The Temperature Field Induced by the Plastic Dissipation in the Infinite Medium with Line of Discontinuity

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ABSTRACT

In the present paper, the transient temperature field induced by the plastic dissipation in an infinite medium with line of discontinuity is investigated. In the author's paper (1988), the closed form of the solution for the stress and strain field near the tip region under longitudinal shear in the hardening materials have been deduced. Based on these solutions, the analytical expresions of the temperature field induced by the plastic dissipation as a heat source are calculated by means of the supperposition method according to the fundamental solution for the transient temperature field corresponding to unit heat source. The distribution of the heat source has a singular type and the line of discontinuity is considered as thermal insulation. Finally, the influences of the mechanical and thermal behaviour of the materials on the temperature distributions near the tip region are discussed.

KEYWORDS

Temperature fields; Plastic dissipation; Near tip field; Line of discontinuity.

INTRODUCTION

The investigation of the temperature fields whithin stationary near tip region of the line of discontinuity is important for the fracture mechanics and fracture physics. The examination of energy dissipation mechanisms near tip region is very interest for understanding of the macroscopic and microscopic process for the damage and fracture phenomena.

In the mettallic materials energy is generated during the deformation and fracture or fatigue process, most of the energy appear in form of heat.

Sih (1965) has considered the temperature field for the steady state of the region around the line of the discontinuities in an infinite elastic medium prescribed the remote heat flux. Dreilich and Gross (1984) has solved the steady heat conduction equation generated by unit heat source in an infinite elastic plate with thermal isolated curved crack as a fundamental analytical solution. In the paper of Huang et al (1985), the heat fields of elasticplastic deformation were calculated by using the numerical analysis and a thermovition system has been used to give a dynamical partern of the temperature fields of strainless notched plate during tension. Hennig, Michel and Sommer (1985) have used the finite element technique to solve the coupled thermomechanical system of field equations in an high-nonlinear equation. Most of the work for this topic are concentrated in the numerical simulations. Rice and Levy (1969) had investigated the temperature elevations in the plastic zone of a crack, they had deduced the temperature rise at the tip of a stationary crack and runing crack for the plane strain by means of the Prandtl slip line model and Dugdale model. The shape of the plastic zone in two models are approximately. As a first approximation, the influence of temperature field on the stress and strain can be neglected.

In this paper, we consider the transient temperature distribution induced by the plastic dissipation under longitudinal shear near the line of discontinuity as a crack or ribbon-like inclusion. An analytical solution is obtained. In the author's paper (1988) a closed-form solution for the stress and strain fields near the tip region for a crack or inclusion under longitudinal shear in the hardening materials has been deduced. Based on this solution, the plastic dissipation considered as heat generation coupling in the heat conduction equation is known, then the temperature field including a singular heat source is calculated by means of supperposition method. The line of the discontinuity is assumed as thermal isolation. Finally, the influence of the mechanical and thermal behaviour of the materials on the temperature distribution near the tip region is discussed.

TRANSIENT HEAT CONDUCTION EQUATION OF LINE OF DISCONTINUITY COUPLING WITH PLASTIC DISSIPATION

Assuming that, Fourier thermal conduction law is valid with the elastic-plastic deformation. According to nondimensional notation for the unsteady case, the heat conduction equation can be written as

$$\frac{\partial T^*}{\partial t} - \varepsilon^{*2} \nabla^2 T^* = \frac{\dot{Q}}{cL_o \rho} \frac{t_o}{T_o} \tag{2.1}$$

The dimensionless variables as following

$$X^* = X/L_o, \quad y^* = y/L_o, \quad t^* = t/t_o, \quad T^* = T/T_o, \\ \dot{Q}^* = \dot{Q}/\dot{Q}_o, \quad \nabla^2 = \partial^2/\partial X^{*2} + \partial^2/\partial y^{*2}, \quad \varepsilon^{*2} = \varkappa t_o/L_o \tag{2.2}$$

x, y—Cartesian coordinates, t_o, T_o and Q_o —— reference time, temperature and heat flux respectively, $\varkappa = k/(c\rho)$, k is the thermal conductive coeficient. Where t is time, ρ is the density and c is specific heat of the material. $\dot{Q}$ is the rate of the internal heat generation per unit volume. $\dot{Q} = \dot{Q}_e + \dot{Q}_p$, $\dot{Q}_e$ and $\dot{Q}_p$ are rates of heat generation per volume

during elastic and plastic deformation respectively. We neglect $\dot{Q}_e$ compared with the plastic dissipation $\dot{Q}_p$, then we have

$$\dot{Q} = \dot{Q}_p = \bar{\sigma}\, \dot{\varepsilon}_p \tag{2.3}$$

$\bar{\sigma}$ — Stresses intensity and $\dot{\varepsilon}_p$ — effective plastic strain rate. For the power law hardening material

$$\bar{\tau} = \tau_o (\gamma / \gamma_o)^N \tag{2.4}$$

γ_o — τ_o/G, τ_o — yielding stress for simple shear, N — hardening exponent. G — shearing modulus.

For the longitudinal shear

$$\bar{\sigma}\dot{\varepsilon}_p = \bar{\tau}\dot{\gamma} \tag{2.5}$$

We use the exact solution near the tip field for the crack and inclusion problem under longitudinal shear obtained by Yu and Gross (1988)

$$\gamma(r,\phi) = \gamma_o \left(\frac{K^2}{2\pi \tau_o^2 r} \sqrt{\beta^2 + 2\beta\cos 2\phi + 1} \right)^{n/(n+1)} \tag{2.6}$$

$$\beta = (n-1)/(n+1), \qquad n = 1/N$$

$$Q_p = \bar{\sigma}\cdot\varepsilon_p = \frac{K\dot{K}}{\pi G r} \sqrt{\beta^2 + 2\beta\cos 2\phi + 1} \left(\frac{n}{n+1} \cdot \right) \gamma_o^{1/n(n+1)} \tag{2.7}$$

K is the stress intensity factors of the line of the discontinuiity under longitudinal shear. According to the Fig.1, the relation between ϕ and θ is given by

$$\cos 2\phi = \cos\theta \sqrt{1 - \beta^2 \sin^2\theta} - \beta \sin^2\theta \tag{2.8}$$

The field quantities for the inclusion at a given angle θ^* are the same as the corresponding quantities for a crack at the angle $\theta = \pi \pm \theta^*$

For the case of ideal-plastic materials, $n \to \infty$, $\beta \to 1$. $\phi = \theta$.

$$\dot{Q}_p = 2K\dot{K}\cos\theta/(\pi G r) \tag{2.9}$$

For the stationary crack or flat rigid inclusion, the heat generation have the singularity with the order $O(r^{-1})$. We assume the line of the discontinuity is thermal isolation, the boundary condition and the initial condition can be shown as following

$$\partial T^* / \partial y^* = 0, \quad \text{at } y^* = 0 \tag{2.10}$$

$$T^* \longrightarrow T_o \quad \text{at } r = (x^{*2} + y^{*2})^{1/2} \longrightarrow \infty \tag{2.11}$$

$$T^* = T_o \quad \text{at} \quad t^* = 0 \tag{2.12}$$

The mathematical formulation of the heat conduction problem can be described by the equation (2.1) take into account the equations (2.7) (2.8) and the boundary conditions (2.10)(2.11), initial condition (2.12).

THE ANALYTICAL SOLUTION FOR THE TRANSIENT TEMPERATURE FIELDS NEAR THE TIP REGION

Substituding the equation (2.7) into (2.1), the heat conduction equation can be expressed as

$$\frac{\partial T^*}{\partial t^*} - \varepsilon^{*2}\nabla^2 T^* = P(r^*,\phi,t^*)\frac{t_o}{c\rho T_o} \tag{3.1}$$

$$P(r^*,\phi,t^*) = \frac{n}{n+1}\gamma_o^{1/n(n+1)}\frac{K\dot{K}}{GL_o\pi}\frac{1}{r^*}(\beta^2+2\beta\cos 2\phi+1)^{1/2} \tag{3.2}$$

Carslaw and Jaeger (1963) had described a fundamental solution for an infinite plane with a point heat source shown in fig.2 and expressing in nondimensional quantities:

$$T^*(Z^*,\bar{Z}^*,t^*) = \frac{\dot{Q}^*}{4\pi\varepsilon^{*2}t^* c\rho}\,\frac{\dot{Q}_o}{L_o^2 t_o}\exp\left\{-(Z^*-\zeta^*)(\bar{Z}^*-\bar{\zeta}^*)/(4\varepsilon^{*2}t^*)\right\} \tag{3.3}$$

Rewritten the equation (3.3) in the polar-coordinate system shown in the Fig.2 and use the supperposition method.
The temperature field induced by distributed sources of heat proportional to the plastic work rate in the plastic zone expressed as equation (3.2) under small scale yielding can be calculated as following

$$T^*(r^*,\theta,t^*) = \int_0^{t^*}\int_0^{R_p(\psi)}\int_{-\theta_0}^{\theta_0} P(\sigma,\psi,\tau)a_o \exp\left\{-\frac{1}{4\varepsilon^{*2}(t^*-\tau)}\cdot\right.$$

$$\left.[r^{*2}+\sigma^2-2r^*\sigma\cos(\theta-\psi)]\right\}\sigma\, d\sigma\, d\psi\;\frac{d\tau}{4\pi\varepsilon^{*2}(t^*-\tau)}$$

$$a_o = \frac{\dot{Q}_o}{L_o^2\, T_o\, c\rho}, \qquad \sigma = \rho/L_o \tag{3.4}$$

$R_p(\psi)$ is the radius of the plastic zone. The shape of the plastic zone of the line of the discontinuity under longitudinal shear in the hardening materials is a circle pointed by Rice (1967).

We consider the temperature rise of the tip of the line of the discontinuity, substituding r=0 into the equation (3.4), we have

$$T^*(o,o,t^*) = \int_0^{t^*}\int_0^{R_p(\psi)}\int_{-\theta_0}^{\theta_0} P(\sigma,\psi,\tau)a_o\exp\left[-\frac{\sigma^2}{4\varepsilon^{*2}(t^*-\tau)}\right]\sigma\, d\sigma\, d\psi\cdot\frac{d\tau}{4\varepsilon^{*2}\pi(t^*-\tau)} \tag{3.5}$$

DISCUSSION

1. Compare the near tip fields between the crack and inclusion problems as shown in Fig.3 obtained by authors (1988). They have a character as mirror image about y* axis with the same stress intensity factors under small scale yielding. Owing to this near tip field, the plastic work rate, heat generation and the temperature distributions for the crack and flat rigid inclusion also exist the mirror reflection with the same stress intensity factor. So we need only to solve one of the problems of the line of the discontinuity (crack or flat inclusion).

2. As a limiting case, when the Fourier number ε^* is very small, as $\varepsilon^*\to 0$. It means no heat conduction effect, and can be explored as adiabatic case, we have

$$T^*(r^*,\theta,t^*) = \frac{1}{\pi G(1+n)}\, n\gamma_o^{1/n(n+1)}\,\frac{t_o}{T_o c\rho r^*}(\beta^2+2\beta\cos\phi+1)^{1/2}\cdot\int_0^{t^*}K(\tau)\dot{K}(\tau)d\tau \tag{4.1}$$

It can be founded in equation (4.1), the temperature fields of the near tip region have quite steep temperature gradient in the radial direction than the circumferential. This result is consistent with the temperature boundary effect as pointed by Yu (1979).

3. For the perfectly-plastic materials corresponding to hardening exponent $n\to\infty$, according to equation (2.9)

$$P(\sigma,\psi,\tau) = \frac{2K(\tau)\dot{K}(\tau)}{GL_o\pi\sigma}\cos\psi \tag{4.2}$$

For the infinite plate with a crack under remote load as $\tau_y = \tau^\infty$, if the applied load rises linearly with time up to maximum load

$$K(\tau) = \tau^\infty(\tau)\sqrt{\pi a} \qquad \tau^\infty(\tau) = \tau_1\tau \tag{4.3}$$

Substituding (4.2) into (3.5) and integrating in ψ, consider a very short time to attain maximumload as a rapid loading cases when ε^* becomes small, the temperature rise at the crack tip can be deduced as

$$T^*(o,o,t^*) = \frac{2}{\pi^{3/2}}\,\frac{\dot{Q}_o\sqrt{t_o}}{L_o^2T_o^2}\,\frac{1}{G}\,\frac{K^2_{max}}{\sqrt{k c\rho t^*}} \tag{4.4}$$

The temperature at the crack tip becomes proportional to K^2_{max} and inversely proportional to the mechanical behaviour G and loading time $\sqrt{t^*}$ as can be seen from the equation (4.4). The thermal properties influencing the temperature at the crack tip appear in the factor ($\sqrt{c\rho k}$). These results are similar with Rice and levy (1969) deduced from approximate assumptions about the shape of plastic zone under plane strain.

CONCLUTION

The analytical solution of the temperature fields generated by the plastic dissipation for the infinite hardening medium with line of discontinuity under longitudinal shear are deduced. The results of the calculation indicate that for the infinite medium containing a crack or flat rigid inclusion, the temperature fields exist mirror reflection character about y* axis. The influence of the mechanical and thermal behaviour of the materials on the temperature distribution of the near tip region is discussed.

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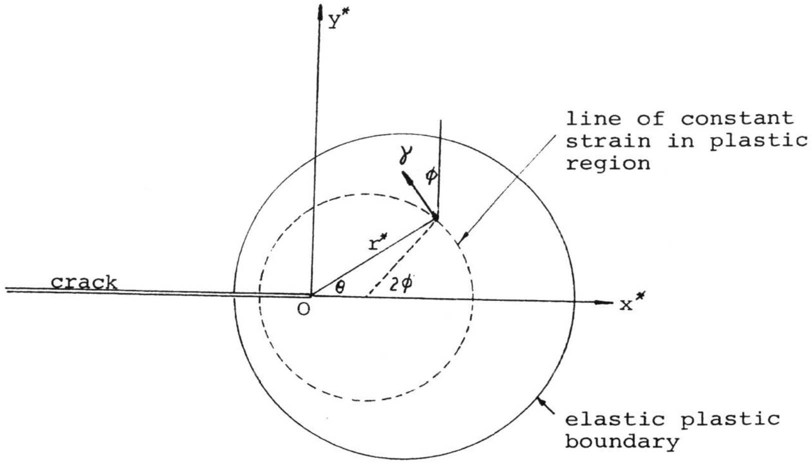

Figure 1

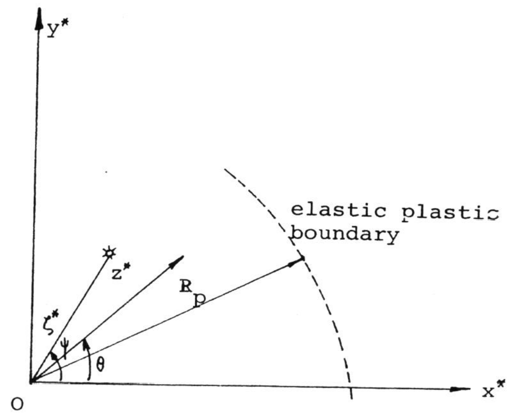

Figure 2

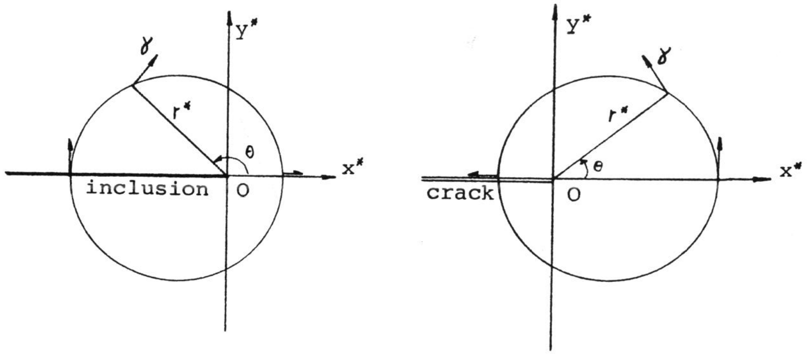

Figure 3