Weight Functions for Rigid Line Inclusions

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ABSTRACT

A theory is developed for weight functions for rigid line inclusions in a direct correspondence with cracks. For Mode I deformation the weight function is obtained.

KEYWORDS

Weight functions; rigid line inclusions;

Introduction

Weight functions for cracks have been obtained by Bueckner (1970) for the computation of the stress intensity factors as a weighted average of applied fractions. Rigid line inclusions have also singular fields at the tip (with the same square root singularity as the crack). So we should like to obtain the stress intensity factors for the stress fields at the tip as a function of the applied "loading" as well. In this paper we formulate the rigid line inclusion problem as a direct analogy to cracks and define the forcing functions as the displacement gradients at the inclusion faces..

Stress-intensity factor for rigid line inclusion

The formulation follows the approach suggested by Dundurs and Markenscoff (1988) in which the unknown function is a distributed line load $p_x(x)$ for what we define as mode I loading. The line load $p_x(x)$ should satisfy the integral equation:

$$\frac{1}{\pi}\int_{-a}^{+a}\frac{p_x(\xi)}{\xi-x}\,d\xi = 2\mu\,\frac{\kappa+1}{\kappa}\,\frac{\partial u_x}{\partial x}(x,0) \qquad |x|\le a \tag{1}$$

so that the prescribed displacement gradients $\frac{\partial u_x}{\partial x}(x,0)$, be canceled on the inclusion (-a, a). The solution of (1) is square-root singular at $x = \pm a$ and the intensity factor for $p_x(x)$ is defined according to

$$L_I = \lim_{x \to \mp a} (x\pm a)^{1/2}\, p_x(x)$$

from which it follows that the stress intensity factors for the stresses σ_{xx}, σ_{xy}, σ_{yy} respectively are:

$$L_{xx} = -L_I\,\frac{3+\kappa}{2(\kappa+1)}$$

$$L_{xy} = \pm\frac{L_I}{2}$$

$$L_{yy} = L_I\,\frac{\kappa-1}{2(\kappa+1)}$$

In a similar way as for cracks we can also obtain energy "release" rates as $\frac{\partial W}{\partial a}$, the energy absorbed to increase the length of the inclusion by Δa, which are found to be

$$\frac{\partial W}{\partial a} = \frac{\pi}{4\mu}\,\frac{\kappa}{\kappa+1}\,L_I^2 \tag{2}$$

with $L_I = -\varphi_x\,\frac{2(\kappa+1)}{\kappa}\left(\frac{a}{2}\right)^{1/2}$

where $\varphi_x = \mu\frac{\partial u_x}{\partial x}(x,0) = \text{const}$

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The derivation may follow Rice (1972) in a completely analogous manner, and obtain:

$$L_I^{(2)} = \frac{-H}{2L_I^{(1)}}\,\frac{d(C_{21})}{d(2a)} = \frac{-H}{2L_I^{(1)}}\,\frac{d}{d(2a)}\int_{-a}^{+a}\vec{t}^{\,(1)}\cdot\vec{u}^{\,(2)}\,dx \tag{3}$$

$$= \frac{H}{2L_I^{(1)}}\,\frac{d}{d(2a)}\int_{-a}^{+a}\left(\frac{\partial}{\partial a}\int \vec{t}\,dx\right)\cdot\frac{\partial \vec{u}_x}{\partial x}\,dx$$

where $H = \frac{4\mu(\kappa+1)}{\pi\kappa}$

From Eqn. (3), the weight function for mode I deformation of rigid line inclusion is

$$\vec{h}_I = \frac{H}{2L_I^{(1)}}\,\frac{d}{d(2a)}\int_{-a}^{x}\vec{t}\,dx$$

The tractions $\vec{t}$ on the rigid line inclusions are the line loads $\vec{p}_x(x)$. Considering the loading $\vec{t}$ to be simple tension in the direction of the inclusion we can obtain the weight function

$$h_{I_x} = \frac{\sqrt{2}\,(\kappa+1)\,\mu a^{1/2}}{\pi\kappa\,(a^2-x^2)^{1/2}}$$

The weight functions for mode II deformation can be obtained by considering the rotation $\frac{\partial u_y}{\partial x}$ of the inclusions.

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