

# Fatigue Crack Growth in Mild Steel Under Cyclic Compressive Loads

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## ABSTRACT

The aim of this work has been to study the closure of cracks created by compressive loading, and to correlate this with the kinetics of crack growth under compressive cyclic loading. A model will be presented to account for the kinetics of growth, expressed in terms of the loads required partially to close the crack. It is based on the principle that crack growth in the presence of residual stress is controlled by the change in the effective (or tensile) stress intensity at the crack tip. The threshold value obtained for this parameter appears to be significantly smaller than values obtained for ferritic steels by load-shedding experiments.

## KEYWORDS

Fatigue; crack growth; residual stress; compression; crack closure; growth kinetics.

## INTRODUCTION

Load-bearing artefacts invariably contain stress-concentrating features such as holes, keyways, threads or other changes in section. When they are loaded, plastic deformation occurs preferentially at these features, producing a non-uniform strain distribution, which, on unloading, induces a state of residual stress. When the loading is cyclic, these features are preferred sites for the nucleation of fatigue cracks so it follows that crack nucleation generally occurs in a region of residual stress. Any model of crack nucleation and growth should take residual stress into account. In some circumstances the effect of the residual stress can be quite striking: for example, if the residual stress is tensile, subsequent cyclic loading in compression may cause fatigue cracks to nucleate and grow at sites of stress concentration, in spite of a widespread belief among engineers that compressive loading does not cause fatigue. Crack growth under cyclic compression was first reported (Gerber and Fuchs, 1968), followed in the 1970's by several other authors (Hubbard, 1969, Saal, 1972, Reid et al., 1979). In the 1980's there has been renewed interest in the effect, as shown by a number of recent publications (Wu-Yang Chu, 1984, Hermann and Reid, 1984, Fleck et al, 1985, Suresh, 1985). All the authors agree that the effect is consistent with the principle that some cyclic tensile loading is required for fatigue and that this is achieved by superposition of the residual and the applied stresses.

## MATERIALS AND METHODS

A mild steel has been used; its chemical composition and properties are given in Table 1. The material was supplied in the form of rolled bar which was machined into the form of standard compact tension (c.t.) testpieces conforming to British Standards Specification 5447 (thickness  $B = 25$  mm; width  $W = 50$  mm; with a  $60^\circ$  v-notch of tip radius 0.1 mm). The line of the notch tip lay perpendicular to the rolling plane of the bar. Prior to testing, samples were given a sub-critical anneal (1 hour at 600 C); this produced an equiaxed ferrite grain structure with an average grain diameter of 40  $\mu$ m.

Table 1 Analysis and Properties

C	% by weight		strength/MPa		elongation %	hardness/ $H_V$
	Si	Mn	yield	tensile		
0.16	0.05	0.81	320	460	20	150

An electrical resistance strain gauge was bonded to the centre of the back face of each testpiece with its axis normal to the notch plane. This enables the compliance (strain per unit load) of a testpiece to be measured. Following an established method (Deans and Richards, 1979), the compliance was used to infer the length over which the crack is open. The relation between the compliance  $\emptyset$  (in units of  $10^{-9} N^{-1}$ ) and the normalised crack length,  $a/W$ , was obtained by calibration and found to be:

$$\frac{a}{W} = 0.08796 + 0.01635 \emptyset - 2.3705 \times 10^{-4} \emptyset^2 + 1.8284 \times 10^{-6} \emptyset^3 - 5.5164 \times 10^{-9} \emptyset^4 \quad (0.45 < \frac{a}{W} < 0.65) \quad (1)$$

This is almost identical to that published by Deans and Richards. For the purpose of calibration, the notch was extended in steps of 1 mm using a spark erosion cutter, 0.25 mm in diameter. Loading was carried out on servo-hydraulic machines according to the procedure shown in Figure 1; it consisted of a static compression followed by cyclic compression. The growth of cracks from the notch tip was monitored on the polished side surfaces of the specimen by a travelling microscope graduated in units of 0.01 mm.

## RESULTS AND ANALYSIS

### Crack Growth.

When a pre-loaded testpiece was subjected to cyclic compression, a crack formed at the notch tip after a number of load cycles. The rate of growth (in mm per cycle) reached a maximum in the early stages of growth and thereafter the rate decreased steadily until it became effectively zero (see Figure 2). The initial growth often occurred at accelerating rates; this is thought to be due to the increase in stress intensity which accompanies the transition from a short crack to what is effectively a long crack (the notch and crack combined). It has been estimated (Smith and Miller, 1977) that this transition occurs at a crack length of  $[0.13 (D\rho)^{0.5}]$ , where  $D$  is the notch length (22.5 mm) and  $\rho$  is the radius of the notch root (0.1 mm). This gives a transition crack length of about 0.2 mm, in reasonable agreement with the region of accelerating growth. The value of the final crack length increased with the magnitude of the pre-load, as shown in Figure 3, for fixed cyclic loading conditions. The onset of general yielding was found to occur at a load  $F_{GY}$  of -50 kN, in excellent agreement with the theoretical value (-50.5 kN) obtained by multiplying Equation 3 (Merkle and Corten, 1974) by a plastic constraint factor of 1.26. Loads in excess of  $F_{GY}$  caused no further increase in the final crack length. Notice that under the cyclic loading conditions employed ( $F_{min} = -15$  kN), even when no pre-load was used a

fatigue crack was formed (albeit a short one). In this case the first cycle of compressive load acted as a "pre-load" of -15 kN. It can also be seen in Figure 3 that the final crack length lies in between the plane strain and the plane stress estimates of the radius of the plastic zone at the notch tip.

The final crack length (for a given value of pre-load) was found to depend also on the value of the maximum (i.e. least negative) cyclic load used. A sample preloaded to -50 kN was subjected to a sequence of load cycles having the same minimum value (-40 kN) but increasing maximum values (Table 2). When loaded cyclically at a given range, the crack extended at a decreasing rate until eventually it stopped at the value of  $\frac{a}{W}$  given in Table 2. The final crack length increased with the value of the maximum cyclic load,  $F_{max}$ .

The shape of the final crack front was unusual: it curved towards the notch with the result that the final crack length was 15-25% longer at the surface of the testpiece than it was in the interior (see Figure 4). This effect suggests that the tensile residual stress set up by preloading extended to a greater distance from the notch at the surface than it did in the interior. This is consistent with the accepted view that the plastic zone is larger near the surface, due to the reduced degree of constraint on yielding which prevails at the surface compared to that in the interior. This explanation was advanced in an earlier paper (Reid et al, 1979) to account for a much larger variation in crack length through the thickness of testpieces. Subsequent unpublished work showed that this enhanced variation was due to the existence of a tensile residual stress in the surfaces of the testpieces induced by the rolling of the bar from which the testpieces had been machined. A stress-relieving heat treatment had not been used in this earlier work.

### Crack Closure.

Cracks that had stopped growing under constant cyclic loading ( $-8 \pm 7$  kN) were subjected to an increasing compressive load, while monitoring the attendant changes in back face strain. The compliance was observed to decrease during compressive loading as the "gaping" crack progressively closed up, starting at its tip, and during unloading an almost identical load-strain curve was obtained (see Figure 5). By fitting a polynomial for strain to this curve and differentiating it with respect to load, the change in compliance with load was obtained. Using Equation 1, changes in compliance were converted to changes in  $\frac{a}{W}$  to give a relation between

the load  $F_c$  and the crack length,  $\frac{a}{W}$  (Figure 6). This relation is changed by subjecting the cracked testpiece to compressive loads greater than those used during cyclic loading ( $-8 \pm 7$  kN); the graph in Figure 6 is shifted to larger values of  $F_c$  by loading the crack to -50 kN. The relation obtained also depended on the value of the initial compressive load used and graphs are presented in Figure 7 for preloads of -35, -43 and -50 kN. The lines are approximately straight and parallel, and are displaced upwards by increasing the pre-load. These graphs indicate the load  $F_c$  required partially to close up the crack to any given length,  $\frac{a}{W}$ . They also indicate that the final crack length corresponds to a small value of the closure load ( $-1.5$  kN at  $\frac{a}{W} = 0.571$  for the lower line in Figure 6).

### Analysis

It is proposed that when the growing crack has a given length  $\frac{a}{W}$ , the tip of the crack is open only when the applied load exceeds  $F_c$ , the closure load corresponding to the value of  $\frac{a}{W}$ . This will be so provided that  $F_{max} > F_c$  (remembering that both loads are negative). The load range

over which the crack tip is open is the smaller of  $(F_{\max} - F_{\min})$  and  $(F_{\max} - F_c)$  and this can be regarded as  $\Delta F_{\text{eff}}$ , the load range that is effective in causing the growth of a fatigue crack. In the early stages of crack growth, when  $F_{\min} > F_c$ , the whole of the applied load cycle is effective but as the crack grows beyond the point at which  $F_{\min} = F_c$ , the effective load range becomes  $(F_{\max} - F_c)$ , only a fraction of the applied load range. As the crack grows,  $F_c$  becomes less negative and  $(F_{\max} - F_c)$  decreases. The corresponding change in the stress intensity,  $\Delta K$ , is given by the standard expression:

$$\Delta K = \frac{Y \Delta F_{\text{eff}}}{B \sqrt{W}} \quad (2)$$

where the coefficient  $Y$  is a function of  $\frac{a}{W}$  (see BS 5447).

It is suggested that the crack stopped growing when its tip reached a location where the change in stress intensity became equal to the threshold value,  $\Delta K_{\text{TH}}$ . In these experiments,  $F_{\max} = -1$  kN, so the closure load from the lower line in Figure 6 (-1.5 kN) indicates that the effective load range is 0.5 kN, giving a threshold stress intensity  $\Delta K_{\text{TH}} = 1.1 \text{ MPa}\sqrt{\text{m}}$ . Four other similar testpieces gave  $\Delta K_{\text{TH}}$  values in the range  $1.5 \pm 0.5 \text{ MPa}\sqrt{\text{m}}$ . This value is much smaller than the published values (6-13  $\text{MPa}\sqrt{\text{m}}$ ) for ferritic steels at small values of stress ratio  $R$  (Lindley and McCartney, 1981).

The graph of  $F_c$  against  $\frac{a}{W}$  can be used to predict the length of crack that would grow under any given compressive load cycle. Growth is expected to occur only if  $F_c < F_{\max}$  (i.e. if the crack opens during the load cycle), and it is expected to cease when  $\Delta K = \Delta K_{\text{TH}}$ ; since this is small, the final crack length is approximately the crack length for which  $F_c = F_{\max}$ . Using this approach, the final crack lengths for the tests referred to in Table 2 have been predicted and the values compared with those observed; it can be seen that there is reasonable agreement between the predicted and observed lengths.

Table 2. The effect of various load cycles on the final crack length of a testpiece preloaded to -50 kN.

load cycle/kN		final crack length/ $\frac{a}{W}$	
$F_{\max}$	$F_{\min}$	observed	predicted
			( $\frac{a}{W}$ for which $F_c = F_{\max}$ )
-25	-40		0.455
-20	-40		0.460
-15	-40		0.495
-10	-40		0.526
-5	-40		0.554
-1	-40		0.579
			0.470
			0.487
			0.508
			0.531
			0.555
			0.577

#### Prediction of Growth Kinetics.

As argued above, the effective part of the load range  $\Delta F_{\text{eff}}$  is the smaller of  $(F_{\max} - F_{\min})$  and  $(F_{\max} - F_c)$ , and this must be positive (i.e. the change in load must alter the crack opening).

$F_c$ , the load required to close the crack tip, varies as the crack grows, so  $F_c = F_c \left( \frac{a}{W} \right)$ . At given values of  $\frac{a}{W}$ ,  $F_{\max}$  and  $F_{\min}$ ,  $\Delta F_{\text{eff}}$  can be evaluated and used to obtain  $\Delta K_{\text{eff}}$  from Equation 2.

The amount of crack growth per cycle is given by the growth law for ferritic steels (Pook, 1983):

$$\frac{da}{dN} = 2.4 \times 10^{-9} (\Delta K_{\text{eff}})^{3.3} \quad (3)$$

where  $a$  is measured in mm and  $\Delta K$  in  $\text{MPa}\sqrt{\text{m}}$ . This increment of growth can be calculated for each of a series of closely-spaced values of  $a$  and the number of cycles required for each increment obtained. By summing the increments of  $a$  and  $N$ , a graph of  $a$  versus  $N$  can be obtained, such as that in Figure 8 which was obtained using a growth increment of 0.1 mm, an initial crack length of 0.2 mm, and closure loads (in kN) which, for a preload of -50 kN, were found to obey the equation:

$$F_c = 0.015 + 131.0 \frac{a}{W} - 217.5 \left( \frac{a}{W} \right)^2 \quad (4)$$

The initial crack length was chosen to be 0.25 mm. The calculated  $a$  versus  $N$  graph in Figure 8 shows excellent agreement with the observed growth curve for the same specimen from which the crack closure loads were obtained. This agreement supports the proposition that the effective load range is the smaller of  $(F_{\max} - F_{\min})$  and  $(F_{\max} - F_c)$ .

#### DISCUSSION

The main proposition of this paper is that the load  $F_c$  required to close the tip of a crack of given length  $\frac{a}{W}$  can be evaluated by employing compliance measurements from a compressed testpiece containing an arrested crack. Values of  $F_c$  can then be used to calculate the effective stress intensity range  $\Delta K_{\text{eff}}$  and thence from the Paris equation, the rate of crack growth. The close agreement between the predictions of this proposition and the observations (Table 2 and Figure 8) adds weight to the proposition. This is the same approach as that taken by other workers (Fleck et al., 1985, Christman and Suresh, 1986), but the method used here to measure  $F_c$  differs from that used by these authors.

Application of this approach to the arrest of a crack under cyclic compressive loading indicates values of  $\Delta K_{\text{TH}}$  ( $1.5 \pm 0.5 \text{ MPa}\sqrt{\text{m}}$ ) below those found in ferritic steel from tensile load shedding experiments (Lindley and McCartney, 1981). In such experiments, the stress intensity range is reduced in finite steps whereas in this work, it is reduced continuously, by the residual tensile stress field. Low values of  $\Delta K_{\text{TH}}$  obtained under cyclic compression have been reported (Suresh, 1985).

It was found (Figure 6) that the relation between  $F_c$  and  $\frac{a}{W}$  was affected by increasing the compressive loading. It is suggested that this loading causes some flattening of asperities on the fracture surfaces, and thereafter an increased load is required in order to cause crack closure. It

is therefore important that the compressive loads within the cyclic range should not be exceeded when determining the compliances and hence the values of  $F_C$ .

The growth of fatigue cracks under compressive cyclic loading is probably not uncommon in engineering products. For example, there have been unpublished reports of cracking at stress raisers on the top surfaces of aircraft wings; in some cases these fatigue cracks have developed sufficiently to cause failure of the wing during proof tests. The possibility of such accidents occurring deserves to be appreciated more widely by engineers.

### CONCLUSIONS

1. Fatigue cracks can be grown from the tip of a notch by cyclic loading in compression in the presence of a tensile residual stress induced by pre-loading in compression.
2. Superposition of a static, tensile residual stress and a cyclic compressive applied stress gives a tensile cyclic stress which may cause fatigue cracks to grow. The rate of growth depends on the range of the stress intensity for which the crack is open,  $\Delta K_{eff}$ .
3. Compliance measurements have been used on compressed specimens containing arrested cracks to determine  $\Delta K_{eff}$  at various stages of crack growth.
4. Growth of cracks ceases when  $\Delta K_{eff}$  becomes equal to the threshold value; crack arrest in mild steel under compressive cyclic loading occurs at unusually small values of  $\Delta K_{TH}$  ( $1.5 \pm 0.5 \text{ MPa}\sqrt{\text{m}}$ ).
5. Accurate predictions of the growth kinetics have been obtained by utilising measured crack closure loads to indicate the range of applied loads over which a crack of given length is open.

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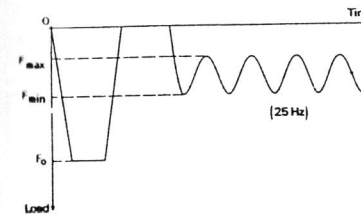


Fig. 1. The loading sequence used.

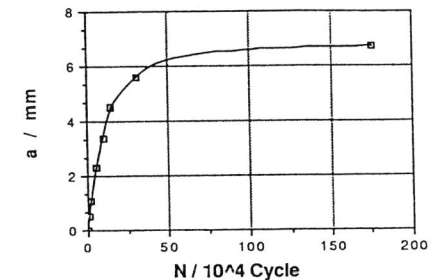


Fig. 2. A plot of the crack length against the number of compressive cycles.

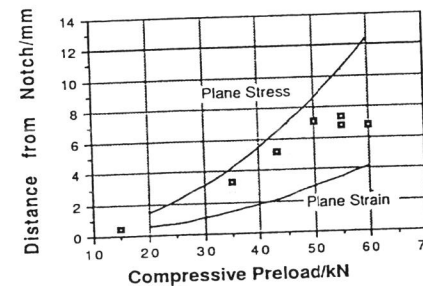


Fig. 3. The final crack length (points) and calculated plastic zone sizes (lines) as a function of compressive preload.

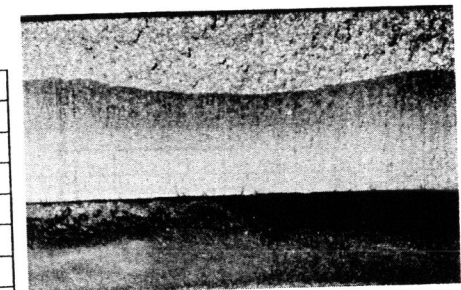


Fig. 4. The arrested fatigue crack with its front uppermost.

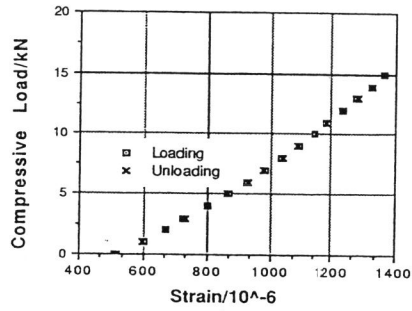


Fig. 5. A plot of load against back face strain for a testpiece preloaded to -50kN, containing an arrested crack.

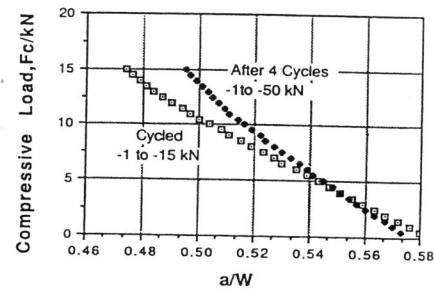


Fig. 6. The closure load  $F_c$  plotted against  $a/W$  for a testpiece preloaded to -50kN.

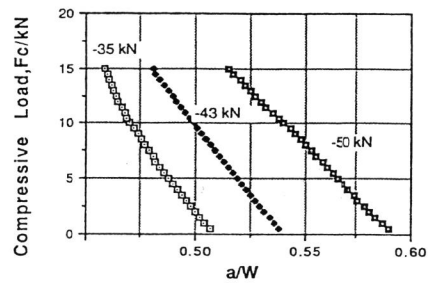


Fig. 7. The closure Load  $F_c$  plotted against  $a/W$  for three values of preload.

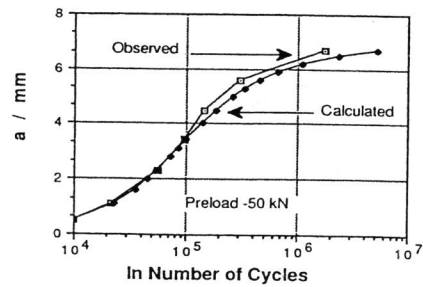


Fig. 8. Comparison of the observed crack growth kinetics with those predicted from closure loads obtained on the same testpiece.