Fatigue Crack Growth Under Random Loading the Equivalent Loading Approach

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ABSTRACT

The study concerns fatigue crack growth under stationary Gaussian loading processes. Specific fatigue testing has been performed to verify the validity of using an equivalent loading to describe the crack growth under random loading. The statistical moment of m^{th} order (m: exponant of the PARIS law) of the load distribution was formally introduced as the equivalent load.

KEYWORDS

Fatigue crack growth, random loading, equivalent stress.

INTRODUCTION

The prediction of fatigue crack growth under random loading still causes problems to the engineers faced with the design of mechanical components or large structures. Whilst numerous experimental and theoretical studies have been undertaken, the complexity of the subject, the multiplicity of parameters to take into account and the knowledge acquired to date do not allow the engineer to face all the actual practical problems.

Two typical approaches are currently investigated by researchers:

i) cycle-by-cycle analysis with interaction effects between loads of different levels,

ii) global analysis using statistical parameters of the load sequence or an equivalent loading.

The idea of using an equivalent constant amplitude loading to predict the fatigue life under random loading has been developed in the early sixties by Paul PARIS (1964). Based on this work, the present study is an attempt to validate this type of approach, at least for stationary Gaussian processes.

In these lines of thought, strictly controlled variable amplitude loading has been applied in specific fatigue crack growth tests. The experimental results obtained have been compared to the predictions made by formal analysis.

EQUIVALENT STRESS INTENSITY RANGE CONCEPT FOR RANDOM LOADING

The use of an equivalent loading is a simple idea which at a first sight appears quite satisfactory: replace a load history of variable amplitude by an equivalent loading, such that if applied as a constant amplitude loading, the same fatigue life is obtained.

On the way, we should note that by continuity such a definition implies that this equivalent stress is the applied stresse itself in case of a constant amplitude loading.

Under constant amplitude loading the fatigue crack growth is recognised as a function of the stress intensity range. The most widely used relationship is the PARIS law:

$$\frac{da}{dN} = C.\Delta K^m \qquad [1]$$

Where da/dN is the crack growth increment per cycle C and m are material constants and ΔK is the stress intensity range defined as:

$$\Delta K = f(a/W)\ \Delta P\ a^{1/2} \qquad [2]$$

ΔP being the stress range, a the crack length and f(a/W) a function which depends on the geometry of the body and the applied loading.

For random loading, if the interaction effects are omitted, the crack growth increment corresponding to the application of the stress ΔP_i is:

$$\delta a_i = C\left[f(a/W)\ \Delta P_i\ a^{1/2}\right]^m \qquad [3]$$

Now let us apply to the loading sequence a correction factor α which depends on the crack length such that a transition from a level i to a level j, anywhere in the sequence, leads to the same value of ΔK whatever the crack length.

If the load applied is $\alpha\Delta P$

$$\text{with } \alpha = \frac{a_o^{1/2}\ f(a_o/W)}{a^{1/2}\ f(a/W)},$$

a_o being the initial crack, then

$$\Delta K = a_o^{1/2}\ f(a_o/W)\ \Delta P = C^{te}.\Delta P$$

Equation [3] can then be written:

$$\delta a_i = C(C^{te}.\Delta P_i)^m \qquad [4]$$

In that way, ΔK no longer depends on the crack length and this allows us to make sense of the notion of average crack growth rate during the load sequence. Then if the number of times stress ΔP_i occurs is n_i, the increase in crack length is:

$$C(C^{te})^m \sum_i n_i\ (\Delta P_i)^m \qquad [5]$$

The average crack growth rate is obtained by dividing by N the number of cycles of the sequence:

$$\frac{da}{dN} = C(C^{te})^m \left[\frac{1}{N}\sum_i n_i\ \Delta P_i^{\ m}\right] \qquad [6]$$

The equivalent constant amplitude stressing ΔP_{eq} which would produce this crack growth rate would be:

$$\frac{da}{dN} = C(C^{te})^m\ \Delta P_{eq}^{\ m} \qquad [7]$$

The value of ΔP_{eq} is derived from equations [6] and [7] to yield

$$\Delta P_{eq} = \left[\frac{1}{N}\sum_i n_i\ \Delta P_i^{\ m}\right]^{1/m} \qquad [8]$$

From equation [8], we note that for a constant amplitude loading $\Delta P_{eq} = \Delta P$.

At this stage it is interesting to note that the equivalent constant amplitude loading is the statistical moment of m^{th} order of the random loading sequence, where m is the slope of the PARIS law under constant amplitude loading, equation [1].

Of course all the mathematical development above is purely formal when m is not an integer.

In the original work by PARIS (1964) the exponent of the crack propagation law was 4, therefore he proposed to use the fourth order moment for the equivalent loading.

In the particular case where m equals 2 the equivalent loading is the second order of the sequence which is also called the Root Mean Square (RMS) value of the sequence.

Let us recall that for a finite series of N numbers the Root Mean Square is given as:

$$X_{RMS} = \left[\frac{1}{N}\sum_i^N X_i^{\ 2}\right]^{1/2} \qquad [10]$$

The RMS value of the statistical distribution is often used (Barsom, 1976; Hudson, 1981) without any formal justification but just because it is widely used as a statistical parameter of random processes.

On the way let us remark on the ambiguity linked to the use of the RMS value: for a sinusoïdal constant amplitude signal, $s_{(t)} = S_o \sin(2\Pi/T)t$ an effective value (or true value) is currently used, in acoustics for example:

$$S_{eff}^{\ 2} = \frac{1}{T}\int_o^T s^2(t)\,dt = S_o/\sqrt{2}$$

This value is also called RMS.

For a triangular signal this "RMS" value will be $S_o/2$, and will tend towards zero for impulse signal. Obviously such a definition is not convenient to describe the fatigue phenomenon.

EXPERIMENTAL PROCEDURE

Load Sequence

The load sequence used for the fatigue test has been developed at LBF and IABG Laboratories (Haibach et al., 1976). It corresponds to a stationary Gaussian process and the cumulative frequency distribution of level crossings is given in figure 1.

The global characteristics are:

- number of cycles of the sequence or return period: 10^6 cycles,
- irregularity factor $I_R = 0.9$,
- crest factor $q = \frac{P_{max}}{P_{RMS}} = 4$ (RMS defined in eq [10]).

The sequence is defined as a succession of numbers between 1 and 32 which corresponds to the interval $[0, P_{max}]$. This interval is cut into 31 equal segments which define 32 values for which the signal will present extremes (relative minima or maxima). Therefore the loads applied to the specimens are:

- level 1 : $P = 0$,
- level 32: $P = P_{max}$,
- level i : $P_i = \frac{P_{max}}{31}(i - 1)$.

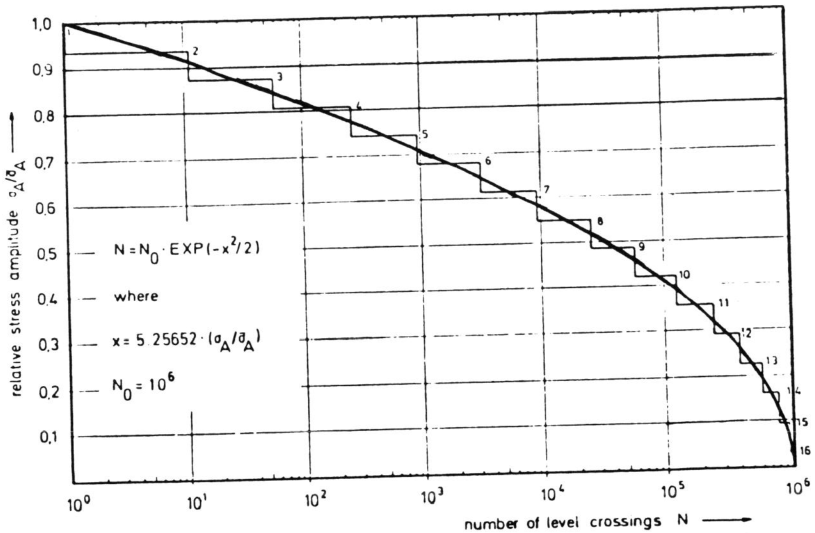

Fig. 1. Cumulative frequency distribution of level crossings

The sequence is applied to the testing machine with the transition matrix technique. Each element s_{ij} of the matrix represents the number of transitions between level i and level j in the sequence of 10^6 transitions. In the following, a cycle is defined as the values of the signal between two adjacent minima or maxima without these being necessarily of the same level.

The algorithm for the generation of the signal is such that the same sequence can be reproduced as many times as desired.

Material and specimens

The material is structural steel E460. It is quenched by accelerated cooling after hot rolling and tempering.

The chemical composition and mechanical properties are given in tables 1 and 2.

TABLE 1
Chemical composition of the E460 steel (weight %)

C	Mn	Si	S	P	Ni	Cr
0.17	1.3	0.34	0.001	0.022	0.22	0.22

TABLE 2
Mechanical properties for the E460 steel

σ_{ys} (MPa)	UTS (MPa)	Elong %	KCV - 40°C (J/cm^2)
560	665	21.4	103

Two types of specimens, in the LT orientation, have been used: CT specimens W = 80 mm, B = 20 mm for the "ΔK_{eq} = constant" tests (see next section) and CCT specimens 2W = 95 mm, B = 10 mm for the "ΔP_{eq} = constant" tests.

The tests have been carried out on a servo-electrohydraulic machine at the frequency of 10 Hz using a sine wave form.

FATIGUE BEHAVIOUR

Two types of fatigue tests have been performed.

i) **"ΔK_{eq} = constant" test**, where the load sequence is corrected by a coefficient α which depends on the crack length as described in the previous section, following the work by M. TRUCHON (1982).

Using $K = \dfrac{S}{B\sqrt{W}} Y(a/W)$

Where B and W are the thickness and the width of the specimen, a the crack length, Y(a/W) the weight function, S the load sequence.

If αS is applied instead of S with $\alpha = Y(a_o/W)/Y(a/W)$ where a_o is the initial crack length it becomes:

$$K = \frac{\alpha S}{B\sqrt{W}} Y(a/W) \quad = \frac{S}{B\sqrt{W}} Y(a_o/W) = C^{te} S$$

In that way, K no longer depends on the crack length. This makes sense with the notion of an average crack growth rate during the load sequence. With this test, a PARIS law for random loading can be built by point using several levels of the load sequence.

The crack length was followed optically by the experimenter. The test frequency was 10 Hz and sequences of about 300 000 cycles were applied to the specimens without interruption.

It has been calculated that the statistical parameters of the sequence are stabilized above 150 000 cycles; thus the sequences of 300 000 cycles can be considered as representative of the whole sequence, for crack growth analysis purposes. Seven sequences of about 300 000 cycles were realized using two CT specimens. The first forty cycles of the sequence are shown in figure 2. The R_{RMS} ratio is 0.58.

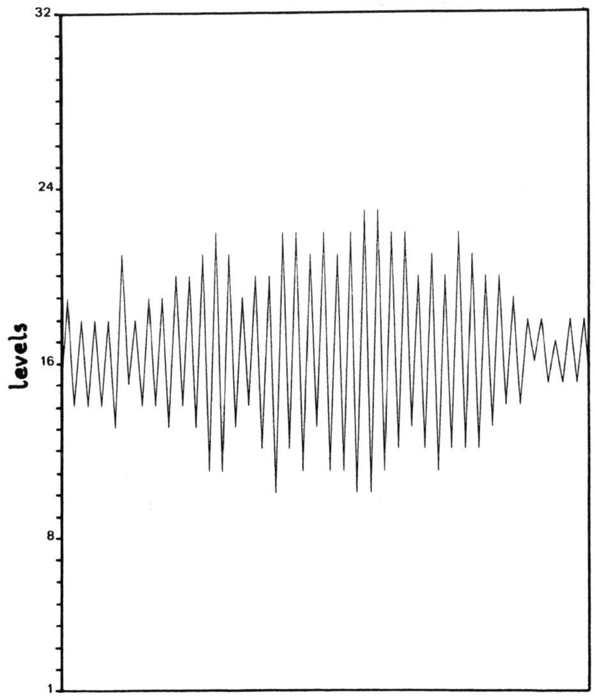

Fig. 2. First forty cycles of the sequence

The measured data are reported on a diagram a = f(N) in figure 3. The average crack growth sequence derived from these data are reported in figure 4 in the form of $\frac{da}{dN}$ versus ΔK equivalent. Two definitions of ΔK equivalent are used:

- ΔK_{RMS} using the second order moment of the sequence,
- ΔK_{RMm} using the mth order moment of the sequence (eq [8]) with m = 2.8, exponent of the constant amplitude PARIS law.

The results are compared to the constant amplitude PARIS law at R = 0.7 which is:

$$\frac{da}{dN}\ (m/c) = 8.392.10^{-12}\ \Delta K^{2.8}\ (MPa\ \sqrt{m}) \qquad [9]$$

From figure 4, it appears that using equation [8] (which represents the mth order moment of a load distribution when m is an integer) as an equivalent load, the PARIS law, eq [9], correctly describes the crack growth rate under random loading. For material with an exponent m = 2 the RMS value can be the correct equivalent but for m > 2 the RMS equivalent will lead to an underestimated equivalent ΔK.

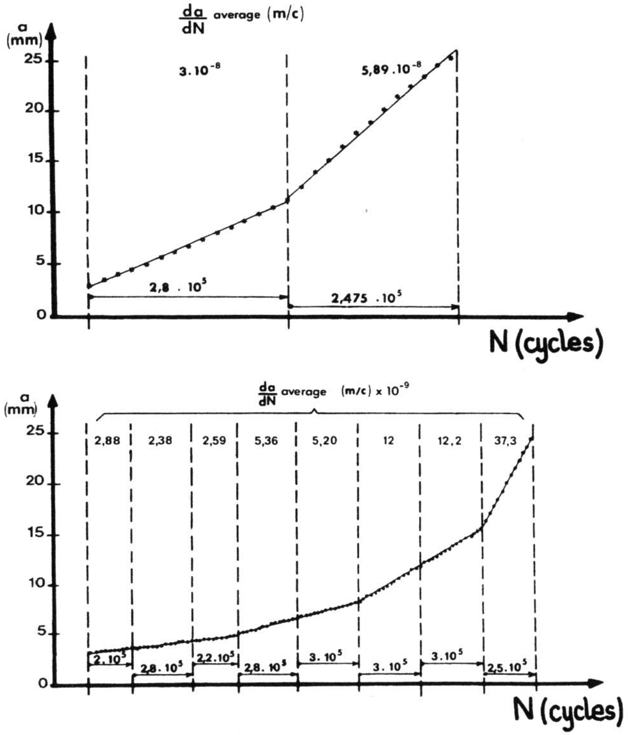

Fig. 3. Crack growth in "Constant ΔK_{eq}" testing for the two specimens

ii) **"ΔP equivalent = constant"** tests were performed to verify if the concept of equivalent stress intensity factor can reasonably describe the crack growth under a given load sequence. In this case, K depends on the stresses but also on the crack length, $K = f(a/W).S.a^{1/2}$.

Two tests were conducted on CCT specimens with two different levels of the load sequence to cover the crack growth range 10^{-9}-10^{-7} m/c. In a first test eight sequences were applied. For the second test, twelve sequences were applied. Table 3 gives the load level and the number of cycles per sequence.

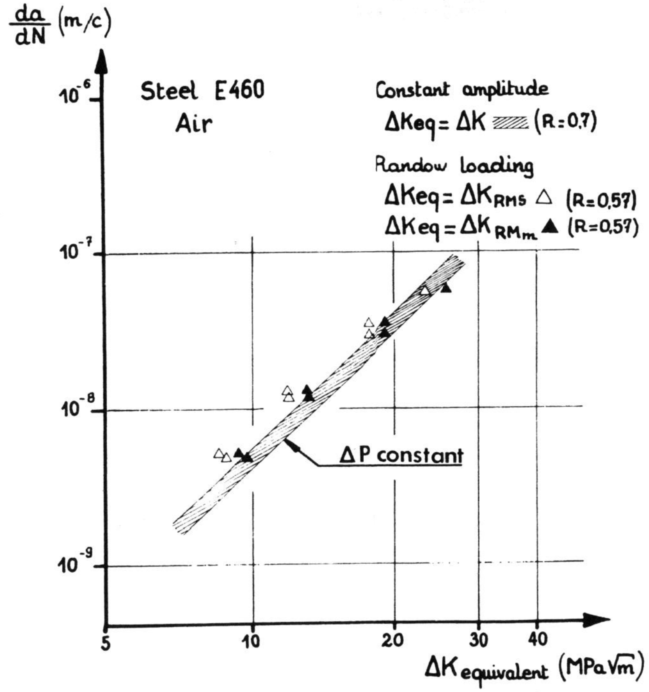

Fig. 4. Fatigue crack growth rate versus ΔK_{eq}

Compared with constant amplitude results

TABLE 3
Load level and number of cycles per sequence

Specimen	P_{max} of the Sequ. (N)	ΔP_{RMS} (N)	R_{RMS}	ΔP_{RMm} (N) (m = 2.8)	R_{RMm}
1	154 306	41 513	0.58	44 898	0.54
2	97 213	26 154	0.58	28 286	0.54

Number of cycles per sequence (10^5 cycles)

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}
1	-	3.09	2.92	2.98	2.83	0.95	0.92	0.64	-	-	-	-
2	-	6.66	3.85	5	2.35	6.55	5	4	4	4	4	4

The results given in figure 5 show the experimental crack growth $a = f(N)$ and the values predicted by the PARIS law, equation [9], with ΔK expressed using the equivalent loading defined in equation [8].

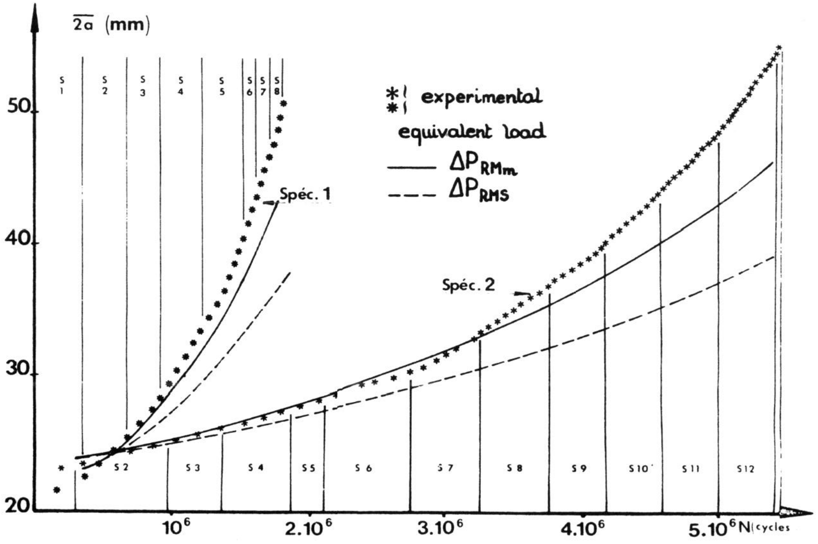

Fig. 5. Evolution of the crack length versus the number of cycles in the two ΔP_{eq} = Constant tests. Comparison with the predicted values

For comparison purposes the equivalent loading has been calculated with the moment of order 2.8 and 2, the latter giving the RMS value.

The experimental results exhibit crack growth a little bit larger than the predicted value from the equivalent loading calculation. The use of RMS value obviously results in poor prediction in the present examples.

In figure 6 are represented the measured crack growth rates versus the crack evolution $\frac{da}{dN} = f(a)$ in comparison with the value predicted through the PARIS law using equivalent K calculation.

The crack growth prediction based on the m^{th} order moment of the long-term load distribution is acceptable as long as the average crack growth rate is not too fast.

When the growth rate increases (say above 5.10^{-8} m/c), the hypotheses made to write equations [5] to [8] in previous section are not satisfied and the average crack growth rate at a given crack length, based on long-term analysis of the sequence, loses its significance.

Short-term variations in the probability density function have then a major effect. Therefore, crack evolution would not be predicted correctly from the long term load distribution.

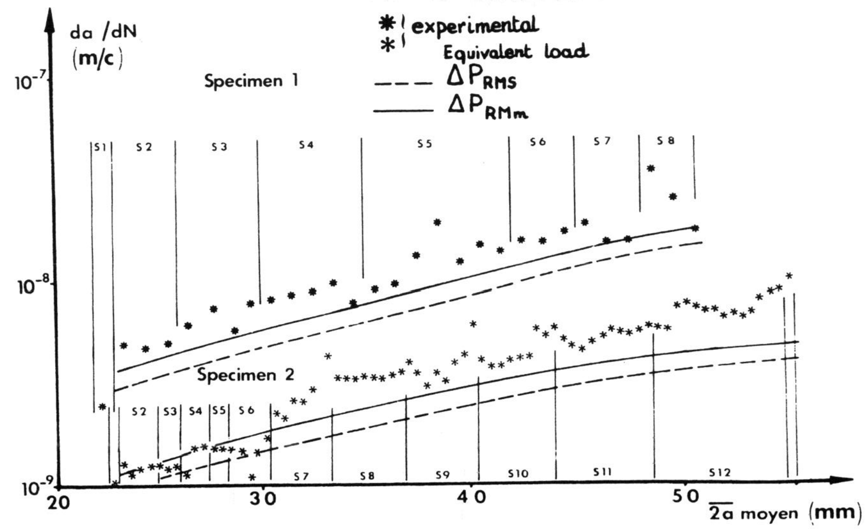

Fig. 6. Evolution of crack growth rate in the two ΔP_{eq} = Constant tests

The model used in this work does not take into account load interaction effects. An attempt to introduce such effects has been proposed by Mc KEE and HANDCOCK (1978) and leads to crack growth rate very much less than the non-interacting model. These authors mention that an important assumption made in the interacting model is that the loads occur in purely random sequences and this is a stringent requirement since real spectra show major deviations from this condition. These deviations which may be expressed as short-term variations in the probability density function have a major effect on the occurrence of load interactions and tend to minimise the interaction effect in a spectrum, so that the fatigue crack growth rate approaches its non-interacting value. If, as it is widely believed, load interactions are important in crack growth, it is also important to quantify the deviations from random behaviour in real spectra, and in the load histories used for test purposes.

CONCLUSIONS

Fatigue crack growth under random loading has been analysed using an equivalent constant amplitude loading.

The equivalent loading can be formally described as the statistical moment of the m^{th} order of the loading sequence, m being the exponent of the PARIS law under constant amplitude loading; this being purely formal when m is not an integer.

This approach allows a prediction of the crack evolution by using the PARIS law for constant amplitude loading as long as it does not grow too quickly. In this case the number of cycles which leads to a significant increment of growth is not representative of the long-term load distribution and short-term variations in the probability density function have a major effect.

Obviously, this type of approach cannot be used for any type of spectrum loading. In aeronautics for example it is known that due to the particularity of the loading, large number of small cycles with periodic overload, the retardation induced by overloading has a major effect.

Nevertheless, the equivalent loading approach is appropriate to describe the crack growth under other types of spectrum loading with less sudden changes in stress evolution. This is the case of sea states loading applied to offshore structures, in civil engineering structures, earth moving machinery.

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