# Fatigue Crack Tip Deformation and Singularity Fields

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## ABSTRACT

Fatigue crack growth models use extensively the analytical elastic-plastic field descriptions for the stationary crack problem. Power relationships characterize spatial distributions of both strain and crack opening. Near-tip displacements and strains obtained in a fatigued aluminum alloy specimen by moirè interferometry are presented and assessed on the basis of theoretical fields, previous experimental results obtained by different techniques and a finite element SSY model of a stationary crack. From various crossed comparisons, it appears that a a live-loaded fatigue crack resembles more a tearing crack than a stationary crack.

## KEY-WORDS

Fatigue crack growth; small scale yielding; HRR field; tearing crack; nonlinear finite elements; moirè interferometry; strains; opening profiles.

## INTRODUCTION

Intermediate Mode I fatigue crack growth (FCG) rates are usually interpreted in terms of the stress intensity factor range $\Delta K_I$

$$da/dN = C\,(\Delta K_I)^m \qquad (1)$$

where C and m are two material constants to be experimentally determined. (The subscript I will be dropped from now on). FCG models attempt to correlate growth rates to other mechanical properties. Under the assumption that crack growth occurs for material separation after ductility exhaustion due to severe cyclic plastic deformation, several investigators have considered the possibility of determining the FCG material properties on the basis of low-cycle fatigue properties obtained with smooth specimens. Knowledge of the strain distribution inside the crack tip plastic zone is a fundamental ingredient of this approach (see, for example, Liu and Iino (1969), Majumdar and Morrow (1974) and, more recently, Kujawski and Ellyin (1984)). Another approach to crack propagation involves the adoption of the plastic sliding-off mechanism of crack advance reviewed recently by McEvily (1983). In this case, the crack tip opening displacement is the fundamental mechanical parameter.

Material deformation about fatigue cracks has been studied by various techniques. The moirè method has been employed on aluminum alloys by H.W. Liu and coworkers and reviewed by Liu (1981). Strain evaluations were, however, confined outside the cyclic plastic zone due to grating and image deterioration. Davidson and Lankford (1980) and Lankford and Davidson (1983) have developed and used two scanning electron microscope(SEM)-based techniques for measuring material deformation at fatigue crack tips.

The aim of the present work is a critical interpretation of localized deformation at a fatigue crack tip in a high strength aluminum alloy (7075-T6) as revealed by moirè interferometry, Nicoletto (1988b). The interpretation is based on theoretical crack tip fields, previous experimental results obtained by different techniques as well as on a numerical simulation of a stationary crack under small-scale-yielding (SSY) conditions. Attention is devoted to strain distribution ahead of the crack tip since it is basic to the cumulative damage approach to FCG and to near-tip flaw profiles which are associated to CTOD-based models of FCG.

## SINGULARITY FIELDS

FCG models make extensive use of theoretical field descriptions which are briefly reviewed as far as strain distributions ahead and crack opening behind of the crack tip are concerned. Since SSY conditions pertain to the fatigue loading conditions examined, local coordinates are appropriately normalized with respect to $(K/\sigma_o)^2$, stress and strain with respect to $\sigma_o$ and $\varepsilon_o=\sigma_o/E$, respectively, and in-plane displacements to $(K^2/E\sigma_o)$. $\sigma_o$, $\varepsilon_o$, and E are the yield stress, the yield strain and the Young's modulus of the material. Furthermore, equations and data are introduced in log-log plots so that the exponents of a power-type distribution is readily associated to the slope of a straight line.

### LEFM solution

The linear elastic field description for a stationary Mode I crack is characterized by the inverse-square-root singularities of the leading terms of the stress series solution, Williams (1957). For a state of plane stress, (i.e. experimental measurements are taken on the free surface), the radial strain distribution ahead of the crack tip is given by

$$\varepsilon_{22}/\varepsilon_o = (2\pi\,\overline{x_1})^{-\frac{1}{2}} \qquad (2)$$

in which $\overline{x_1} = x_1/(K/\sigma_o)^2$, see Fig.1a. The plane stress flaw profile is immediately retrieved by substitution of $\Theta = \pi$ in the LEFM $u_2$-equation and it is given by

$$u_2/(K^2/E\sigma_o) = (8\overline{x_1}/\pi)^{\frac{1}{2}} \qquad (3)$$

### SSY solution

For plastically yielding materials, the LEFM field description can control the development of a plastic region provided its size be small compared to other physical dimensions (i.e. crack length). The LEFM field, however, does not yield information on the strain distribution inside the plastic zone. Following Majumdar and Morrow (1974) approach, one may extend the antiplane SSY work-hardening solution due to Rice (1967) to the Mode I case. For a solid with a workhardening response given by

$$\varepsilon/\varepsilon_o = (\sigma/\sigma_o)^{1/n} \qquad \text{for } \sigma > \sigma_o \qquad (4)$$

where n may vary from n=1 for an elastic material to n=0 for an elastic-perfectly plastic material, the principal strain orthogonal to the crack plane ahead of the crack tip is given by

$$\varepsilon_{22}/\varepsilon_o = [(n+1)\,\pi\,\overline{x_1}]^{-1/n+1} \qquad (5)$$

Eq. (5) is the solution inside a small plastic zone which is surrounded by the elastic singular field. On the same basis, the antiplane SSY solution for a material characterized by a bilinear stress-strain relationship (Rice (1967)) can be extended to Mode I

$$\varepsilon_{22}/\varepsilon_o = (2\pi\,\beta\,\overline{x_1})^{-\frac{1}{2}} \qquad \text{for } \overline{x_1} \longrightarrow 0 \qquad (6)$$

where $\beta = E_t/E$ and $E_t$ is the tangent modulus.

### HHR field

The elastic-plastic solution for a stationary crack in a power-law strain hardening material obeying a Ramberg-Osgood equation was obtained by Hutchinson (1968) and Rice and Rosengren (1968). It is known as the HRR field. In this case, the strain distribution ahead of the crack tip for SSY is given by

$$\varepsilon_{22}/\varepsilon_o = (\tilde{\sigma}_{22} - \nu\,\tilde{\sigma}_{11})(\alpha\,I_N\,\overline{x_1})^{-1/(N+1)} + \alpha\,\tilde{\varepsilon}_{22}\,(\alpha\,I_N\,\overline{x_1})^{-N/(N+1)} \qquad (7)$$

where $\tilde{\sigma}_{11}$, $\tilde{\sigma}_{22}$ and $\tilde{\varepsilon}_{22}$ are known dimensionless constants dependent on the hardening exponent N, $\alpha$ is a fitting constant of the stress-strain equation and and $I_N$ is a function of N and stress state. Since N and n of Eq. (4) are related (i.e. $N = 1/n$), the hardening coefficient N varies from N = 1 for the elastic case to $N \longrightarrow \infty$ for elastic-perfectly-plastic solid. The plane stress crack opening very close to the crack tip is given by

$$u_2 / x_1 = \tilde{u}_2\,[K^2/(E\,\sigma_o\,I_N\,\overline{x_1})]^{N/(N+1)} \qquad \text{for } \overline{x_1} \longrightarrow 0 \qquad (8)$$

where $\tilde{u}_2$ is a constant dependent on $\Theta$, N and stress state. The interesting feature of this description is that the near tip strains and crack opening displacements vary with $x_1$ as a power of $-1/(N+1)$ and a power of $N/(N+1)$, respectively. For the limit case of a perfectly plastic material, Rice (1968), strain varies as $1/x_1$ and a finite opening occurs at the crack tip.

### Tearing crack

The previous theoretical descriptions pertain to a stationary crack. The question, however, may arise whether a fatigue crack, at least for a part of the loading cycle, resembles a tearing crack. The strain singularity ahead of the crack tip is weaker in this case than for a stationary crack since the crack propagates into material that has already deformed plastically. For a plane strain, elastic-perfectly-plastic solid, the asymptotic form of the plastic strain has the form, (Rice (1968b) (1982))

$$\varepsilon_{ij}/\varepsilon_o = D_1 \ln (R_o/r) \qquad r \longrightarrow 0 \qquad (9)$$

while for plane stress, Rice (1975)

$$\varepsilon_{ij}/\varepsilon_o = D_2 \ln^2 (R_o/x_1) \qquad \text{for } \Theta = 0 \text{ and } r \longrightarrow 0 \qquad (10)$$

where $D_1$ and $D_2$ are constants and $R_o$ is a reference length undetermined by the asymptotic analysis. For both plane strain and plane stress, elastic-perfectly-plastic solid, the asymptotic form of the crack opening has the form, (Rice (1968b) (1982))

$$u_2/(K^2/E\sigma_o) = D_3\, x_1 \ln(R_o/x_1) \qquad (11)$$

where $D_3$ is a constant. Dean and Hutchinson (1980) and Fleck and Newman (1986) confirmed numerically Eq. (11) for steady crack growth in SSY.

## NUMERICAL MODEL

Since the moirè interferometric technique has been described in its application to FCG elsewhere by Nicoletto (1987), (1988b), it is only reminded that the experimental evidence has the form of interference fringe patterns depicting with high sensitivity in-plane displacement fields on free surfaces. The components of the strain tensor are determined indirectly by numerical or optical displacement field differentiation, Post (1987).

A finite element modelling as a stationary crack of the boundary conditions occuring in the moirè interferometric experiments was developed to help in the interpretation of the experimental results. Material discretization for crack tip deformation followed the boundary layer approach developed for SSY conditions by Rice (1968b). To increase the resolution at the crack tip a rezoning technique was also adopted. No attempt was made to force a singularity at the crack tip by using special elements. The finite element grids used in the analyses are shown in Figs. 1b and 1c.

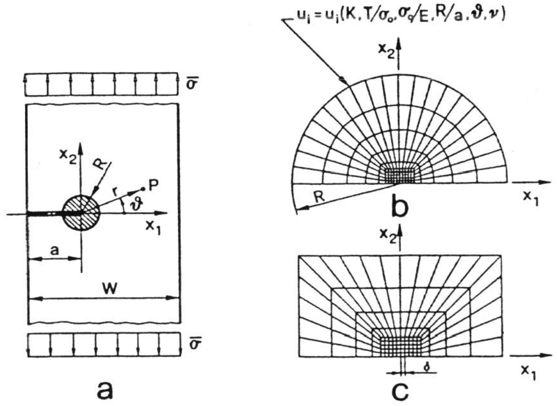


Fig. 1. (a) specimen geometry and coordinate system; (b) initial finite element mesh; (c) rezoning.



The minimum element size δ of the inner mesh was 1/200 of the outer radius where the LEFM asymptotic displacements were prescribed. Nicoletto (1988a) directly compared the fringe patterns obtained by moirè interferometry with simulated fringes obtained by postprocessing the numerical results. The limitations involved in modelling a fatigue crack with a stationary crack subjected to monotonic loading and the effect of the T-term (see Rice (1974)) were there pictorially revealed and assessed. A bilinear stress-strain response was considered based on the uniaxial tensile test for the present material (i.e. 7075-T6). The following mechanical properties were used in the analysis: E = 70 GPa, $\sigma_o$ = 480 MPa, Poisson's ratio $\nu$= 0.3, $E_t$ = 3322 MPa. A displacement based formulation was employed with the incremental finite element equilibrium equations derived from the principle of virtual work by linearization, Bathe (1982). Their solution for each time step used the Newton-Raphson method.

## CRACK TIP DEFORMATION

### Moirè interferometry vs. other experimental techniques

The evolution with applied load of the strain distribution ahead of a fatigue crack tip by moirè interferometry is presented in Fig. 2. The reference $\Delta K$ = 10 MPa$\sqrt{m}$ and R = 0.05. The adimensional moirè interferometric results reveal a complex evolution during the load cycle and regions in the strain distributions which are characterized by an the inverse power-law type. They, however, do not hold close to the crack tip. The correlation with the LEFM prediction is accurate outside the monotonic plastic zone while the LEFM strains apparently are upper bounds to the experimental strains over three orders of magnitude. A similar trend was observed in a yet unpublished study of deformation in a Ni Cr Mo V steel.

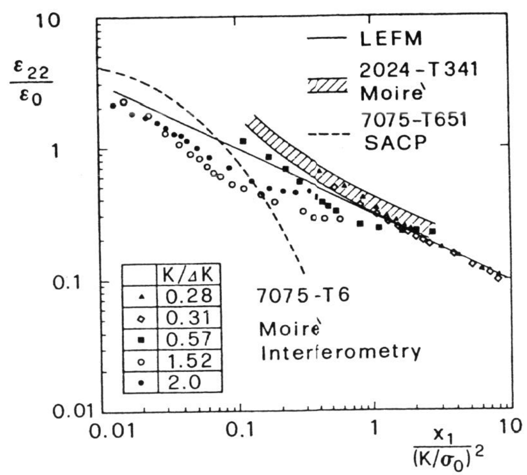


Fig. 2. Normalized radial distribution of strain ahead of the crack tip.

In order to assess the significance of these results, analogous data obtained by moirè and SEM-based techniques in similar aluminum alloys were examined. The moirè analysis of the strains ahead of a fatigue crack in a center cracked tensile panels (CCP) made of 2024-T341 aluminum alloy was reported by Liu (1981). The normalized strains for six load levels are summarized in the shaded band of Fig. 2. Comparison of these results with those obtained by the present technique demonstrates that moirè interferometry allows an order-of-magnitude increase in resolution at the crack tip. Liu's results belong to the monotonic plastic zone while present moirè interferometric measurements are taken within the cyclic plastic zone. Furthermore, it appears that the trend in the strain field identified by moirè relatively away from the crack tip may not be confirmed when measurements are taken inside the cyclic plastic zone.

In order to assess the value of the moirè interferometric results close to the crack tip, recent investigations on aluminum alloys using the scanning electron microscope (SEM) were considered. Davidson and Lankford (1980) and Lankford and Davidson (1983) have developed the selected area electron channeling pattern analysis technique (SACP) and the stereoimaging technique in order to determine strains at fatigue crack tips. These techniques have revealed that crack tip strains (i.e. within 10 microns) are smaller than predicted by theoretical descriptions. Furthermore, Davidson (1986) recently assumed the validity of Eq. (9) for correlating the effective total strain within the cyclic plastic zone as measured by stereoimaging and determined the unknown constants by best-fit. He found a superior correlating capability of the logaritmic function over the power function which is associated to a stationary crack. His experimental cyclic strain vs. $x_1$ equation for the 7075-T651 Al-alloy, after normalization, is introduced in Fig. 2 and it is found to reasonably correlate with the trend in the moirè interferometric results close to the crack tip. Davidson (1986) verified this equation with SACP measurements as well.

The opening profiles behind a fatigue crack tip as determined by moirè interferometry are presented in Fig. 3 and compared with the LEFM plane stress prediction.

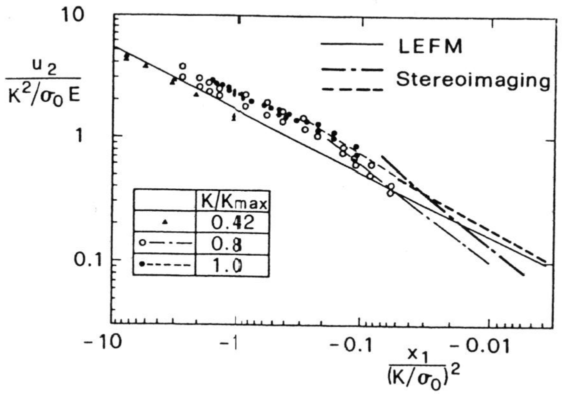


Fig. 3. Normalized crack profiles behind the crack tip.

Two regions are identified in the experimental data: away from the crack tip the slope are less steep than for the LEFM case; near the tip, however, the slopes become steeper than 1/2. The flaw profiles obtained by the stereoimaging technique for the same Al-alloy and similar stress intesity ranges (i.e. 8 and 10 MPa$\sqrt{m}$) by Lankford and Davidson (1983) are also reported in Fig. 3. While their measurements are very close to the crack tip, the near-tip moirè interferometric results and corresponding extrapolations are in very good agreement with their data, especially in terms of slope.

From the foregoing discussion ensues that moirè interferometry bridges the gap between microstructure-affected techniques (i.e. characterized by large scatter) and less sensitive continuum-type methods such as coarse moirè. It, therefore, represents a valuable experimental counterpart to numerical modelling of the mechanics of fatigue crack growth.

## Moirè interferometry vs. theoretical descriptions

The experimental strain distributions ahead of a fatigue crack tip are compared in Fig. 4 with the smoothed numerical results obtained by monotonic loading of the finite element model. The elastic plastic finite element analysis shows that the plastic zone extension correlates favourably with $\overline{x_1} = 1/\pi$ prediction for an elastic-perfectly-plastic solid, Rice (1968b). Inside the plastic zone, the numerical near-tip strains are found to stabilize close to the crack tip to an inverse square root distribution (i.e. parallel to the LEFM prediction) in accordance with Rice's (1967) prediction for a bilinear material (see also Eq. (6)).

The SSY and the HRR strain distributions (i.e. Eqs. (5) and (7)) for n =0.1 and N = 10 are also presented in Fig. 4. While these two solutions tend to merge when approaching the crack tip, both descriptions are found to overestimate the actual strains ahead of a fatigue crack. Interestingly, however, the linear portions of the moirè interferometric distributions correlate favourably with the 1/(n+1) (or N/(N+1)) slopes of these descriptions. If, however, each experimental distribution is considered as a whole, it is characterized by a nonlinear trend (in a log-log plot). An explanation of this behavior may be given by the significance of the tearing crack problem (i.e. see Fig.3) for a live-load fatigue crack.

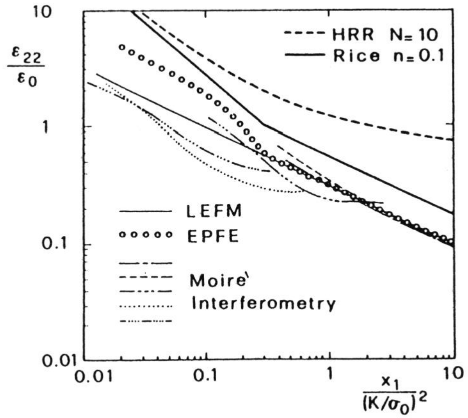


Fig. 4. Normalized radial distribution of strain ahead of the crack tip.

This would confirm one of the conclusions which Fleck and Newman (1986) arrived at during a detailed finite element modelling of fatigue crack growth. To investigate the subject further, the experimental results for the crack opening behind the crack tip are compared in Fig. 5 with the LEFM and the smoothed numerical results. The finite element model basically confirms the HRR description which predicts a slope of 1/(N+1) in the near-tip displacements (see Eq. (8)) although not supported by the trend in the moirè interferometric results. The profile for a steady-state tearing crack obtained numerically by Dean and Hutchinson (1980) and basically confirmed by Fleck and Newman (1986) is, therefore, included in Fig. 5. As expected, it shows less opening displacement than its corresponding stationary crack. Its accordance with the moirè interferometric results is remarkably good, thus supporting earlier conclusions based on observations of near-tip strain distributions on the relevance of the tearing crack problem even for a live-loaded fatigue crack.

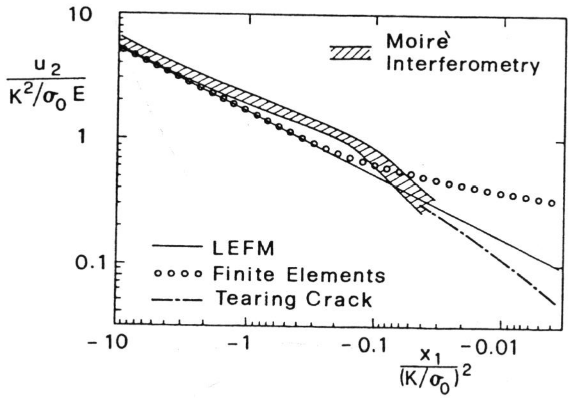


Fig. 5. Normalized crack profiles behind the crack tip.

CONCLUSIONS

The present work critically interpreted localized deformation at a fatigue crack tip in a high strength aluminum alloy (7075-T6) as revealed by moirè interferometry, Nicoletto (1988b). Comparison of experimental strains and opening displacements with previous experimental results obtained with different techniques confirmed the value of moirè interferometry for fracture studies. Theoretical descriptions and a finite element modelling of a stationary crack in a bilinear material under SSY conditions provided the necessary reference near-tip strain and opening displacement distributions. From various crossed correlations, it is concluded that a live-loaded fatigue crack resembles more a tearing crack than a stationary crack. This is in accordance with a conclusion of Fleck and Newman (1986) based on a detailed finite element analysis.

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