. Galperin, Regression models for prediction and control of processes of unknown structure. Proc. 21st IEEE CDC, Vol. 3, USA, 1982, pp. 1341-1346.
11. E.A. Galperin, Regression models for non-experimentable stationary control systems. Methods of Oper. Research, 45, Verlag A. Hain, 1983, pp. 87-95.
12. E.A. Galperin and A. Zinger, Stationarity conditions for linear models. Int. J. Mathem. Modelling, Vol. 4, No. 6, 1983, pp. 501-514.
13. E.A. Galperin and R. Labonté, Problème d'identification de modèles de régression et sa solution par la programmation linéaire. Annales des Sciences Mathématiques du Québec, Vol. 8, No. 1, 1984, pp. 29-53.
14. E.A. Galperin, Deterministic regression models for prediction and control. Int. J. Mathematical Modelling, Vol. 6, No. 2, 1985, pp. 157-171.